Kinematic Analysis of Clavel's "Delta" Robot

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1. INTRODUCTION

In 1988 Clavel introduced a three degree of freedom(dof), three identical legged manipulator he called "Delta". Its end effector(EE) or "foot" executes pure spatial translation.

Description

The fixed frame(FF) or "pelvis" supports three actuated revolute(R) jointed "hips". These R-axes form an equilateral planar triangle. The "knee" end of each "thigh" supports another R-joint whose axis is parallel to the one at the hip. The foot also supports three R-joints whose axes form another triangle which is similar to and maintains the same orientation as the one on FF. The EE triangle R-axes are held parallel to those on FF because the "shin" is a parallelogram four bar linkage whose R-axes are all perpendicular to the hip, knee and "ankle" R-axes. One pair of linkage R-axes intersects the knee R-axis, the other intersects the ankle R-axis.

Kinematic Geometry

When a thigh angle is set by the actuator, the R-axis of the ankle, if disconnected from EE, would be free to move in the parallel line bundle of the hip R-axis. Note also the three points D_i, E_i C_i, $i = 1, 2, 3$, at hip, knee and ankle of each leg as shown in Fig. 1. D_i is the midpoint of a FF R-joint axis triangle side. E_i is the point on a knee R-joint axis midway between the parallel axis R-joint pair of the four bar while C_i is midway on the opposite link, coincident with an EE R-joint axis triangle side. If disconnected from the foot, C_i moves on the sphere centred on E_i. Similarly, if EE were fixed and E_i were freed at the knee then E_i moves on one centred on C_i.

Rationale

Why rehash old research? "Delta" is mentioned in a recent book by Angeles[1]. We found the elegant symmetry of this robot quite compelling. Inverse kinematics of "Delta" was treated by Pierrot[2]. His closure equation approach is simple but his direct problem was based on three of these quadratic equations which require numerical solution. Hervé[3] designed a similar translational robot, using prismatic rather than revolute actuation. It has screw actuated P-jointed hips. Its inverse kinematic analysis was done using the notion of intersecting Schönflies displacement subgoups so as to explain the liason of groups. The purpose of *this* article is to provide a clear kinematic analysis, useful in programming these 3-legged robots, based on a line and sphere intersection model.

2. ANALYSIS

Consider the inverse and direct kinematics via geometric constructions. Computation is based on similar, but not quite identical, geometry.

Inverse Kinematic Construction

Fig. 1 shows a top view of the two triangular platforms. The points, D_i, E_i, C_i, are clearly visible. The centre of EE is displaced by (x, y, z) from origin O at the centre of FF. Note the design constants. e is the side length of EE, f the side of FF, r_e the distance C_iE_i and r_f the distance D_iE_i. A sphere, radius r_e, centred on C_i gives the locus of E_i. A second constraint is imposed by the circular trajectory of E_i at radius r_f from centre D_i. The plane of this circle cuts the sphere. This small circle appears in auxiliary views. The other solid arc is the circle centred on D_i. The intersection of the arcs yield E_i, the solution. The desired actuated R-joint angles can be measured as θ_i.

Inverse Kinematic Computation

Here, a line will be intersected with the sphere centred on D. The homogeneous coordinates $E\{w : x : y : z\}$ of a point on it are

$$(x_d w - x)^2 + (y_d w - y)^2 - z^2 - r_f w^2 = 0 \qquad (1)$$

Which line? The one through the two desired solutions for E, obtained by intersecting the plane of a thigh circle centred on D with the plane of a circle produced by the intersection of the sphere given by Eq. 1 and one of radius r_e centred on C. The homogeneous coordinates of the three vertical thigh planes, $\pi\{W_\pi : X_\pi : Y_\pi : Z_\pi\}$ on O can be written by inspection.

$$\pi_1\{0 : 1 : 0 : 0\}, \quad \pi_2\{0 : 1 : -\sqrt{3} : 0\}, \quad \pi_3\{0 : 1 : \sqrt{3} : 0\}$$

Coordinates of the plane of the circle of intersection between the spheres centred on C and D are the coefficients of the linear equation which is the difference between the two sphere equations. Its plane coordinates are $\{W_i : X_i : Y_i : Z_i\}$. Explicitly, a thigh and shin sphere intersection circle plane has coordinates

$$\{(r_e^2 - r_f^2 + x_d^2 - x_c^2 + y_d^2 - y_c^2 - z_c^2)/2 : (x_c - x_d) : (y_c - y_d) : z_c\}$$

The next step is to compute Plücker coordinates of the line.

$$\left| \begin{array}{cccc} W_\pi & X_\pi & Y_\pi & Z_\pi \\ W & X & Y & Z \end{array} \right| \Rightarrow \{p_{01} : p_{02} : p_{03} : p_{23} : p_{31} : p_{12}\}$$

The point-on-line relationship is

$$\begin{bmatrix} 0 & p_{23} & p_{31} & p_{12} \\ -p_{23} & 0 & p_{03} & -p_{02} \\ -p_{31} & -p_{03} & 0 & p_{01} \\ -p_{12} & p_{02} & -p_{01} & 0 \end{bmatrix} \begin{bmatrix} w \\ x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \qquad (2)$$

The second and third lines of Eq. 2 are substituted into Eq. 1 to produce Eq. 3, a quadratic in $z = z_e$, as needed to find $\theta = \sin^{-1}(z_e/r_f)$.

$$
\begin{gathered}
(R^2 + S^2)z^2 - 2(RT - SU) + Rx_d + Sy_d + z_d)wz \\
+[(T + x_d)^2 + (U - y_d)^2 + z_d^2 - r_f^2]w^2 = 0 \\
R = p_{01}/p_{03},\ S = p_{02}/p_{03},\ T = p_{31}/p_{03},\ U = p_{23}/p_{03}
\end{gathered}
\tag{3}
$$

Computational expense is similar to that of Pierrot's[2] solution but no rotation matrix is necessary.

