Singularity Analysis of 3-DOF Planar Parallel Mechanisms

Ilian A. Bonev, Dimiter Zlatanov, and Clément M. Gosselin

Département de Génie Mécanique, Université Laval, Québec, QC, Canada G1K 7P4

Abstract

This paper presents the results of a systematic study of the singular configurations of 3-DOF planar parallel mechanisms with three identical serial chains. Only prismatic and revolute kinematic pairs are considered. From the point of view of singularity analysis, there are ten different architectures. All of them are examined; the nature of all possible singular configurations are discussed and the singularity loci for a constant orientation of the mobile platform are found. For some architectures, simplified designs with easy to determine singularities are proposed.

1. Introduction

Singularity analysis of only several 3-DOF planar parallel mechanisms (PPMs) with three identical serial chains have been presented in the literature [1-3]. Furthermore, no systematic analysis of all possible architectures has ever been discussed. Although some of those architectures are unlikely to be used in practice, such a detailed study brings an insight into the kinematics of PPMs. It also points out several novel architectures with valuable properties.

To describe the various types of serial chains, we will denote a revolute joint by *R* and a prismatic joint by *P*. When a joint is actuated, its corresponding symbol will be underlined. Since we are interested only in 3-DOF fully-parallel mechanisms, each serial chain will be composed of two passive and one actuated joints and will be denoted by a sequence of three symbols starting from the base joint.

There are 21 3-DOF serial chains in total (Table 1). Three of them (marked with $^\times$) cannot be used in our PPMs since they lead to mechanisms with only one controllable DOF. There are also eight pairs of symmetrical chains, where each pair leads to two PPMs that are kinematically equivalent. Therefore, we can eliminate eight more chains (the ones marked with $^\sim$) which leaves us with only ten architectures to examine. The latter are presented on Fig. 1 with the notation assigned to each architecture.

$\underline{R}RR$	$\underline{R}PR$	$\underline{R}PP^\times$	$\underline{P}RR$	$\underline{P}RP$	$\underline{P}PR$	$\underline{R}RP$
$R\underline{R}R$	$R\underline{P}R$	$R\underline{P}P$	$P\underline{R}R$	$P\underline{R}P^\times$	$P\underline{P}R^\sim$	$R\underline{R}P^\sim$
$RR\underline{R}^\sim$	$RP\underline{R}^\sim$	$RP\underline{P}^\sim$	$PR\underline{R}^\sim$	$PR\underline{P}^\sim$	$PP\underline{R}^\times$	$RR\underline{P}^\sim$

Table 1: All possible serial chains

2. Singularity Analysis of the Ten Architectures

The singular configurations of parallel mechanisms can generally be described by the degeneracies of the corresponding velocity equation which is of the form

$$\mathbf{J}_q\dot{\mathbf{q}} + \mathbf{J}_\Theta\dot{\mathbf{\Theta}} = \mathbf{0}, \tag{1}$$

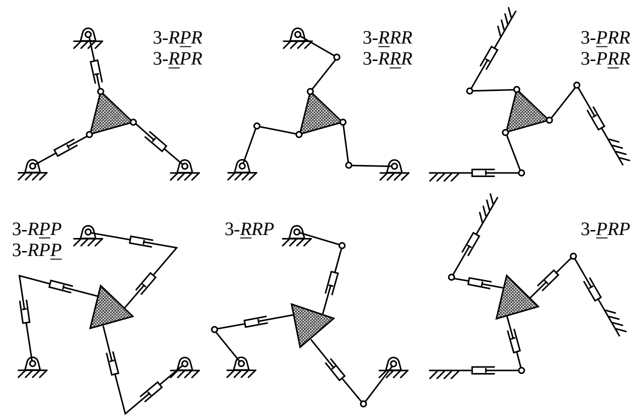

Figure 1: The basic 3-DOF planar parallel mechanisms with identical serial chains

where $\mathbf{q} = [x, y, \phi]^T$ is the vector of *generalized coordinates* defining the pose of the mobile platform, $\mathbf{\Theta}$ is the vector of *input coordinates*, while $\mathbf{J}_q$ and $\mathbf{J}_\Theta$ are 3×3 Jacobian matrices. Two main types of singularities are defined in [4]: Type I when $\det\mathbf{J}_\Theta = 0$ and Type II when $\det\mathbf{J}_q = 0$. However, the study of the singularities of a parallel mechanism is not completed solely with the study of the two latter matrices. Indeed, it was noted in [5] that the above equation is always necessary but not always sufficient to describe the instantaneous motion of a parallel mechanism.

The conventional process of deriving eq. 1 consists in differentiating the inverse kinematics equation. Generally, the process is tedious and leads to possible parametrization errors. A much better approach is the use of reciprocal screws. They provide a better geometrical insight into the problem and allow the description of additional types of singularities [5]. Once the reciprocal screws are found for a given design, the two Jacobian matrices can be obtained directly in coordinate-invariant form.

3-R$\underline{P}$R PPMs A detailed study of this popular architecture has already been published [1]. The reciprocal screws are the line vectors passing through the *R* joints in each chain. Type II singularities occur when the three screw axes intersect or are parallel. For a constant orientation (CO) of the platform, the singularity loci form a quadratic curve. For some designs (with congruent base and platform), there is an orientation at which the PPM is singular for any position. When the two *R* joints of a serial chain coincide, there is a Type I singularity. It is important to note that in this singularity, there are two linearly independent reciprocal screws for the degenerate serial chain. This singularity allows an uncontrollable passive motion (RPM type) and the three input velocities cannot be chosen independently (II type) [5].

