PASSIVE MECHANISMS WITH MULTIPLE EQUILIBRIUM CONFIGURATIONS

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Abstract

From a practical point of view, it is of interest to design mechanisms with multiple equilibrium configurations. In this paper, passive spring mechanisms with multiple equilibrium configurations are proposed. The total potential energy function of the system is defined to describe the behavior of these mechanisms. Knowing that the first derivative of this function is zero in the equilibrium positions of the mechanisms, the latter configurations can be determined by applying classical stability theory. Using this method, two examples are given in order to show that these mechanisms have multiple equilibrium configurations. It is remarkable that the springs used in the first example are practical, i.e., the undeformed length of the springs is not equal to zero. Therefore, these results have significance in practical equilibrator design.

1 Introduction

The balancing of mechanisms has been an important research issue for several decades [1], since balanced mechanisms have better dynamic characteristics and less vibrations caused by motion. Static and dynamic balancing of planar linkages have been studied extensively by some scholars [2-6]. A comprehensive review is given in [7]. For complex parallel manipulators, static balancing has been studied in [8-14]. Conventionally, there are two approaches of static balancing, namely i) using counterweights and ii) using springs. Both methodologies have their own merit and provide static equilibrium throughout the entire range of motion.

Up to now, numerous authors have discussed applications of springs to achieve statically balanced machinery. The idealized springs with zero free length were first introduced by Carwardine in 1932 [15]. Hain presented the relationship between spring and linkage parameters of a rotating link in 1961 [16]. In 1985, based on Hain's work, Nathan introduced a new concept called perfect spring balancing and addressed perfect equilibration of an n link open or closed-loop revolute joint kinematic chain [17]. Streit et al. introduced perfect equilibrator design and a general approach for serial manipulators using springs [18-21].

According to the practical design specifications, various design objectives can be required, including: (1) one equilibrium position (2) several equilibrium positions, (3) equilibrium at every position, throughout the range of motion ("perfect" equilibrator design). However, all of the above mentioned authors have studied the third case. In some cases, such as an automotive hood, "perfect" equilibrator design is not desirable. In that case, the potential energy can be specified as some nonconstant function. Thus, design of mechanisms with several equilibrium positions is also an interesting topic. Therefore, the purpose of this paper is to design mechanisms with several equilibrium configurations using linear springs.

When springs are used, static equilibrium positions of the system can be defined as the set of conditions for which the total potential energy in the mechanism —including gravitational energy and the elastic energy stored in the springs— reaches an extremum for several specific configurations of the mechanism. Mathematically, this condition is equivalent to the first derivative of the total potential energy function being zero.

In the equilibrator designs presented by all the above mentioned authors, zero free length springs are used. In articulated mechanical systems, the assumption that the effective undeformed length of the springs be equal to zero does not represent any particular practical problem, since the physical springs can extend beyond the attachment points using guiding systems or pulleys and wires. However, in some applications it may not be possible or desirable to impose the zero free-length condition. If this condition is not met, then perfect balancing is often not possible, although some exceptions can be found [7]. Therefore, non-zero free-length springs will be used in this paper.

2 Introduction to the Equilibrium of Spring Systems

The theory is used here for finding equilibrium positions and investigating them for stability is just the general procedure used for conservative n degree-of-freedom (DOF) mechanical system under gravity and spring forces.

Let a general spatial n DOF mechanism with elastic springs and rigid members be composed of n_b moving bodies, one fixed link and n_s elastic elements. The total potential energy of the system, denoted V, is defined as the sum of the gravitational and elastic potential energy and can be written as

$$V = g\mathbf{e}_z^T \sum_{i=1}^{n_b} m_i \mathbf{c}_i + \frac{1}{2} \sum_{j=1}^{n_s} k_j (s_j - s_j^o)^2 \tag{1}$$

where g is the magnitude of the gravitational acceleration, $\mathbf{e}_z^T$ is an upward unit vector oriented in the direction of gravity, m_i is the mass of the ith moving body, $\mathbf{c}_i$ is the position vector of the center of mass of the ith moving body with respect to a fixed reference frame, k_j is the stiffness of the jth elastic element, s_j is the length of the jth elastic element and s_j^o is its undeformed length.

