2-DOF Loss Velocity Degeneracies of the Spherical-Revolute-Spherical Manipulator

Scott B. Nokleby and Ron P. Podhorodeski
Robotics and Mechanisms Laboratory, University of Victoria, Victoria, B. C., Canada

1. Introduction

This paper presents the two-DOF (degree-of-freedom) loss velocity-degenerate configurations of the 7-jointed spherical-revolute-spherical manipulator. The degeneracies are found using the reciprocity-based method of Nokleby and Podhorodeski outlined in [1].

The D & H parameters [2] for the spherical-revolute-spherical manipulator are presented in Table 1. Figure 1 shows the layout of the manipulator.

Table 1: D & H Parameters

F_{i-1}	θ_i	d_i	a_i	α_i	F_i
F_0	θ_1	0	0	$\frac{\pi}{2}$	F_1
F_1	θ_2	0	0	$-\frac{\pi}{2}$	F_2
F_2	θ_3	g	0	$\frac{\pi}{2}$	F_3
F_3	θ_4	0	0	$-\frac{\pi}{2}$	F_4
F_4	θ_5	h	0	$\frac{\pi}{2}$	F_5
F_5	θ_6	0	0	$-\frac{\pi}{2}$	F_6
F_6	θ_7	0	0	0	F_7

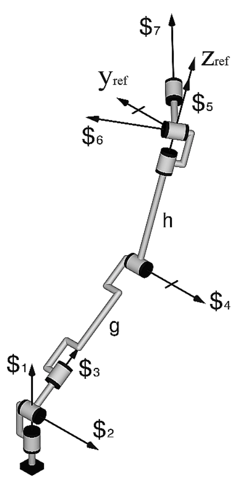

Figure 1: Manipulator Layout

Choosing a reference frame that is located at the intersection of the wrist spherical group and oriented with z_{ref} in the direction of $\$_5$ and y_{ref} in the opposite direction to that of $\$_4$ allows the joint screws to be found as:

$$
{}^{ref}\$_1 = \left\{ \begin{array}{c} S_2C_3C_4 + C_2S_4 \\ -S_2S_3 \\ -S_2C_3S_4 + C_2C_4 \\ -S_2S_3C_4g - S_2S_3h \\ -S_2C_3g - S_2C_3C_4h - C_2S_4h \\ S_2S_3S_4g \end{array} \right\}
$$

$$
\begin{aligned}
{}^{ref}\$_2 &= \{\ -C_4S_3,\ \ -C_3,\ \ S_3S_4;\ \ -C_3C_4g - C_3h,\ \ S_3g + S_3C_4h,\ \ C_3S_4g\ \}^{\mathrm{T}} \\
{}^{ref}\$_3 &= \{\ S_4,\ \ 0,\ \ C_4;\ \ 0,\ \ -S_4h,\ \ 0\ \}^{\mathrm{T}} \\
{}^{ref}\$_4 &= \{\ 0,\ \ -1,\ \ 0;\ \ -h,\ \ 0,\ \ 0\ \}^{\mathrm{T}} \\
{}^{ref}\$_5 &= \{\ 0,\ \ 0,\ \ 1;\ \ 0,\ \ 0,\ \ 0\ \}^{\mathrm{T}} \\
{}^{ref}\$_6 &= \{\ S_5,\ \ -C_5,\ \ 0;\ \ 0,\ \ 0,\ \ 0\ \}^{\mathrm{T}} \\
{}^{ref}\$_7 &= \{\ -C_5S_6,\ \ -S_5S_6,\ \ C_6;\ \ 0,\ \ 0,\ \ 0\ \}^{\mathrm{T}}
\end{aligned}
$$

where $C_{ij} = \cos(\theta_i + \theta_j)$ and $S_{ij} = \sin(\theta_i + \theta_j)$. The Jacobian for the manipulator is:

$$
{}^{ref}\mathbf{J} = {}^{ref}[\ \$_1\ \ \$_2\ \ \$_3\ \ \$_4\ \ \$_5\ \ \$_6\ \ \$_7\ ] \tag{1}
$$

