REPRESENTATION OF THE SINGULARITY LOCI OF A SPECIAL CLASS OF SPHERICAL 3-DOF PARALLEL MANIPULATOR WITH REVOLUTE ACTUATORS

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1. INTRODUCTION

It is well known that singular configurations are an inherent nature of mechanical systems and have important influences on their properties. Therefore, these special configurations of the system are usually avoided in design and application of mechanisms.
For simple 2-DOF and 3-DOF planar manipulators, the singularity loci have been obtained in [1, 2]. For spherical 3-DOF parallel manipulators with prismatic actuators, the expression for the singularity loci has been provided in [3]. In this paper, the representation of the singularity loci of a spherical 3-DOF parallel manipulator with revolute actuators will be studied. According to the analysis presented in [4], there are three types of singularities for parallel manipulators. For the type of manipulator studied here, the simple expression for the singularity conditions is derived by means of the kinematic solutions. These expressions are then used to construct the singularity loci.

2. A SPECIAL CLASS OF SPHERICAL 3-DOF PARALLEL MANIPULATOR

As shown in Fig. 1, the prototype of the spherical parallel manipulator studied here was described in [5] and the solution of the direct kinematic problem of this manipulator has been reported in [6]. For the spherical 3-DOF parallel manipulator, the expression of the Jacobian matrix is now recalled from [5]. We have the following equation

$$\mathbf{A}\boldsymbol{\omega} + \mathbf{B}\dot{\boldsymbol{\theta}} = \mathbf{0} \tag{1}$$

where $\dot{\boldsymbol{\theta}}$ is the vector of actuated joint rates and $\boldsymbol{\omega}$ is the angular velocity of the end effector. Matrices $\mathbf{A}$ and $\mathbf{B}$ are

$$\mathbf{A} = \begin{bmatrix} (\mathbf{w}_1 \times \mathbf{v}_1)^T \\ (\mathbf{w}_2 \times \mathbf{v}_2)^T \\ (\mathbf{w}_3 \times \mathbf{v}_3)^T \end{bmatrix} \tag{2}$$

and

$$\mathbf{B} = diag(\mathbf{w}_1 \times \mathbf{u}_1 \cdot \mathbf{v}_1, \mathbf{w}_2 \times \mathbf{u}_2 \cdot \mathbf{v}_2, \mathbf{w}_3 \times \mathbf{u}_3 \cdot \mathbf{v}_3) \tag{3}$$

Since both the direct and the inverse kinematic problems have already been solved for this special kind of architecture in [6], the notation used here will be the same as the one used in [6]. In short, vectors $\mathbf{u}_i$, $\mathbf{w}_i$ and $\mathbf{v}_i$ are the unit vectors defined along the three joints of each leg. Moreover, the axes of the base and the platform joints form an orthogonal base and all link angles are equal to 90^o [5].

3. SINGULARITY ANALYSIS

For the spherical parallel manipulator with the above mentioned special geometry, the expressions of the vectors $\mathbf{w}_i$

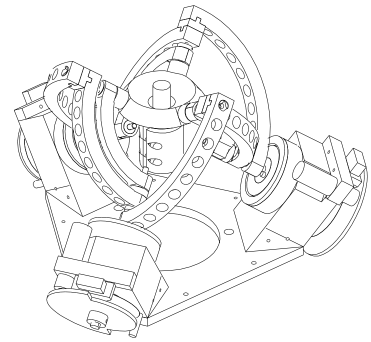

Figure 1: A special class of 3-DOF spherical parallel manipulator with revolute actuators.

and $\mathbf{v}_i$ are given in [6]. They are

$$\mathbf{w}_1 = \begin{bmatrix} 0 \\ -s\theta_1 \\ c\theta_1 \end{bmatrix}, \mathbf{w}_2 = \begin{bmatrix} -s\theta_2 \\ c\theta_2 \\ 0 \end{bmatrix}, \mathbf{w}_3 = \begin{bmatrix} c\theta_3 \\ 0 \\ -s\theta_3 \end{bmatrix} \tag{4}$$

$$\begin{aligned} \mathbf{v}_1 &= [-c_1 s_2 s_3 - s_1 c_3, s_1 s_2 s_3 - c_1 c_3, c_2 s_3]^T \\ \mathbf{v}_2 &= [-c_1 c_2, s_1 c_2, -s_2]^T \\ \mathbf{v}_3 &= [c_1 s_2 c_3 - s_1 s_3, -s_1 s_2 c_3 - c_1 s_3, -c_2 c_3]^T \end{aligned} \tag{5}$$

where c_i and s_i stand for $\cos\phi_i$ and $\sin\phi_i$ and $c\theta_i$ and $s\theta_i$ stand for $\cos\theta_i$ and $\sin\theta_i$. Angles ϕ_i and θ_i, $i = 1, 2, 3$ are respectively the Euler angles (Cartesian coordinates) and the input angles (joint coordinates).
The second type of singularity occurs when $\det(\mathbf{A}) = 0$. In this case, nonzero angular velocities of the gripper are possible even if the three input velocities are zero. Substituting eqs. (4) and (5) into eq. (2), the expression for matrix $\mathbf{A}$ is obtained.

