Dynamic Modelling of Electro-Mechanical Multibody Systems

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1. INTRODUCTION

Over the past two decades, a number of approaches have been developed for systematically formulating the equations of motion for multibody systems. Principles of analytical and vectorial mechanics have been combined with topological representations, so that the dynamics of a wide range of mechanical systems can be automatically and efficiently analyzed [1].

Several authors have recently proposed extensions to the Principle of Virtual Work (and/or Lagrange's Equations) so that electrical components can be included in a model of a "mechatronic" system [2-4]. In these papers, the mechatronic system consists of rigid multibody sub-systems and electrical networks of analog components (resistors, capacitors, etc). Although linear graph theory is used to generate Kirchoff's laws for the electrical sub-systems, it is misperceived as being inefficient [2] and is dismissed as a unified modelling theory.

In fact, linear graph theory provides a natural representation of multi-disciplinary problems and, when combined with principles of mechanics, results in efficient models for electro-mechanical multibody systems. The application of graph theory to electrical networks has long been established [5] and, more recently, graph theory has been combined with principles of vectorial [6] and analytical mechanics [7] to obtain systematic formulations for rigid and flexible multibody systems. The extension of these methods to electro-mechanical systems is natural and straight-forward.

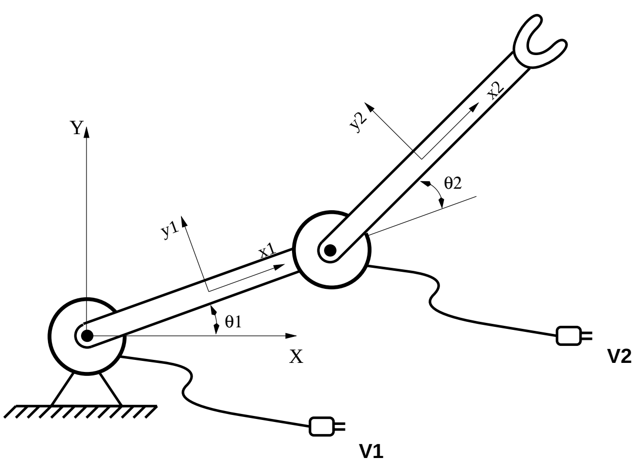

Figure 1. Two-Link Robot Driven by DC Motors

2. SYSTEM MODELLING

To demonstrate this, consider the example in Figure 1 in which a two-link robot arm is being driven by two DC-motors powered by voltage sources V_1 and V_2. The topology of this electro-mechanical system is encapsulated by the linear graph representation shown in Figure 2.

In contrast with other representations, e.g. bond graphs, the linear graph is relatively simple and bears a striking resemblance to the physical system. This is emphasized by overlaying the graph with the links and motors in dashed lines. The edges of the graph correspond directly to physical components: $J1$ and $J2$ are the two revolute joints, $r3$–$r6$ represent the location of these joints relative to body-fixed reference frames, $M7$ and $M8$ are the two motors, and $V1$ and $V2$ are the voltage sources.

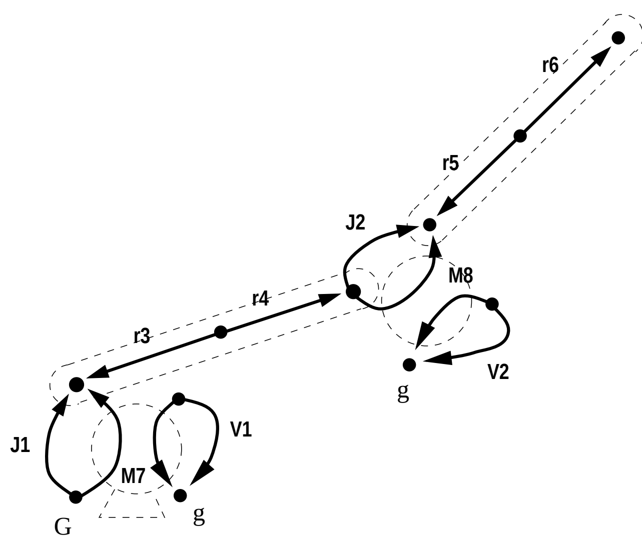

Figure 2. Linear Graph of Two-Link robot

Note that the electro-mechanical transducers, the DC-motors, are represented by two edges — one in the mechanical system and one in the electrical network. The dynamic equations for the two sub-systems are coupled by the constitutive equations for the motors:

$$V_i = K_i\dot{\theta}_i + R_iI_i + L_i\dot{I}_i \quad (1)$$

$$T_i = B_i\dot{\theta}_i + J_i\ddot{\theta}_i - C_iI_i \quad (2)$$

where V_i and I_i are the voltage across and current through motor Mi $(i = 7, 8)$, T_i and $\dot{\theta}_i$ are the motor torque and speed, R_i and L_i are the armature resistance and inductance, K_i and C_i are the voltage and torque constants, and B_i and J_i are the damping coefficient and inertia of the motor shaft.

Using graph-theoretic topological equations and principles of mechanics, the dynamic equations for the mechanical sub-system can be systematically formulated in absolute coordinates, joint coordinates, or some combination of these and other coordinates [6]. Furthermore, the mechanical equations can be expressed in either recursive or non-recursive formats. For this example, the dynamic equations are automatically generated in terms of the joint coordinates θ_1 and θ_2, using symbolic Maple routines [7] that exploit the topological equations to reduce the number of variables and equations, and virtual work to eliminate non-working joint reactions.

