THE CONCEPTUAL DESIGN OF EPICYCLIC CAM TRAINS

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ABSTRACT

The speed reducers dubbed Speed-o-Cam (SoC), as alternatives to current gear transmissions are being developed at the Centre for Intelligent Machines (CIM) for the past six years. Their objective is to overcome the drawbacks of gear trains, such as *Coulomb friction and backlash.* This paper introduces new types of SoC systems, which can be applied to both simple trains and epicyclic trains. Moreover, the computation of the total pressure ratio, a generalization of the pressure angle, of such a system is given in detail.

La conception créative de trains épicycloidaux à cames

Des réducteurs de vitesse surnommés Speed-o-Cam ont été recherchés au Centre de recherches sur les machines intelligentes (CIM) depuis six ans. Ces réducteurs visent la résolution des problèmes posés par des engrenages, soit le *frottement à sec* et le *jeu.* Cet article présente de nouveaux types de systèmes SoC, qui trouvent des applications dans la conception de train simples ainsi qu'épicycloïdaux. En outre, le calcul du rapport de pression, une généralisation du concept d'angle de pression, de ces systèmes, est étudié en grand détail.

1 Background

Although the technology of gear-train manufacturing, production, and utilization is well established [1], a family of novel mechanical transmissions for speed reduction, SoC, is being developed as a viable alternative [2]. This transmission is based on the layout of pure-rolling indexing cam mechanisms, for it is intended to eliminate backlash and friction, which are the main drawbacks of gear transmissions [2]. Planar and Spherical prototypes of SoC have been produced, as shown in Fig. 1 (a) and (b).

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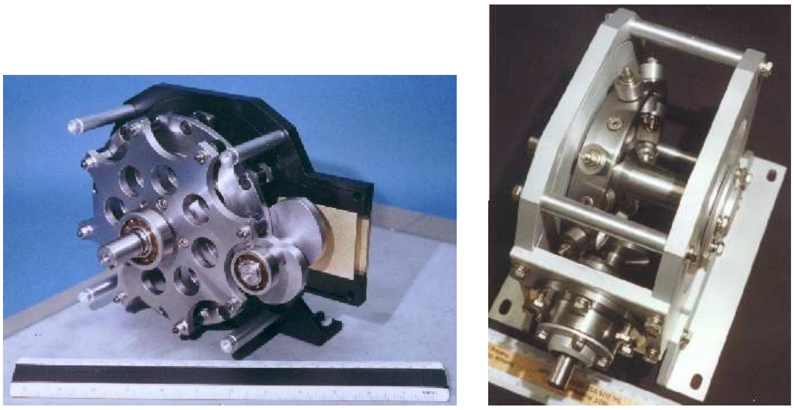


(a) (b)

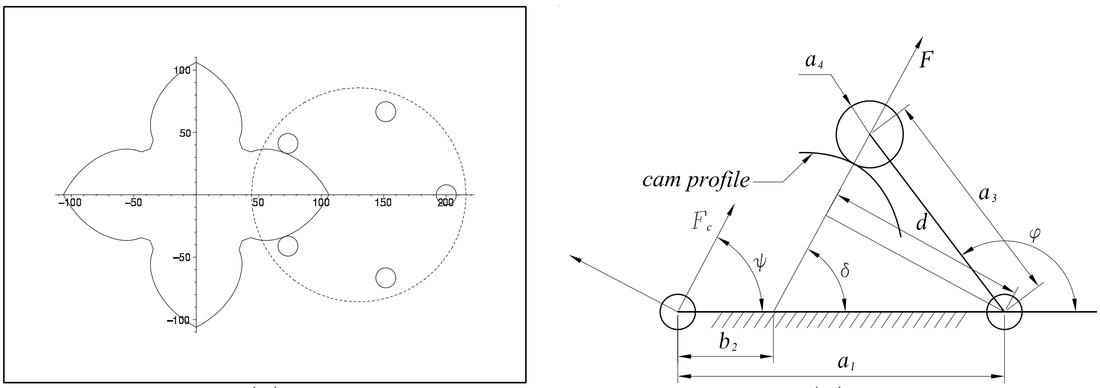
Figure 1: The prototypes of: (a) planar SoC (b) spherical SoC

However, the prototypes exhibit new drawbacks, such as high pressure angle and double thickness due to the conjugate cams, besides a host of manufacturing problems. In order to improve the performance of SoC, we are developing two new types of SoC mechanisms, the *sun cam* and the *ring cam*.