$$a_{11} = -s\theta_1 c_2 s_3 + c\theta_1 c_1 c_3 - c\theta_1 s_1 s_2 s_3 \tag{6}$$
$$a_{12} = -c\theta_1 s_1 c_3 - c\theta_1 c_1 s_2 s_3 \tag{7}$$
$$a_{13} = -s\theta_1 s_1 c_3 - s\theta_1 c_1 s_2 s_3 \tag{8}$$
$$a_{21} = -c\theta_2 s_2 \tag{9}$$
$$a_{22} = -s\theta_2 s_2 \tag{10}$$
$$a_{23} = -s\theta_2 s_1 c_2 + c\theta_2 c_1 c_2 \tag{11}$$
$$a_{31} = -s\theta_3 c_1 s_3 - s\theta_3 s_1 s_2 c_3 \tag{12}$$
$$a_{32} = s\theta_3 s_1 s_3 - s\theta_3 c_1 s_2 c_3 + c\theta_3 c_2 c_3 \tag{13}$$
$$a_{33} = -c\theta_3 c_1 s_3 - c\theta_3 s_1 s_2 c_3 \tag{14}$$

where a_{ij} is the ijth element of $\mathbf{A}$.

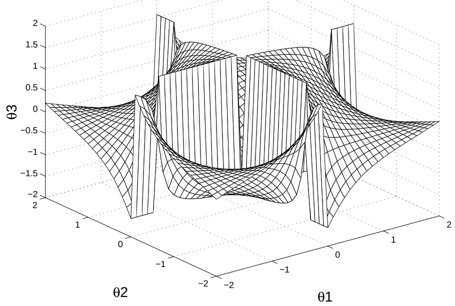

Figure 2: Singularity loci of 3-DOF spherical parallel manipulator with revolute actuators in the joint space.

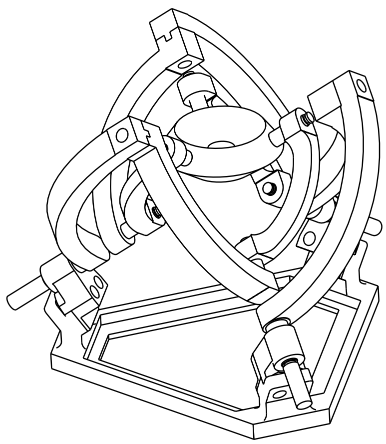

Figure 3: First singular configuration of the special 3-DOF spherical parallel manipulator with revolute actuators.

Using the direct kinematic solutions, the condition for this type of singularity in the joint space can be obtained as

$$c\theta_1 c\theta_2 c\theta_3 + s\theta_1 s\theta_2 s\theta_3 = 0 \tag{15}$$

From eq. (15), the locus of singularities can be plotted in the joint space, as shown in Fig. 2. It can be shown that the singularity locou has a period of π. Using the inverse kinematic solutions, the condition for the second type of singularity in the Cartesian space can be obtained as

$$c\phi_2(1 + c2\phi_1 c2\phi_3 - s2\phi_1 s\phi_2 s2\phi_3) = 0 \tag{16}$$

It is not difficult to prove that the two solutions of eq. (16) have the same geometric interpretation.When $c\phi_2 = 0$, the mobile platform has only four singular configurations. The first singular configuration is shown in Fig. 3. Since this manipulator is symmetric, Fig. 4 shows the singular configuration corresponding to the other three cases.

4. CONCLUSION

Singularity loci for a spherical 3-DOF parallel manipulator with special architecture have been studied for the first time in this paper. The concise analytical expressions describing the singularity loci of the manipulator have been obtained by using the kinematic solutions. These expressions have

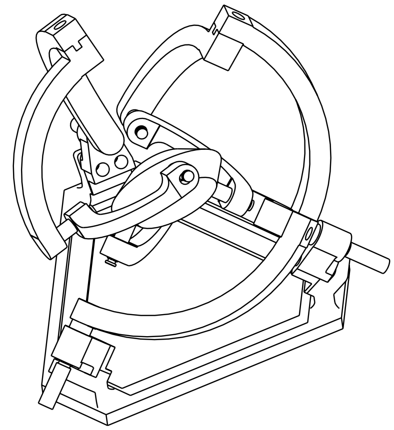

Figure 4: Second singular configuration of the special 3-DOF spherical parallel manipulator with revolute actuators.

been used to obtain plots of the singularity loci of the manipulator in the joint and in the Cartesian spaces. For this manipulator, the derivations presented here have focussed only on the second type of degeneracy. It is noted that the manipulator studied here has only four singular configurations in the Cartesian space. Moreover, it can be shown that the singularity locus is the same for any branch of the inverse kinematic problem.

References

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