Kinematic Analysis of Planar Parallel Mechanisms Actuated with Cables

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Abstract

This paper presents a general and systematic analysis of planar parallel mechanisms actuated with cables. The equations for the velocities and the forces in the cables are presented. Then, a detailed analysis of the workspace is performed and an analytical method for the determination of the boundaries of an x-y two-dimensional subset is proposed. The new notion of dynamic workspace is defined, as its shape depends on the accelerations of the end-effector. We demonstrate that any subset of the workspace can be considered as a combination of three-cable sub-workspaces, with boundaries being of two kinds: two-cable equilibrium loci and three-cable singularity loci.

1. Introduction

Many studies have been performed concerning various types of parallel mechanisms, but few of these involve manipulators actuated with cables. However, the advantages of this type of actuation are numerous and incontestable [1]. Firstly, cables allow incomparable motion range, that is much larger than that of conventional actuators, and, as the cables are being used only in tension, they are, for a similar task, much thinner and lighter than most conventional actuators. Also, being much more flexible, they provide a kind of natural protection in the case of interference.

The unilaterality of actuation imposed by the use of cables involves a workspace considerably different from the workspace corresponding to standard manipulator with a similar geometry. This workspace does not depend so much on actuator length limits but more on the inability of cables to generate compression. The fact that cables can generate tension only imposes the utilization of actuation redundancy if full control of all degrees of freedom is desired. Redundancy has the positive effect of reducing or completely eliminating singularities in the mechanism.

2. Generalities

A moving object is actuated in the plane by n links (k actuated links, e.g. cables, and $n-k$ passive links, e.g. springs). These n links connect respectively points $P_1 \dots P_n$ on the base and $V_1 \dots V_n$ on the end-effector. The position vector of point P_i is defined by vector $\mathbf{p}_i$ expressed in the fixed reference frame, attached to the base, whereas the position vector of point V_i is defined by vector $\mathbf{v}_i$ expressed in the mobile reference frame, attached to the end-effector, the origin of which is at the center of mass of the end-effector. The position of the mobile frame's origin, point C, is defined by vector $\mathbf{c}$, while its orientation relative to the fixed frame, is given by angle ϕ.

3. Velocity Equations

Starting from the length of the link i, ρ_i, connecting point P_i to point V_i, $\rho_i^2 = (\mathbf{c}+\mathbf{Q}\mathbf{v}_i-\mathbf{p}_i)^T(\mathbf{c}+\mathbf{Q}\mathbf{v}_i-\mathbf{p}_i)$, we obtain the following matrix form of the velocity equations:

$$\mathbf{A}\dot{\boldsymbol{\rho}} = \mathbf{B}\mathbf{t} \tag{1}$$

where the Jacobians $\mathbf{A}$ and $\mathbf{B}$ and vectors $\dot{\boldsymbol{\rho}}$ and $\mathbf{t}$ are

$$\begin{aligned}
\mathbf{A} &= \begin{bmatrix} \rho_1 & 0 & & 0 \\ 0 & \rho_2 & & 0 \\ & & \ddots & \\ 0 & 0 & & \rho_n \end{bmatrix} = \begin{bmatrix} (\mathbf{A}_a)_{k\times k} & \mathbf{0} \\ \mathbf{0} & \mathbf{A}_s \end{bmatrix} \\
\mathbf{B} &= \begin{bmatrix} \mathbf{b}_1^T \\ \mathbf{b}_2^T \\ \vdots \\ \mathbf{b}_n^T \end{bmatrix}_{n\times 3} = \begin{bmatrix} \Big(\mathbf{B}_a\Big)_{k\times 3} \\ \Big(\mathbf{B}_s\Big)_{(n-k)\times 3} \end{bmatrix} \\
\mathbf{b}_i^T &= \begin{bmatrix} \Big(\mathbf{c}+\mathbf{Q}\mathbf{v}_i-\mathbf{p}_i\Big)^T & \Big((\mathbf{c}-\mathbf{p}_i)^T\mathbf{E}\mathbf{Q}\mathbf{v}_i\Big) \end{bmatrix} \\
\dot{\boldsymbol{\rho}} &= \begin{bmatrix} \dot\rho_1 & \dot\rho_2 & \cdots & \dot\rho_n \end{bmatrix}^T, \mathbf{t} = \begin{bmatrix} \dot x & \dot y & \dot\phi \end{bmatrix}^T
\end{aligned}$$

Indices a and s refer respectively to *actuators* and *springs*.