The pressure ratio is a generalization of the pressure angle, which is defined as the ratio between the *pressure radius* and the *characteristic radius* [3]. It is applied here for the transmission quality because the pressure angle fails to evaluate the mechanical transmission system such as follower-cam, cam-cam [3]. The total pressure ratio, which provides more precise measure than the pressure angle does, is applied to two types of systems: serial and parallel. For serial systems, the total pressure ratio is the smallest one among the individual pressure ratios [3]. For parallel systems, the total pressure ratio is the biggest one, which is equivalent to choosing the smallest pressure angle as the total pressure angle. This principle has been used in the past [2]. Although the approach for parallel systems is not completely accurate, because the force produced by the contact with higher pressure angle cannot be overlooked sometimes, it is still a reasonable means to conduct the performance evaluation of the mechanism.

2 The Sun Cam

The *sun cam* is a new type of SoC, as shown in Fig. 2 (a), in which the small circles indicate the follower rollers. Features of this mechanism, as compared with the previous SoC prototypes, are static and dynamic balance of the cam and a lower pressure angle, by sacrificing the speed reduction ratio. The notation, as shown in Fig. 2 (b), is needed to define the sun cam profile:



(a) (b)

Figure 2: (a) the sun cam and its roller-follower with $M_s = 4$ and $N = 5$, (b) the notation

ψ: angle of rotation of the cam
ϕ: angle of rotation of the follower
ϕ': derivative of ϕ w.r.t. ψ
a_1: distance between input and output axes
a_3: distance between output and roller axes
a_4: radius of the roller
N: number of the rollers on the follower
M_s: number of the stages of the sun cam

The input-output function is given by

$$\phi := -\pi(1 - 1/N) - M_s\psi/N$$

The pitch curve is given by the position of a typical point P_p of the curve, of coordinates (u_p, v_p); these coordinates are

$$u_p = a_1 \cos(\psi) + a_3 \cos(\phi - \psi) \quad (1a)$$
$$v_p = -a_1 \sin(\psi) + a_3 \sin(\phi - \psi) \quad (1b)$$

The coordinates of the contact point are, in turn, given by:

$$u_c = u_p + a_4 \cos(\delta - \psi + \pi) \quad (2a)$$
$$v_c = v_p + a_4 \sin(\delta - \psi + \pi) \quad (2b)$$
$$b_2 = \frac{M_s}{M_s + N} a_1 \quad (2c)$$
$$\delta = \arctan\left(\frac{a_3 \sin\phi}{a_3 \cos\phi + a_1 - b_2}\right) \quad (2d)$$

Equation (2c) is derived from Aronhold-Kennedy Theorem [4, 5].

