# Generation of Parallel Manipulators with Three Translational Degrees of Freedom Based on Screw Theory

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**Abstract**

A method is proposed based on screw theory to generate parallel manipulators with three translational degrees of freedom (PMTTDOFs). The generation of PMTTDOFs is reduced to the generation of serial chains with $n$-infinite-pitch (IP)-wrench systems. Serial chains with $n$-IP-wrench systems can be easily generated. Taking all the combinations of serial chains generated, making sure that the union of the wrench systems for all the serial chains in each combination constitutes a 3-IP-wrench system, and discarding the combinations without full cycle degrees of freedom, we get all the PMTTDOFs. In addition to the PMTTDOFs proposed up to now, some new PMTTDOFs are generated.

## 1. Introduction

Parallel manipulators with three translational degrees of freedom (PMTTDOFs) have a wide range of applications such as assembly. Several types of PMTTDOFs have been proposed ([1-5]). An approach is proposed systematically in [1, 2] to generate PMTTDOFs based on the displacement group, while some PMTTDOFs are proposed based on screw theory in [3-5]. The generation of PMTTDOFs is still an open problem to investigate.

In this paper, we concentrate on the systematic generation of PMTTDOFs based on screw theory [6, 7]. At first, some relevant results from screw theory are reviewed. Then, the fundamental idea of a method to generate PMTTDOFs is proposed. Serial kinematic chains with $n$-IP-wrench systems are generated in section 4. In section 5, the generation of PMTTDOFs is dealt with. At last, conclusions are drawn.

## 2. Twist Systems and Wrench Systems of Serial Manipulators and Parallel Manipulators

A screw is called a twist if it represents an instantaneous motion of a rigid body, or a wrench if it represents a system of forces and couples acting on a rigid body. The first three components of a twist represent the angular velocity and the last three components represent the linear velocity of a point in the rigid body which is instantaneously coincident with the origin of the reference frame. On the other hand, the first three components of a wrench represent the resultant force and the last three components represent the resultant moment about the origin of the reference frame.

(1) The twist system of the end effector for a serial manipulator is the union of the twist systems of each joint in the serial chain. The twist system of the moving platform for a parallel manipulator is the intersection of those of all the serial chains in the parallel manipulator.

(2) The wrench system of the end effector for a serial manipulator is the intersection of the wrench systems of each joint in the serial chain. The wrench system of the moving platform for a parallel manipulator is the union of those of each serial chain in the parallel manipulator.

(3) The twist system of the end effector for a serial (or parallel) manipulator is the reciprocal screw system of the wrench system of the end effector.

## 3. The Fundamental Idea of the Method to the Generation of PMTTDOFs

The twist system of the moving platform for a PMTTDOF is a 3-IP-twist system. It can be found without difficulty that the wrench system of a PMTTDOF is a 3-IP-wrench system. Thus, the generation of PMTTDOFs is reduced to the generation of parallel manipulators with 3-IP-wrench systems. The latter are then reduced to the generation of serial chains with $n$-IP-wrench systems as the wrench system of the moving platform is the union of those of all the serial chains. The serial chains with $n$-IP-wrench systems can be easily generated. Taking all the combinations of serial chains generated, making sure that the union of the wrench systems for all the serial chains in each combination constitutes a 3-IP-wrench system and discarding the combinations without full cycle degrees of freedom, we get all the PMTTDOFs.

## 4. Generation of Serial Kinematic Chains with $n$-IP-Wrench Systems

### *Kinematic Joints Whose Twist Is Reciprocal to $n$ IP Wrenches*

It is easy to find that

(1) Kinematic joints whose twist is reciprocal to three IP wrenches are P (prismatic) joint or $P_a$ (hinged planar parallelogram) joints;

(2) Kinematic joints whose twist is reciprocal to two IP wrenches are R (revolute), H (helical), C (cylindrical) joints whose axes are perpendicular to the axes of the two IP wrenches;

(3) The kinematic joint whose twist is reciprocal to one IP wrench are U (universal) joint the axes of its two R joints are perpendicular to the IP wrench; and

(4) The kinematic joint whose twist is reciprocal to no IP wrench are S (spherical) joint.