2. Velocity Degeneracies

From [3], the one-DOF loss velocity degeneracies, along with their associated reciprocal screws, for the spherical-revolute-spherical manipulator can be summarized as:

(i) $S_4 = 0$
$${}^{ref}\mathbf{W}_1 = \{\ 0,\ \ 0,\ \ 1;\ \ 0,\ \ 0,\ \ 0\ \}^{\mathrm{T}}$$
(ii) $S_2 = 0$ & $C_3 = 0$
$${}^{ref}\mathbf{W}_2 = \{\ 0,\ \ 0,\ \ 1;\ \ 0,\ \ 0,\ \ 0\ \}^{\mathrm{T}}$$
(iii) $S_2 = 0$ & $S_6 = 0$
$${}^{ref}\mathbf{W}_3 = \{\ -S_5,\ \ C_5,\ \ \frac{-C_3C_4S_5 - S_3C_5}{C_3S_4};\ \ C_5h,\ \ S_5h,\ \ 0\ \}^{\mathrm{T}}$$
(iv) $C_5 = 0$ & $S_6 = 0$
$${}^{ref}\mathbf{W}_4 = \{\ -S_5,\ \ C_5,\ \ \frac{-C_3C_4S_5 - S_3C_5}{C_3S_4};\ \ C_5h,\ \ S_5h,\ \ 0\ \}^{\mathrm{T}}$$

Condition (i): $S_4 = 0$

Setting $S_4 = 0$ and ${}^{ref}\mathbf{W}_1^* = \{0, 0, 0; 0, 0, 1\}^{\mathrm{T}}$ allows $\mathbf{J}_{sub_1}^*$ to be defined as:

$$
{}^{ref}\mathbf{J}_{sub_1}^* = [\ \$_2\ \ \mathbf{W}_1^*\ \ \$_4\ \ \$_5\ \ \$_6\ \ \$_7\ ]
$$

with the "redundant joints" being $\$_1$ and $\$_3$. The determinant of $\mathbf{J}_{sub_1}^*$ is:

$$
\left|{}^{ref}\mathbf{J}_{sub_1}^*\right| = -S_3S_6h\,(g + C_4h) \tag{2}
$$

Therefore, if either $S_3 = 0$, $S_6 = 0$, or $g + C_4 h = 0$, then the six "joints" comprising $\mathbf{J}^*_{sub_1}$ are degenerate.

Setting $S_3 = 0$ in $\mathbf{J}^*_{sub_1}$ (with $S_4 = 0$) and equating ${}^{ref}\mathbf{W}_{recip_{1_1}} \circledast {}^{ref}\$_i = 0$, for $i = 2$, 4 to 7 and ${}^{ref}\mathbf{W}_{recip_{1_1}} \circledast {}^{ref}\mathbf{W}^*_1 = 0$ allows $\mathbf{W}_{recip_{1_1}}$ to be found as:

$$ {}^{ref}\mathbf{W}_{recip_{1_1}} = \{\ 0,\quad 1,\quad 0;\quad 0,\quad 0,\quad 0\ \}^{\mathrm{T}} \tag{3} $$

Setting the reciprocal products between $\mathbf{W}_{recip_{1_1}}$ & $\$_1$ and $\mathbf{W}_{recip_{1_1}}$ & $\$_3$ to zero yields:

$$ \begin{aligned} &{}^{ref}\mathbf{W}_{recip_{1_1}} \circledast {}^{ref}\$_1 \\ &= -S_2 C_3 (g + C_4 h) - C_2 S_4 h = 0 \\ &{}^{ref}\mathbf{W}_{recip_{1_1}} \circledast {}^{ref}\$_3 = 0 \end{aligned} \tag{4} $$

Since S_4 and S_3 are assumed to equal zero, if in addition $S_2 = 0$ or $g + C_4 h = 0$, then $\mathbf{W}_{recip_{1_1}}$ is reciprocal to joints $\$_1$ to $\$_7$. Therefore, $S_2 = 0$, $S_3 = 0$, & $S_4 = 0$ and $S_3 = 0$, $S_4 = 0$, & $g + C_4 h = 0$ define two 3-condition families of double-DOF loss velocity-degenerate configurations.