2.1 The Pressure Angle and the Pressure Ratio

The pressure radius, d, as shown in Fig. 2 (b) is given by

$$d = (a_1 - b_2)\sin\delta$$

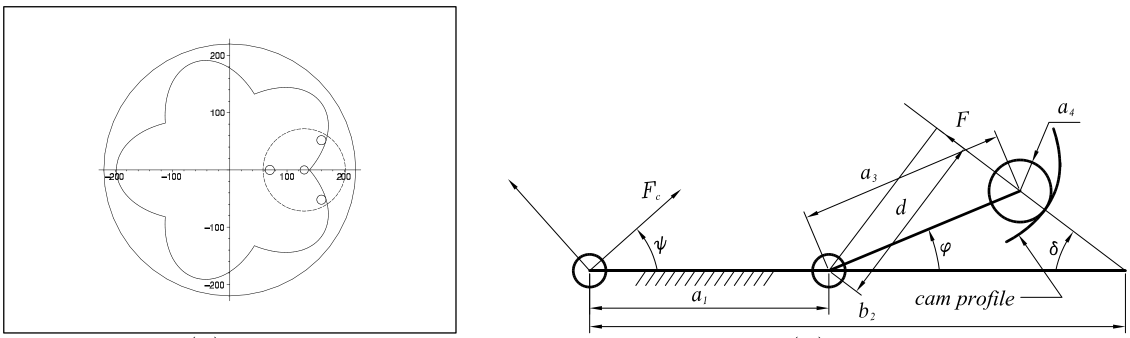
The corresponding pressure ratio p is, in turn,

$$p = \frac{d}{a_3} = \frac{a_1 - b_2}{a3}\sin\delta$$

Since the action profile of the follower-disk is circular, the relationship between the pressure angle α and the pressure ratio is simple [3]

$$p = \cos\alpha$$

3 The Ring Cam



(a) (b)

Figure 3: (a) the ring cam and its roller-follower with $M_r = 5$ and $N = 3$, (b) the notation

The *ring cam* is another new type of SoC, as shown in Fig. 3 (a), which composes an epicyclic cam train with a sun cam for the same roller-follower. The notation used here, as shown in Fig. 3 (b), is the same as that for the sun cam, except for M_r, which is the number of the stages of the ring cam.
The input-output function of the ring cam is given by

$$\phi := -\pi/N + M_r\psi/N$$

The pitch curve and the profile of the *ring cam* are derived from the relations below:

$$u_p = a_1\cos(\psi) + a_3\cos(\phi - \psi) \tag{3a}$$

$$v_p = -a_1 \sin(\psi) + a_3 \sin(\phi - \psi) \tag{3b}$$
$$u_c = u_p + a_4 \cos(-\psi - \delta) \tag{3c}$$
$$v_c = v_p + a_4 \sin(-\psi - \delta) \tag{3d}$$
$$b_2 = \frac{M_r}{M_r - N} a_1 \tag{3e}$$
$$\delta = \arctan\left(\frac{a_3 \sin\phi}{b_2 - a_3 \cos\phi - a_1}\right) \tag{3f}$$

3.1 The Pressure Angle and the Pressure Ratio

The pressure radius, d, as shown in Fig. 3 (b) is given by

$$d = (b_2 - a_1) \sin\delta$$

The corresponding pressure ratio p is, in turn,

$$p = \frac{d}{a_3} = \frac{a_1 - b_2}{a3} \sin\delta$$

The equivalent pressure angle is given by

$$p = \cos\alpha$$

4 Epicyclic Cam Train

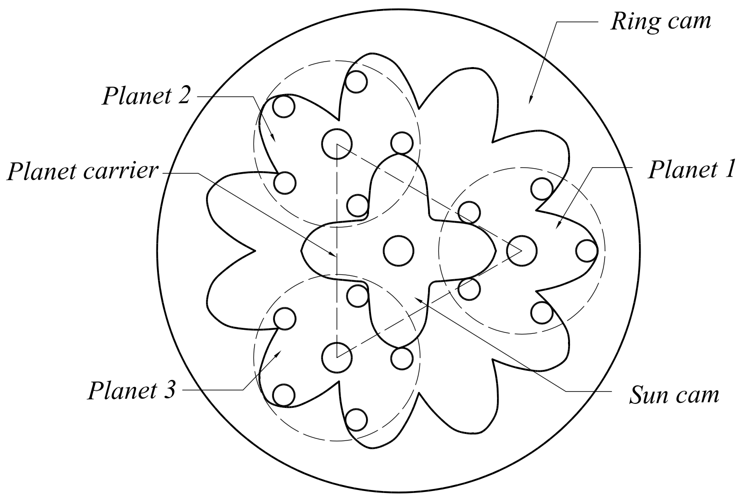


Figure 4: An epicyclic cam train with $M_s = 4$, $M_r = 11$, $K = 3$ and $N = 5$

The epicyclic cam train, as shown in Fig. 4, is composed of a sun cam, a ring cam and three roller-followers as the planets. The assembly of the planets is depended on the number of the planets, K, and the numbers of stages of both the sun cam and the ring cam, M_s and M_r.