For a n DOF system, let x_i, $(i = 1, ..., n)$ be the independent variables describing the configuration of the system. These variables can be assembled in a vector $\mathbf{x}$ such that $\mathbf{x} = [x_1, ..., x_n]^T$. The partial derivative of V with respect to vector $\mathbf{x}$ must be zero for equilibrium, i.e.,

$$\frac{\partial V}{\partial \mathbf{x}} = \mathbf{0} \tag{2}$$

The stability of the system can be determined by the eigenvalues of the Hessian matrix derived from the second derivative of the energy function V. At a nondegenerate critical point, the number of positive eigenvalues and the number of negative eigenvalues must total to n. If the Hessian has p positive eigenvalues, then we say that the critical point is a Morse p-saddle. If $p = n$, then V has a local minimum, namely the system is in the stable position. If $p = 0$, V has a local maximum. The system is unstable. For example, consider $n = 2$, the conditions for equilibrium of the system are

$$\begin{cases} \frac{\partial V}{\partial x_1} = 0 \\ \frac{\partial V}{\partial x_2} = 0 \end{cases} \tag{3}$$

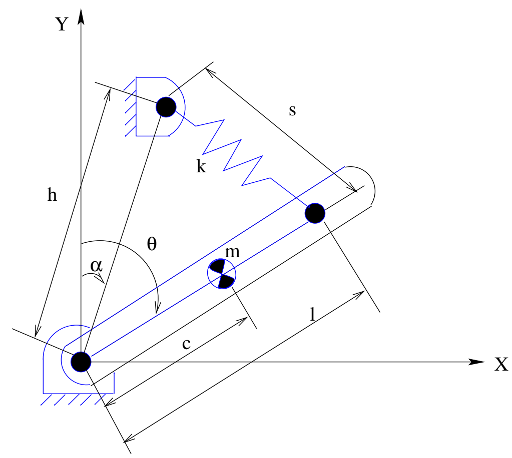


Figure 1: Planar one-link mechanism with a spring.

If the Hessian matrix of the system is

$$\mathbf{H} = \frac{\partial^2 V}{\partial \mathbf{x}^2} = \begin{bmatrix} \frac{\partial^2 V}{\partial x_1^2} & \frac{\partial^2 V}{\partial x_1 \partial x_2} \\ \frac{\partial^2 V}{\partial x_2 \partial x_1} & \frac{\partial^2 V}{\partial x_2^2} \end{bmatrix} \tag{4}$$

then, the conditions for the stable equilibrium of the system, which correspond to the minimum value of V are

$$D_1 = \frac{\partial^2 V}{\partial x_1^2} > 0 \quad \text{and} \quad D_2 = \det\mathbf{H} > 0 \tag{5}$$

Otherwise, the configurations are considered to be unstable. Using the above theory we shall analyze the equilibrium configurations of the following two mechanisms with springs.

3 Equilibrium Analysis of a Planar One-link Mechanism with a Spring

Consider the simple planar 1-DOF mechanism shown in Fig. 1. This system consists of a single link of mass m, rotating in a vertical plane. The center of mass of the link is located at a distance c from the pivot and a spring is attached to the fixed link, oriented at the angle α with respect to the $Y-$axis, at a distance h from the pivot, as well as to the rotating link, at a distance l from the pivot. According to Eq. (1), the total potential energy in the system can be written as

$$V = mgc\cos\theta + \frac{1}{2}k(s - s^o)^2 \tag{6}$$

where k is the spring stiffness, while s and s^o are its length and its undeformed length, respectively. From the law of cosines, one can write

$$s = \sqrt{l^2 + h^2 - 2lh\cos(\theta - \alpha)} \tag{7}$$

Moreover, if the undeformed length of the spring is not equal to zero, i.e., $s^o \neq 0$ the first derivative of the energy function V with respect to the independent variable θ is given by

$$\frac{dV}{d\theta} = -mgc\sin\theta + k(s - s^o)\frac{ds}{d\theta} \tag{8}$$

where

$$\frac{ds}{d\theta} = \frac{lh\sin(\theta - \alpha)}{s} \tag{9}$$

Then, Eq. (9) can be substituted directly into Eq. (8), which leads to the condition of the static balancing configurations of the system, namely