These Maple routines have been extended to include models of electrical networks and a number of electro-mechanical transducers. A graph-theoretic approach again allows some freedom in selecting the system variables; the electrical sub-system equations are automatically formulated in currents or voltages, as desired by the user.

Assuming the links to be rigid in the robot example, our dynamic formulation produces two symbolic second-order differential equations for the multibody sub-system:

$$[M(\theta_1,\theta_2)]\left\{\begin{array}{c}\ddot{\theta}_1\\ \ddot{\theta}_2\end{array}\right\}=\left\{\begin{array}{c}Q_1(\theta_1,\theta_2,\dot{\theta}_1,\dot{\theta}_2,I_7)\\ Q_2(\theta_1,\theta_2,\dot{\theta}_1,\dot{\theta}_2,I_8)\end{array}\right\} \quad (3)$$

Had the links been modelled as elastic beams, additional equations would be generated for the elastic coordinates, which would also appear in the mass matrix $[M]$ and generalized forces $\{Q\}$. Selecting currents as the variables for this problem, two first-order differential equations are obtained for the electrical sub-network:

$$\left[\begin{array}{cc}L_7 & 0\\ 0 & L_8\end{array}\right]\left\{\begin{array}{c}\dot{I}_7\\ \dot{I}_8\end{array}\right\}=\left\{\begin{array}{c}V_1(t)-R_7I_7-K_7\dot{\theta}_1\\ V_2(t)-R_8I_8-K_8\dot{\theta}_2\end{array}\right\} \quad (4)$$

Thus, a minimal number of system equations (3-4) is automatically generated by the graph-theoretic formulation and symbolic implementation. Although the electrical networks in this example are relatively trivial, networks of any complexity can be efficiently treated using graph theory.

With the equations expressed in symbolic form, it is often possible to find closed-form solutions for the generalized inverse dynamics problem. In this case, given desired joint trajectories of $\theta_1=\theta_2=\pi t/8$, i.e. both links rotate through 90 degrees in 4 seconds, one can solve (3-4) to get analytical expressions for the required motor currents:

$$\begin{aligned}I_7(t) &= \frac{1}{64K_{T_7}}\left[8\pi B_7-3\pi^2 m_2 l_5\,(l_3+l_4)\sin\left(\frac{\pi}{8}t\right)\right.\\ &\quad +64\left(m_2 g\,(l_3+l_4)\cos\left(\frac{\pi}{8}t\right)+m_2 g l_5\cos\left(\frac{\pi}{4}t\right)\right.\\ &\quad \left.\left.+m_1 g l_3\cos\left(\frac{\pi}{8}t\right)\right)\right] \quad (5)\\ I_8(t) &= \frac{1}{64K_{T_8}}\left[8\pi B_8+\pi^2 m_2 l_5\,(l_3+l_4)\sin\left(\frac{\pi}{8}t\right)\right.\\ &\quad \left.+64 m_2 g l_5\cos\left(\frac{\pi}{4}t\right)\right] \quad (6)\end{aligned}$$

This solution can be verified by a forward dynamic simulation in which the motor input voltages are regulated by a PD-controller to respond to errors in the joint trajectories. For this case, a numerical integration of equations (3-4) results in the motor currents shown in solid line in Figure 3. As expected, they oscillate about the analytical solutions (5-6) shown in dotted lines.

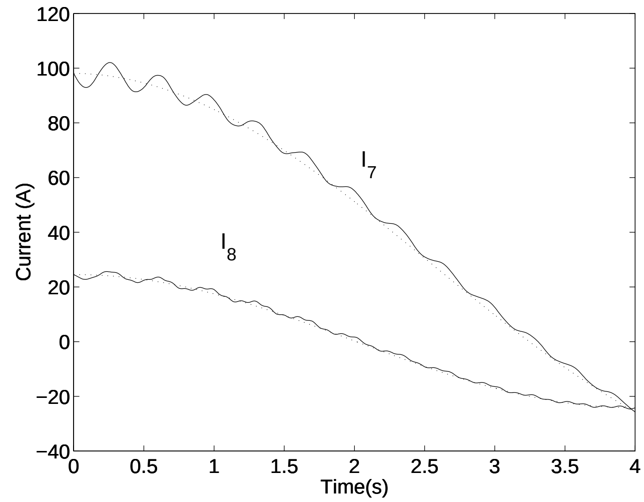

Figure 3. Motor Currents Required for Given Joint Trajectories

3. CONCLUSIONS

In summary, a unified and efficient modelling methodology for electro-mechanical multibody systems has been obtained by combining linear graph theory with principles of mechanics. From a single graph representation, a relatively small number of system equations is generated in a methodical manner that is well-suited for computer implementation. It is also worth noting that a graph-theoretic approach is not restricted to analog components, in contrast to approaches based solely on virtual work [2-4]. Thus, components of a discrete-time nature can be readily included in a model of a mechatronic system containing digital controllers; this appears to be a promising area for future research.

4. ACKNOWLEDGEMENT

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