Similarities Between a New Gradient Method and the Superposition Method for Generating Burmester Curves

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Abstract

There are a variety of methods for generating center-point and circle-point circles for the 3-position, and Burmester curves for the 4-position and 5-position planar motion generation problem. Dyads are synthesized based on these solution sets. However, in general it is difficult or impossible to generalize these methods to 3D mechanisms. This paper begins by briefly reviewing the geometric constructions used in classical Burmester theory. Then, it presents a method using a scalar field to find the center-point and circle-point circles by finding the points where these fields are equal, and shows similarities with the graphical superposition methods. The developed method could be extended to 3D since it does not use poles or other strict 2D entities (the 3D analysis is beyond the scope of this paper). The 2D solution for 4-bar linkages with revolute joints will be presented to support the developed procedure.

1. Introduction

Solution methods for motion generation problems in four bar linkages have been studied for some time beginning with the works of Burmester over a century ago [1]. The general nomenclature used is shown in figure 1.

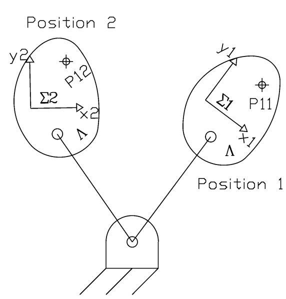

Figure 1: Nomenclature for links in two positions

The type of solution depends on the number of input precision positions. In general, for 3 positions a set of circles, known as center-point and circle-point circles, show the solution. For 4 positions, the solutions lie on a set of Burmester curves. For 5 positions, if a solution exists, the solutions are a finite set of points. Using a purely graphical method, known as the superposition method, one could solve 4- and 5-position problems by first solving for the center-point and circle-point circles for two sets of 3 positions, say for positions 1-2-3 and 1-2-4. These two solutions can yield a solution to the 4-position problem by finding the intersections of the circles and generating a Burmester curve through the intersections. Similarly, the intersections of two 4-position problems can yield a finite set of solution points. The superposition method is purely graphical, and uses the poles and Euclid's chord angle principle to generate the circles and the Burmester curves [2]. Using the poles and other geometric entities, one can sketch the Burmester curve for the 4-position problem [3]. Similar methods exist using complex number theory and other mathematical tools [4]. Graphical solutions generated using a CAD package are shown in figures 2 and 3.

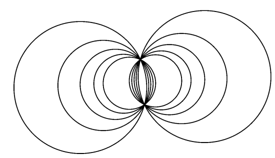

Figure 2: Center-Point-Circles

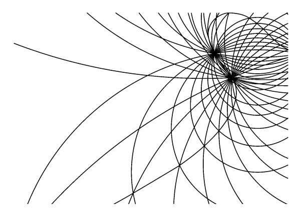

Figure 3: Center-Point-Circles and Burmester curve generated graphically with a CAD package

2. The Scalar Field - β

This section will show that a common parameter used in mechanism synthesis β could be viewed as a scalar function. This gradient approach was first developed by one of the authors (Adams) [5, 6]. However, the similarities between the gradient approach method and the graphical superposition were not discussed. Later, gradient and field theory in the study of Burmester curves were used in [7]. It is important to note that the different center-point-circles for a given 3-position problem will give a different angle β, that the link moves through from position 1 to position 2. For example, if the center-point-circle that was generated using Euclid's chord angle principle and $\beta = 30^o$ was used to specify the ground point, then the link (usually a crank) will rotate 30^o from point 1 to point 2. Figure 4 shows a circle drawn through the 3 positions of a point.

One can solve for the position of the circles given the 3 positions of the point and the displacement matrices by solving the vector equations

$$|r_i - c| - |r_1 - c| = 0 \tag{1}$$

for **c**, and j = 2, 3. According to figure 4

$$L_{12} = \left| r_2 - r_1 \right| \tag{2}$$

and

$$\sin(\beta / 2) = (L_{12} / 2) / R_1 \tag{3}$$

where R_1 can be found from the center to any of the positions of the point.

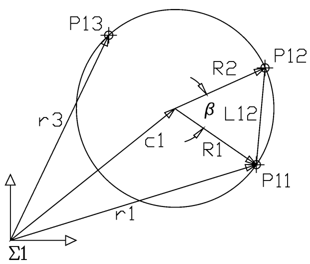

Figure 4: Point in 3 positions, center of circle, and β

Thus, the input variable is the point picked, and the equations for **c**, L_{12}, and β all vary with the input point. Using these equations, one can use a symbolic processor such as Mathematica™ to solve analytically and plot β for the given 3 positions. These plots can then be superimposed similar to the graphical superposition method to provide the Burmester curve for the 4-position problem as shown in Figure 4. Figure 5 shows the center-point-circles, and Figure 6 the superimposed 3-position problems giving the Burmester curve. Reference to figure 6, the gradients are perpendicular to the equi-potential circles, the arrow indicates an intersection of Burmester curve with center-point-circles. Also, the scale in figure 6 is skewed to clarify the intersection.

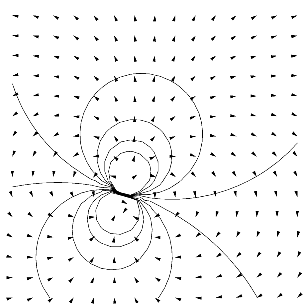

Figure 5: Center-point-circles and scalar field gradients.

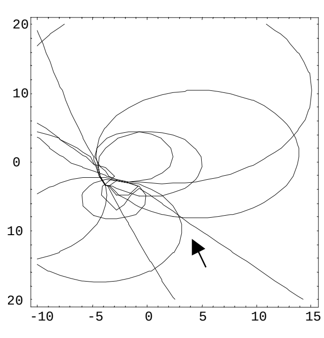

Figure 6: Burmester curve by superposition of two scalar fields.

3. Conclusions:

One must note some important properties of β. First it is a continuous function in 2D space. It is also differentiable in space. Thus, it is similar to a potential function such as those used in fluid mechanics. Thus, the center-point and circle-point circles are analogous to equi-potential curves. The superposition of two such scalar functions for positions 1-2-3, and 1-2-4, gives the Burmester curve for positions 1-2-3-4. Therefore, along the Burmester curve $\beta_{1\text{-}2\text{-}3} = \beta_{1\text{-}2\text{-}4}$. Thus, β can be thought of as a unique scalar function of the mechanism problem. Given the displacement matrices and equations 1-3, one can solve for the scalar function for two sets of 3 positions. The Burmester curve is obtained by finding the equality points at the superposition of the two scalar functions. Also, this approach does no require strict 2D entities, such as poles, to generate these curves. Thus it would be possible to generalize this method to 3D. This is currently an area that we are working on.

4. References

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