Direct Kinematic Construction

Now consider Fig. 2. Here the three angles θ_i are given instead of the position of the EE centre point, shown as O', which must be determined. The solution key is to locate points E_i' the centres of spheres, radius r_e. Their intersections produce two poses. The constructive solution is shown in a second auxiliary view where the circle of intersection on spheres centered on E_1' and E_2' defines a plane which sections the sphere on E_3' on a second coplanar circle. The intersections of the two circles are obtained here by inspection and projected to the first auxiliary view which shows the FF plane in edge or line view. The lower point is chosen as O' and the z-coordinate can be measured here. Projection of this point into the top view provides the other two coordinates. But how are the points $E'i$ located? O' is located from C_i by three displacement vectors $\mathbf{e}_1' = e/(2\sqrt{3})\mathbf{j}$, $\mathbf{e}_2' = [-e/4\mathbf{i}, -e/(4\sqrt{3})\mathbf{j}]$, $\mathbf{e}_3' = [e/4\mathbf{i}, -e/(4\sqrt{3})\mathbf{j}]$ and three equations, like Eq. 1, can be written.

Direct Kinematic Computation

Differences between pairs of these equations provide plane coordinates and the key line coordinates to be employed in the computationally simplest of the three sphere equations, like Eq. 3. This is solved for the least z-coordinate and the second and third lines of Eq. 2 produce the other two coordinates of O'.

3. CONCLUSION

It is claimed that the simplest quadratic direct solution for "Delta" manipulators has been exposed for the first time herein.

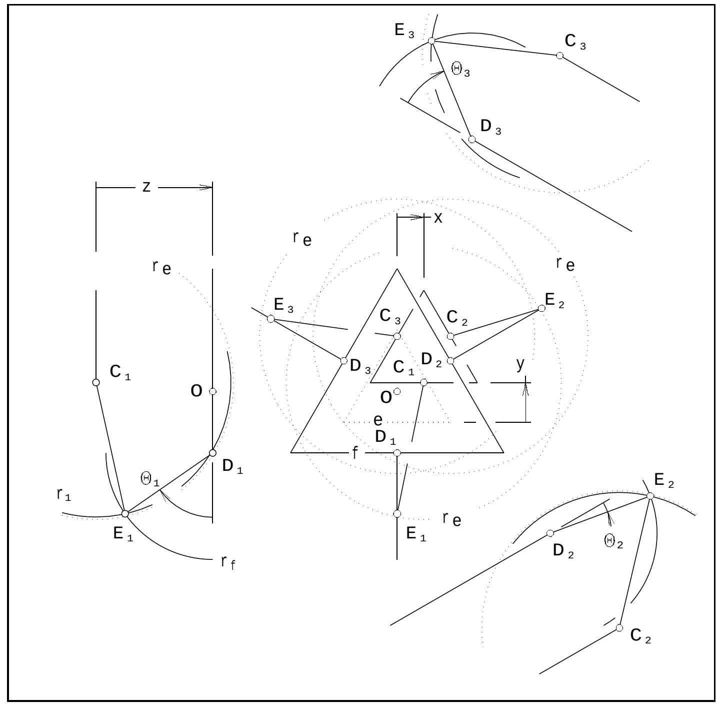

Figure 1. Inverse kinematics

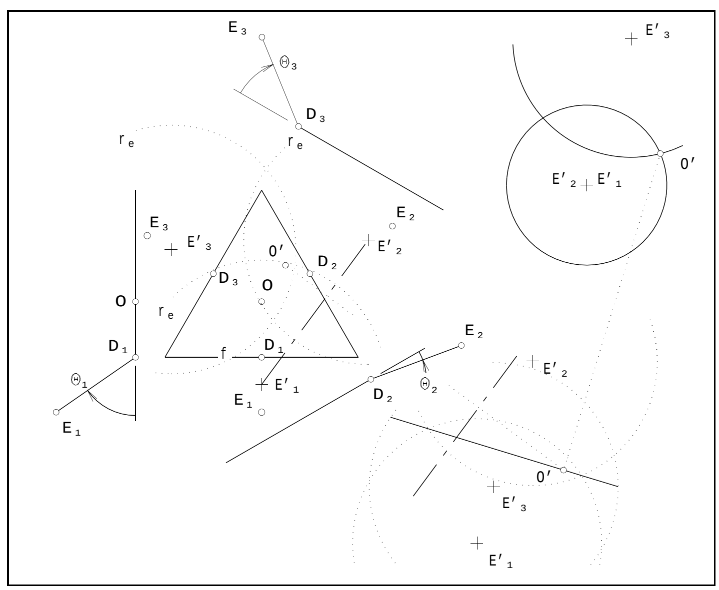

Figure 2. Direct kinematics

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[3] Hervé, J.-M. & Sparacino, F. (1992): "Star, a New Concept in Robotics", *Proc. 3ARK Wkshp.*, Ferrara, (Parenti-Castelli, V. & Lenarĉiĉ, J, .eds.), ISBN 88-86141-00-9, pp.176-183.