In order to assembly all planets around the sun cam, we first install one roller-follower at the position of planet 1, as shown in Fig. 4. Next, we rotate the sun cam around its center $2\pi/K$ clockwise, which causes the follower to rotate $2\pi M_s/(KN)$ counterclockwise. Then, we rotate the whole mechanism around its center $2\pi/K$ counterclockwise, which makes the sun cam to return its initial position while the roller-follower rotates totally $2\pi(N+M_s)/(KN)$ counterclockwise, which is called the *differential angle*, to the position of the planet 2, as shown in Fig. 4. Finally, we can install the second roller-follower on the position of planet 1. The above procedure is repeated to install all the planets into the system.

For the ring cam, as shown in Fig. 4, we apply exactly the same procedure as that of the sun cam, except that the corresponding differential angle is different, which turns out to be $2\pi(N-M_r)/(KN)$.

In order to make sure that the sun cam, the ring cam and all the planets can be installed without interference, the planet at the same position during the assembly with both the sun cam and the ring cam must be in the same pose. That means the difference in the follower-disk differential angle should be a multiple of $2\pi/N$. Then, we have

$$\frac{2\pi(N+M_s)/(KN) - 2\pi(N-M_r)/(KN)}{2\pi/N} = \frac{M_s+M_r}{K}$$

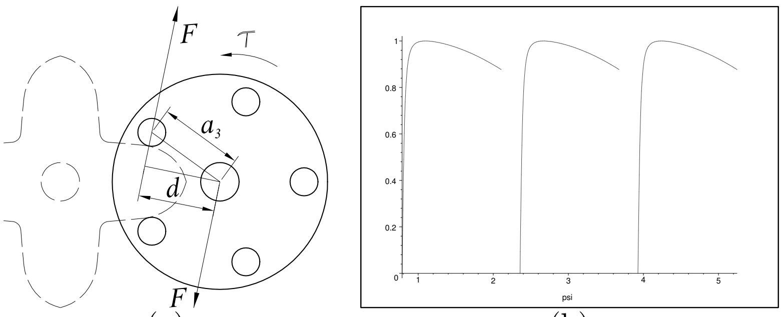
As a result, M_s+M_r must be the multiple of K, which is the necessary condition for assembly.

5 The Pressure Ratio of the Epicyclic Cam Train

In this mechanism, the computation of the pressure ratio is dependent on the input and the output. Hence, we use two configurations to illustrate this issue.

5.1 Configuration One

We consider the sun cam as the input, and the planet 1 as the output. Therefore, we have three parallel trains to transmit the motion from the input to the output: (1) the sun cam to the planet 1 (S-P1); (2) the sun cam to the planet 2, then to the ring cam, and finally to the planet 1 (S-P2-R-P1); (3) the sun cam to the planet 3, then to the ring cam, and finally to the planet 1 (S-P3-R-P1). We will compute the total pressure ratio of each train first, then obtain the total pressure ratio of the system, as described below.



(a) (b)