### *Serial Kinematic Chains with $n$-IP-Wrench Systems*

Note that the wrench system of the end effector for a serial chain is the intersection of those of all the joints in the serial

Table 1: Serial chains with $n$-IP wrench systems

| $n$ | Joint | Type | Geometric constraints | Full cycle mobility condition | reciprocal IP wrenches |
|---|---|---|---|---|---|
| 3 | PPP | PPP | The axes of all the P joints can not be parallel to one plane | | Any three independent IP wrenches |
| 2 | RRRP | RRRP<br>RRPR<br>RPRR<br>PRRR | The axes of all the R joints and C joints are parallel and not perpendicular to all the axes of P joints | | Two independent IP wrenches perpendicular to the axes of R or C joints |
| | RRC | RRC<br>RCR<br>CRR | | | |
| | RRPP | RRPP<br>RPRP<br>RPPR<br>PRRP<br><br>PRPR<br>PRPR<br>PPRR | | | |
| 1 | RRRRR | RRRRR | All the axes of R and C joints are parallel to one plane and not parallel to one line. | The axes of two or three successive inner R joints are parallel, the axes of the other R joints are parallel | The IP wrench perpendicular to all the axes of R and C joints |
| | RRRRP | RRRRP<br>RRRPR<br>RRPRR<br>RPRPR<br>PRRRR | | The axes of two successive R joints or two R joints connected by P joint are parallel to each other, the axes of the other two R joints are parallel to each other | |
| | RRRC | RRRC<br>RRCR<br>RCRR<br>CRRR | | The axes of two inner joints are parallel to each other, the axes of the other two joints are parallel to each other | |
| 0 | omitted | omitted | omitted | omitted | omitted |

chain and that the order of the reciprocal screw system to a screw system of order $n$ is $(6-n)$. All the possible serial kinematic chains with n-IP-wrench systems can be generated from the kinematic joints whose twist is reciprocal to $n$ IP wrenches. The serial chains obtained are shown in Table 1. For brevity, serial chains involving U, H and $P_a$ joints are omitted in Table 1. If needed, one R joint in Table 1 can be replaced by one H joint, one P joint can be replaced by one $P_a$ joint, while two success R joints whose axes are not parallel to each other can be replaced by one U joint.

## 5. Generation of PMTTDOFs

Taking two or three serial chains the union of whose wrench systems constitutes a 3-IP-wrench system, possible PMTTDOFs can be generated. To guarantee that the parallel manipulators have full-cycle mobility, the serial chains should meet some conditions. The full cycle mobility conditions are revealed for the serial chains with $n$-IP-wrench systems and shown in Column 5 in Table 1.

It should be pointed out that for a PMTTDOF, if the total of the orders of wrench systems of all the serial chains is greater than 3, it is an over-constrained mechanism.

Among the PMTTDOFs generated, some are new PMTTDOFs in addition to all PMTTDOFs proposed so far. For example, the 3-RPRRR PMTTDOF composed of three RPRRR serial chains (Fig. 1) is a new PMTTDOF.

## 6. Conclusions

A general approach has been proposed based on screw theory to generate PMTTDOFs. In addition to the PMTTDOFs

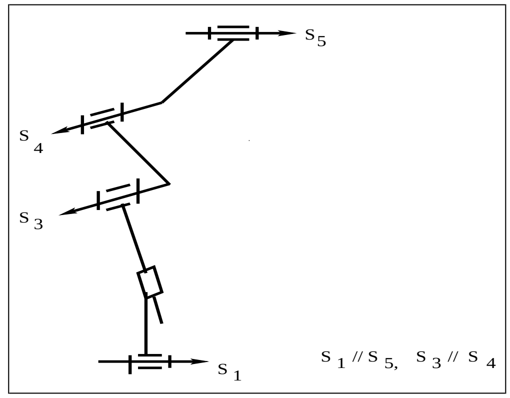


Figure 1: The RPRRR serial chain for PMTTDOFs.

proposed up to now, some new PMTTDOFs are generated. The approach can also be applied to the generation of other types of parallel manipulators with reduced or decoupled degrees of freedom. The results in this paper should be of value to the fast parallel manipulator design and the theory of mechanisms.

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