4. Forces in cables

From the principle of virtual work, one can write $-\mathbf{f}^T\delta\boldsymbol{\rho} = \mathbf{F}^T\delta\mathbf{x}$, where $\mathbf{f}^T = \begin{bmatrix} f_1 & f_2 & \dots & f_n \end{bmatrix}$, $\mathbf{F}^T = \begin{bmatrix} F_x & F_y & \tau \end{bmatrix} = \begin{bmatrix} m(\ddot{\mathbf{c}}+\boldsymbol{g}) & \mathbf{I}\ddot\phi \end{bmatrix}$ and where f_i is the tension in link i, and F_x, F_y, and τ are the forces and torque applied by cables and springs to the end-effector. If the configuration and the accelerations are known, we finally get an equation in the form $\mathbf{U}\mathbf{f}_a = \mathbf{h}$ where $\mathbf{f}_a$ is the vector of forces in cables, $\mathbf{U}_{3\times k} = -\mathbf{B}_a^T\mathbf{A}_a^{-1}$, $\mathbf{h}_{3\times 1} = \mathbf{F}-\mathbf{F}_s$ and $\mathbf{F}_s = -\mathbf{B}_s^T\mathbf{A}_s^{-1}\mathbf{f}_s$, with $\mathbf{f}_s$ being the vector of forces in springs and $\mathbf{F}_s$ is the vector of forces and torque applied by springs on the end-effector.

Two main cases then arise:

- Isostatic case ($k=3$) The only solution is obtained from $\mathbf{f}_a = \mathbf{U}^{-1}\mathbf{h}$.
- Hyperstatic case ($k>3$) There exists an infinite number of solutions. The minimum norm solution is obtained from $\mathbf{f}_a = \mathbf{U}^I\mathbf{h}$ where $\mathbf{U}^I$ is the generalized inverse of $\mathbf{U}$,written as $\mathbf{U}^I = \mathbf{U}^T(\mathbf{U}\mathbf{U}^T)^{-1}$. This solution does not guarantee that the components of $\mathbf{f}_a$ will be positive, i.e. that all the cables are in tension. The problem to solve is then:

$$\left\{\begin{array}{ll} \text{min} & \mathbf{f}_a^T\mathbf{f}_a \\ \text{under} & \left| \begin{array}{ll} \mathbf{U}\mathbf{f}_a = \mathbf{h} & \\ \mathbf{f}_{ai} \geq 0 & i = 1\dots k \end{array}\right. \end{array}\right.$$

Solving this problem is achieved through the use of quadratic programming [2, 3]. The issue addressed in the next section is the determination of the set of configurations for which a solution to this problem exists.

5. Workspace

Since the shape of the workspace depends on the accelerations of the end-effector, we must now refer to dynamic workspace, static workspace being only a particular case for which all accelerations are zero. We now consider the set of configurations for which a specific dynamic equilibrium is possible. We define the dynamic workspace as the set of all configurations $\{x, y, \phi\}$ and dynamic conditions $\{\ddot{x}, \ddot{y}, \ddot{\phi}\}$ for which all cables work in tension.