Figure 5: (a) The FBD of S-P1, (b) the total pressure ratio of the first train

5.1.1 The First Parallel Train

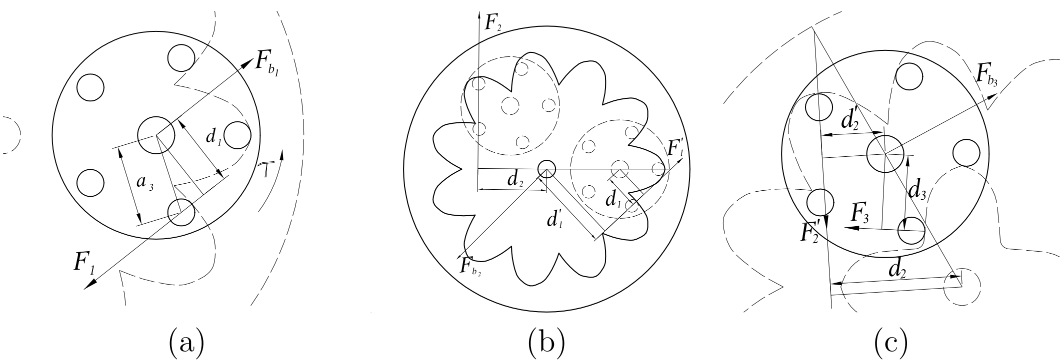
S-P1 is the first parallel train, whose free-body diagram (FBD) is shown in Fig. 5 (a). The pressure radius is d and the characteristic radius is a_3; the pressure ratio is given by

$$p_1 = \frac{d}{a_3}$$

as plotted in Fig. 5 (b).

5.1.2 The Second Parallel Train

S-P2-R-P1 is the second parallel train, which has three stages: (1) the ring cam to the planet 1 (R-P1); (2) the planet2 to the ring cam (P2-R); (3) the sun cam to the planet 2 (S-P2).

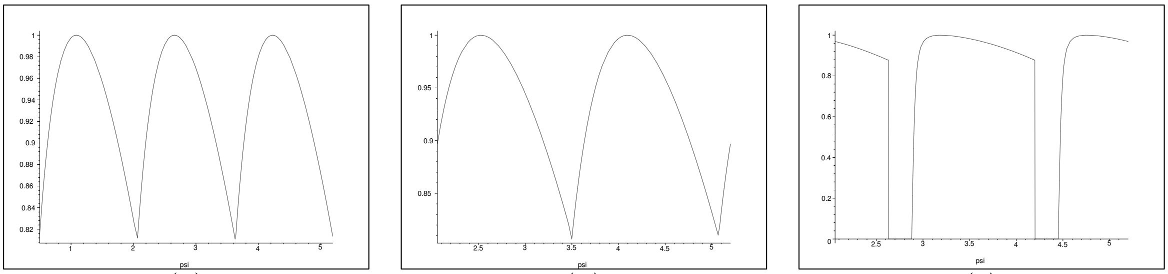


(a) (b) (c)

Figure 6: The FBD of (a) the first stage, (b) the second stage, and (c) the third stage of the second train

The FBD of the first stage is shown in Fig. 6 (a). The pressure ratio is given by

$$p_{2_1} = \frac{d_1}{a_3}$$



(a) (b) (c)

Figure 7: The pressure ratio of (a) the first stage, (b) the second stage, and (c) the third stage of the second train

as plotted in Fig. 7 (a). The FBD of the second stage is shown in Fig. 6 (b).

$$d_1' = \frac{b_2}{b_2 - a_1} d_1 = \frac{M_r}{N} d_1 = \frac{M_r}{N} a_3 p_{2_1}$$

According to the computation of the pressure ratio of a serial train [3], we have

$$p_{2_2} = \frac{F_1'}{F_2} p_{2_1} = \frac{d_2}{d_1'} p_{2_1} = \frac{N d_2}{M_r a_3}$$

as plotted in Fig. 7 (b). The FBD of the third stage is shown in Fig. 6 (c), whereby

$$d_2' = \frac{N}{M_r} d_2 = \frac{N}{M_r} \frac{M_r}{N} a_3 p_{2_2} = a_3 p_{2_2}$$

$$p_{2_3} = \frac{F_2'}{F_3} p_{2_2} = \frac{d_3}{d_2'} p_{2_2} = \frac{d_3}{a_3}$$

as plotted in Fig. 7 (c).