Setting $S_6 = 0$ in $\mathbf{J}^*_{sub_1}$ (with $S_4 = 0$) and equating ${}^{ref}\mathbf{W}_{recip_{1_2}} \circledast {}^{ref}\$_i = 0$, for $i = 2$, 4 to 7 and ${}^{ref}\mathbf{W}_{recip_{1_2}} \circledast {}^{ref}\mathbf{W}^*_1 = 0$ allows $\mathbf{W}_{recip_{1_2}}$ to be found as:

$$ \begin{aligned} {}^{ref}\mathbf{W}_{recip_{1_2}} = \ &\{\ \tfrac{-S_5}{h},\quad \tfrac{C_4 C_5 S_3 h - C_3 C_4 S_5 g}{S_3 h (g + C_4 h)},\quad 0; \\ &C_5,\quad S_5,\quad 0\ \}^{\mathrm{T}} \end{aligned} \tag{5} $$

Setting the reciprocal products between ${}^{ref}\mathbf{W}_{recip_{1_2}}$ & $\$_1$ and ${}^{ref}\mathbf{W}_{recip_{1_2}}$ & $\$_3$ to zero yields:

$$ \begin{aligned} &{}^{ref}\mathbf{W}_{recip_{1_2}} \circledast {}^{ref}\$_1 = S_2 C_4 S_5 g = 0 \\ &{}^{ref}\mathbf{W}_{recip_{1_2}} \circledast {}^{ref}\$_3 = 0 \end{aligned} \tag{6} $$

Since S_4 and S_6 are assumed to equal zero, if in addition $S_2 = 0$ or $S_5 = 0$, then $\mathbf{W}_{recip_{1_2}}$ is reciprocal to joints $\$_1$ to $\$_7$. Therefore, $S_2 = 0$, $S_4 = 0$, & $S_6 = 0$ and $S_4 = 0$, $S_5 = 0$, & $S_6 = 0$ define two 3-condition families of double-DOF loss velocity-degenerate configurations.

Setting $g + C_4 h = 0$ in $\mathbf{J}^*_{sub_1}$ (with $S_4 = 0$) and equating ${}^{ref}\mathbf{W}_{recip_{1_3}} \circledast {}^{ref}\$_i = 0$, for $i = 2$, 4 to 7 and ${}^{ref}\mathbf{W}_{recip_{1_3}} \circledast {}^{ref}\mathbf{W}^*_1 = 0$ allows $\mathbf{W}_{recip_{1_3}}$ to be found as:

$$ {}^{ref}\mathbf{W}_{recip_{1_3}} = \{\ 0,\quad 1,\quad 0;\quad 0,\quad 0,\quad 0\ \}^{\mathrm{T}} \tag{7} $$

Setting the reciprocal products between $\mathbf{W}_{recip_{1_3}}$ & $\$_1$ and $\mathbf{W}_{recip_{1_3}}$ & $\$_3$ to zero yields:

$$ \begin{aligned} &{}^{ref}\mathbf{W}_{recip_{1_3}} \circledast {}^{ref}\$_1 = -S_2 C_3 (g + C_4 h) = 0 \\ &{}^{ref}\mathbf{W}_{recip_{1_3}} \circledast {}^{ref}\$_3 = 0 \end{aligned} \tag{8} $$

Noting that $S_4 = 0$ & $g + C_4 h = 0$, no further conditions are necessary to make $\mathbf{W}_{recip_{1_3}}$ reciprocal to joints $\$_1$ to $\$_7$. Therefore, $S_4 = 0$ & $g + C_4 h = 0$ defines a 2-condition family of double-DOF loss velocity-degenerate configurations.