3-$\underline{R}PR$ PPMs The reciprocal screws are the line vectors passing through the platform *R* joints and normal to the direction of the corresponding *P* joints. Again, the Type II singularity loci for a CO form a quadratic curve. Type I singularities occur for the same configurations as in 3-$R\underline{P}R$ PPMs. However, these are generic Type I singularities, where the input velocities are indeterminate.

3-$R\underline{R}R$ PPMs The reciprocal screws for this architecture are the line vectors passing through the passive *R* joints in a serial chain. Hence, its Type II singularity loci are the same as for 3-$R\underline{P}R$ PPMs. Type I singularities occur, however, when the proximal and distal link in a serial chain are aligned. Therefore, the corresponding singularity loci for a CO are pairs of concentric circles. When the links are of equal lengths, we also have a singularity of the same class (RPM, II, IO [5]) as in 3-$R\underline{P}R$ PPMs. A major difference, however, is that 3-$R\underline{R}R$ PPMs have 8 branch sets, even though, they all lead to exactly the same singularities.

3-$\underline{R}RR$ PPMs The line vectors for this architecture are along the distal links (the ones closer to the platform). They have the same Type I singularities as 3-$R\underline{R}R$ PPMs. However, a recent detailed study of their Type II singularity loci [2] found that they are represented by curves of degree 42. Note that these singularity loci for a CO for a single branch set are represented by a non-polynomial equation, and that the polynomial of degree 42 corresponds to the singularities for all 8 branch sets. The simplified design of a 3-$\underline{R}RR$ PPMs is obtained when two of the platform *R* joints coincide. The Type II singularity loci of such a mechanism reduce to four circles and one sextic and can be determined geometrically.

3-$\underline{P}RR$ PPMs The kinematic properties of this architecture are similar to those of 3-$\underline{R}RR$ PPMs. Type I singularities occur when a distal link is normal to its connecting proximal link. The Type II singularity loci for a CO form a curve of degree 20 and correspond, as in 3-$\underline{R}RR$ PPMs, to all branch sets. Similarly, the simplified design is obtained when two platform *R* joints coincide, which leads to singularity loci represented by two lines, two circles, and an ellipse.

3-$P\underline{R}R$ PPMs The reciprocal screws for this architecture are the line vectors passing through the platform *R* joints and normal to the directions of the corresponding *P* joints. For a CO, the configuration of the three line vectors is fixed, and hence the Type II singularities do not depend on the position of the platform. In fact, there is only one orientation at which the PPM is in a singularity. Similarly to 3-$R\underline{R}R$ PPMs, the choice of branch set does not influence the singularities. The Type I singularities are the same as for 3-$\underline{P}RR$ PPMs.

3-$RP\underline{P}$ and 3-$R\underline{P}P$ PPMs The kinematic properties of both architectures are the same. The reciprocal screw for each serial chain is a line vector passing through the *R* joint and normal to the passive *P* joint. Since the directions of the passive joints are defined by the orientation of the platform, the Type II singularities do not depend on the platform position. In fact, there is only one orientation at which the PPM is in a singularity. There are no Type I singularities.

3-$\underline{R}RP$ PPMs The reciprocal screws are the line vectors passing through the passive *R* joints and normal to the corresponding *P* joints. This is another PPM with 8 branch sets that all lead to different singularities. The Type II singularity loci for all branch sets are represented by a polynomial of degree 6. Type I singularities occur when a proximal link is normal to the direction of the corresponding *P* joint. An interesting simplified design may be obtained by constructing two of the *P* joints to be parallel, while setting the third one, e.g., normal to them. In that case, it is easy to see that the only Type II singular configuration will occur when the passive *R* joints of the first two serial chains coincide.

3-$\underline{P}RP$ PPMs A design of this type was proposed in [3], known under the name *double-triangular manipulator*. The reciprocal screws are the line vectors passing through the *R* joints and normal to the directions of the passive *P* joints. Type I singularities occur when the directions of both *P* joints in a serial chain coincide. In general, the singularity loci for a CO are represented by a line. However, for designs such as the double-triangular PPM for which the angles between the directions of the *P* joints for all three chains are equal, the Type II singularities do not depend on the platform position. For such designs, there are four orientations at which the PPM is singular. In addition, there is another orientation at which all serial chains are singular. It is interesting to note that, once in such a Type I singularity, the mobile platform becomes blocked.

3. Conclusion

The basic results of an exhaustive study on the singularities of 3-DOF PPMs were presented. The compact presentation was possible mainly due to the use of reciprocal screws.

References

[1] Sefrioui, J. and Gosselin, C. M., 1995, "On the Quadratic Nature of the Singularity Curves of Planar Three-Degree-Of-Freedom Parallel Manipulators," *Mech. Mach. Theory*, Vol. 30, No. 4, pp. 533–551.

[2] Bonev, I. A. and Gosselin, C. M., 2001, "Singularity Loci of Planar Parallel Manipulators with Revolute Joints," *2nd Workshop on Computational Kinematics*, Seoul, South Korea, May 20–22.

[3] Daniali, H. R. M., Zsombor-Murray, P. J. and Angeles, J., 1993, "The Kinematics of 3-DOF Planar and Spherical Double-Triangular Parallel Manipulators," *Computational Kinematics*, J. Angeles et al. (eds.), Kluwer Academic Publishers, pp. 153-164.

[4] Gosselin, C. M. and Angeles, J., 1990, "Singularity Analysis of Closed-Loop Kinematic Chains," *IEEE Trans. Rob. Autom.*, Vol. 6, No. 3, pp. 281–290.

[5] Zlatanov, D., Fenton, R. G. and Benhabib, B., 1994, "Analysis of the Instantaneous Kinematics and Singular Configurations of Hybrid-Chain Manipulators," *Proc. ASME 23rd Biennial Mechanisms Conference*, DE-Vol. 72, Minneapolis, MN, Sept. 11-14, pp. 467–476.