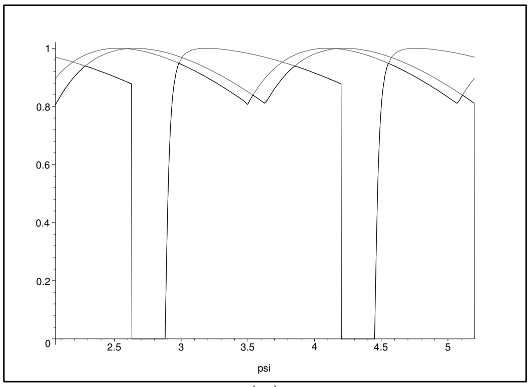
Since these three stages are in a serial train, we choose the smallest pressure ratio as the total pressure ratio in the second train, as plotted in Fig. 8 (a).

5.1.3 The Third Parallel Train

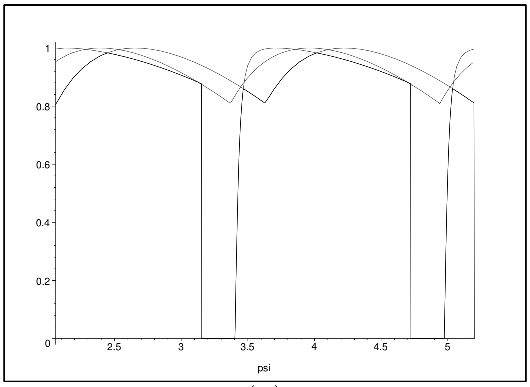
Applying the same procedure as that of the second train yields the total pressure ratio of the third train, as plotted in Fig. 8 (b).

5.1.4 The Total Pressure Ratio in the System

Since these three trains are parallel in this mechanism, we choose the biggest pressure ratio as the total pressure ratio of the mechanism, as shown in Fig. 9. The minimum pressure ratio is 0.842. The equivalent maximum pressure angle is 32.6^o.



(a)



(b)

Figure 8: The total pressure ratio of (a) the second train, and (b) the third train

5.2 Configuration Two

Considering the sun cam as the input, and the planet carrier as the output, the direction of the force exerted on the planet carrier through the planet bearings cannot be determined directly by the geometric configuration of the mechanism. Therefore, neither the pressure angle nor the pressure ratio can be computed in this case [3]. The performance evaluation of such a mechanism is still under research.

6 Conclusions

We introduce here two new types of SoC, the sun cam and the ring cam, for simple cam trains or epicyclic cam trains. In the mechanisms discussed in this paper, no coaxial conjugate cams are needed, so that the thickness of the system is reduced. Both the sun cam and the ring cam are statically and dynamically balanced. Two principles in the computation of the pressure ratio for both serial and parallel trains are applied in the force transmission performance evaluation of this mechanism.

Acknowledgements

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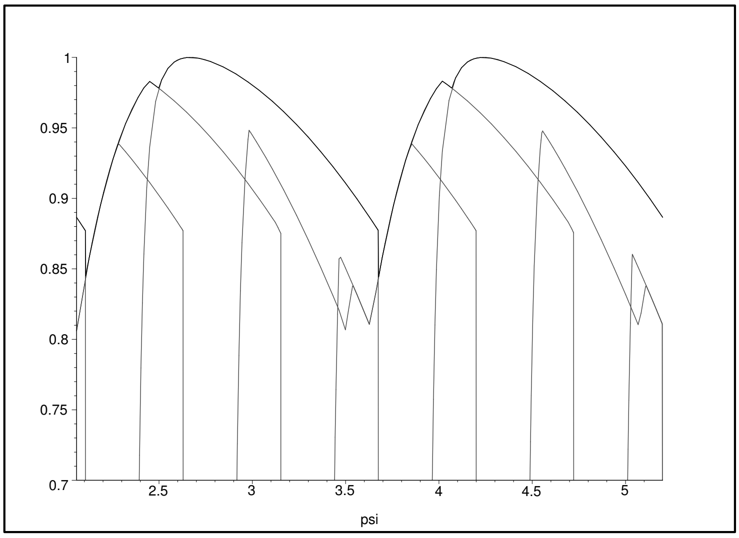

Figure 9: The total pressure ratio of the first configuration of the epicyclic cam train

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