In the case for which $n = k$ (no spring), $\mathbf{h}$ = $\mathbf{F}$ and we obtain $\mathbf{U}\mathbf{f}_a = \mathbf{F}$ that we can write in the following way:

$$f_1\mathbf{u}_1 + f_2\mathbf{u}_2 + \ldots + f_k\mathbf{u}_k = \mathbf{F}\ , \qquad (2)$$

where coefficients f_i (forces in cables) must all be positive or zero—cables cannot work in compression. For some known angle ϕ, the workspace is the set of all possible configurations $\{x, y\}$ for which vector $\mathbf{F}$ can be obtained from any vectorial sum of vectors $\mathbf{u}_i$ (columns of $\mathbf{U}$), but using only positive or zero coefficients, f_i. That means that vector $\mathbf{F}$ must be inside a pseudo-pyramid made up of vectors $\mathbf{u}_i$ in a three-dimensional virtual space, in which the three dimensions are along F_x, F_y, and τ.

Figure 1 is a representation of such a pseudo-pyramid where vectors $\mathbf{u}_2$, $\mathbf{u}_3$, $\mathbf{u}_4$, and $\mathbf{u}_5$ define four planes that make up the faces of the pyramid ($\mathbf{u}_1$, $\mathbf{u}_6$ and $\mathbf{F}$ are inside).

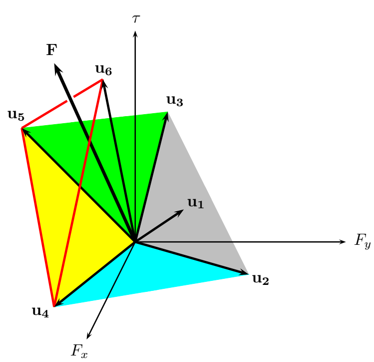

Figure 1: Pseudo-pyramid.

We see from Fig. 1 that every workspace configuration can also be achieved by one particular combination of three cables ($\mathbf{u}_4$,$\mathbf{u}_5$ and $\mathbf{u}_6$ in that example) since any pseudo-pyramid is in reality a union of three-vector basic pyramids. This does not guarantee that this three-cable combination will be optimal with regard to forces but only that any configuration is reachable using only three of the cables. Then, the whole workspace is the union of sub-workspaces corresponding to all 3-cable mecanisms.

When investigating the workspace, we are interested in determining its boundaries. As it can be seen with the pseudo-pyramid, there are two ways for the system to be on one of these boundaries. In the first one, vector $\mathbf{F}$ lie on one of the pyramid faces (plane in the figure). $\mathbf{F}$ can therefore be applied to the effector using only two of the vectors $\mathbf{u}_i$ and, in this case, we face a two-cable equilibrium. In the second one, at least three vectors are coplanar and $\mathbf{F}$ is the only vector on its side of the plane of these vectors. In this case, the system is on a singularity boundary.

6. Determination of the workspace boundaries

The first step of the simple method that we propose is the creation of every 2-cable combination quadratic of equilibrium and every 3-cable combination singularity quadratic [4]. Then, we intersect the curves with each other to create a set of sections. We then test every section to determine if it belongs to the boundary of the workspace or not. Both two-cable equilibrium and singularity sections can be tested easily by simple geometrical conditions. Figure 2 shows the resulting workspace for a particular dynamic state.

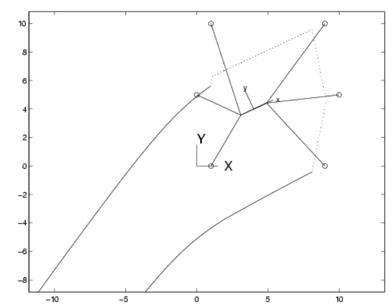

Figure 2: Complete workspace
(ϕ= $\mathbf{25}^\circ$, $\ddot{\mathbf{x}} = \mathbf{7\ m/s^2}$, $\ddot{\phi}$= $\mathbf{-5\ rad/s^2}$).

7. Conclusion

A fundamental and systematic analysis of planar parallel manipulators actuated with cables has been performed. Particularly, the workspace was analyzed by introducing the new concept of dynamic workspace as it depends on the accelerations at the end-effector. An algorithm for the determination of an $x - y$ workspace (a two-dimensional subset of the dynamic workspace) has been proposed.

References

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