Repeating the procedure for degeneracy conditions (ii) to (iv) allows identification of all sets of conditions resulting in a two-DOF loss velocity-degeneracy for the spherical-revolute-spherical manipulator. These velocity-degenerate configurations and their associated reciprocal screws can be summarized as:

(I) $S_4 = 0$ & $g + C_4 h = 0$

$$ \begin{aligned} {}^{ref}\mathbf{W}_{1_1} &= \{\ 0,\ 0,\ 1;\ 0,\ 0,\ 0\ \}^{\mathrm{T}} \\ {}^{ref}\mathbf{W}_{1_2} &= \{\ 0,\ 1,\ 0;\ 0,\ 0,\ 0\ \}^{\mathrm{T}} \end{aligned} $$

(II) $S_2 = 0$, $S_3 = 0$, & $S_4 = 0$

$$ \begin{aligned} {}^{ref}\mathbf{W}_{2_1} &= \{\ 0,\ 0,\ 1;\ 0,\ 0,\ 0\ \}^{\mathrm{T}} \\ {}^{ref}\mathbf{W}_{2_2} &= \{\ 0,\ 1,\ 0;\ 0,\ 0,\ 0\ \}^{\mathrm{T}} \end{aligned} $$

(III) $S_2 = 0$, $S_4 = 0$, & $S_6 = 0$

$$ \begin{aligned} {}^{ref}\mathbf{W}_{3_1} &= \{\ 0,\ 0,\ 1;\ 0,\ 0,\ 0\ \}^{\mathrm{T}} \\ {}^{ref}\mathbf{W}_{3_2} &= \{\ \tfrac{-S_5}{h},\ \tfrac{C_4 C_5 S_3 h - C_3 C_4 S_5 g}{S_3 h (g + C_4 h)},\ 0; \\ &\quad C_5,\ S_5,\ 0\ \}^{\mathrm{T}} \end{aligned} $$

(IV) $S_4 = 0$, $S_5 = 0$, & $S_6 = 0$

$$ \begin{aligned} {}^{ref}\mathbf{W}_{4_1} &= \{\ 0,\ 0,\ 1;\ 0,\ 0,\ 0\ \}^{\mathrm{T}} \\ {}^{ref}\mathbf{W}_{4_2} &= \{\ 0,\ \tfrac{C_4 C_5 S_3 h}{S_3 h (g + C_4 h)},\ 0; \\ &\quad C_5,\ 0,\ 0\ \}^{\mathrm{T}} \end{aligned} $$

(V) $S_2 = 0$, $C_3 = 0$, $C_5 = 0$, & $S_6 = 0$

$$ \begin{aligned} {}^{ref}\mathbf{W}_{5_1} &= \{\ 0,\ 0,\ 1;\ 0,\ 0,\ 0\ \}^{\mathrm{T}} \\ {}^{ref}\mathbf{W}_{5_2} &= \{\ -S_5,\ 0,\ 0;\ 0,\ S_5 h,\ 0\ \}^{\mathrm{T}} \end{aligned} $$

Note that condition (I) is only possible if $g = h$ and $\theta_4 = \pi$.

3. Conclusions

Five two-DOF loss velocity-degenerate configurations exist for the spherical-revolute-spherical manipulator. With the exception of condition (I), all of the two-DOF loss degenerate configurations are based solely on joint angles. Condition (I) requires that the lengths of g and h must be the same and that the value of θ_4 equals π.

References

[1] S.B. Nokleby and R.P. Podhorodeski, "Identification of Multi-DOF Loss Velocity Degeneracies for Redundant Manipulators," *Proceedings of the 2001 CCToMM Symposium on Mechanisms, Machines, and Mechatronics*, (in press).

[2] J. Denavit and R.S. Hartenberg, "A Kinematic Notation for Lower-Pair Mechanisms Based on Matrices," *Transactions of the ASME, Journal of Applied Mechanics*, June, 215-221, (1955).

[3] S.B. Nokleby and R.P. Podhorodeski, "Reciprocity-Based Resolution of Velocity Degeneracies (Singularities) for Redundant Manipulators," *Mechanism and Machine Theory*, (in press).