$$-a_1\sin\theta + a_2\sin(\theta - \alpha) - \frac{a_3\sin(\theta - \alpha)}{s} = 0 \tag{10}$$

where

$$a_1 = mgc \tag{11}$$

$$a_2 = khl \tag{12}$$

$$a_3 = khls^o \tag{13}$$

To solve Eq. (10), one can move the third term containing the square root to the right hand side and then square both sides to eliminate the square root term. Finally, one can convert the equation to an algebraic polynomial using the tangent-of-the-half-angle, namely $\sin\theta = \frac{2t}{1+t^2}$, $\cos\theta = \frac{1-t^2}{1+t^2}$, where $t = \tan\frac{\theta}{2}$. After introducing the substitution and clearing the denominator, a sixth-order polynomial in t is obtained as

$$b_6t^6 + b_5t^5 + b_4t^4 + b_3t^3 + b_2t^2 + b_1t + b_0 = 0 \tag{14}$$

which means that the maximum number of equilibrium configurations of the system is six. Since the expressions of the coefficients b_i, $(i = 0, ..., 6)$ are complicated, they are not given in this paper.

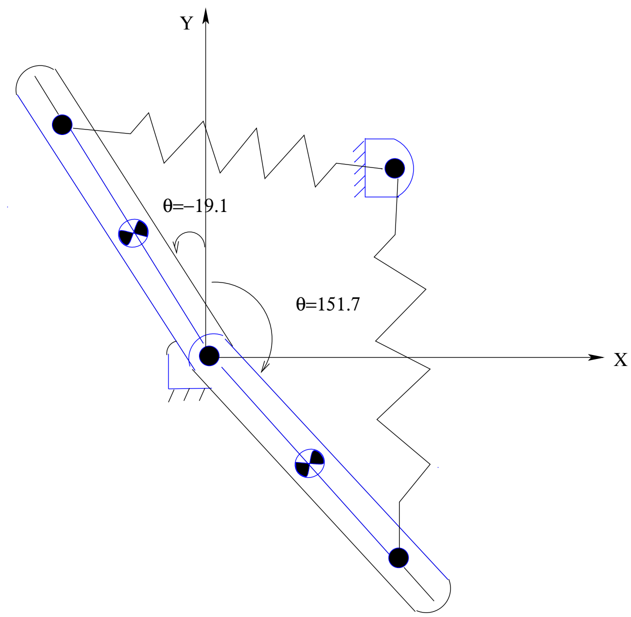
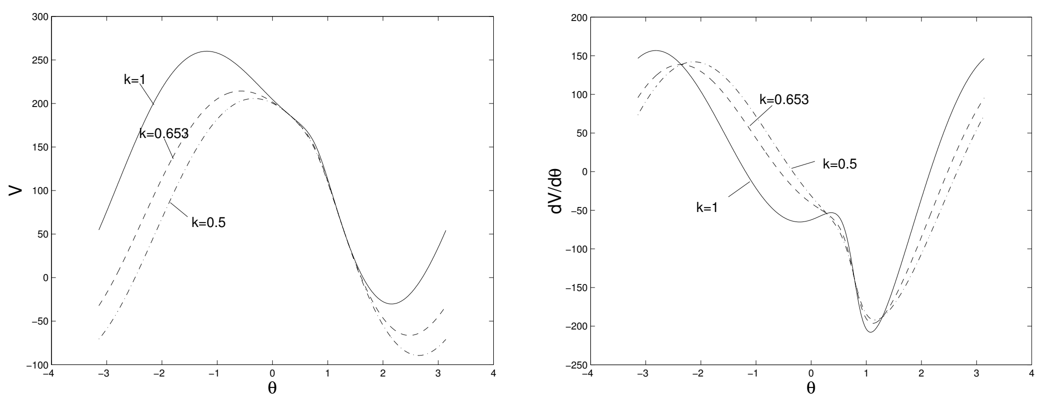


Figure 2: The equilibrium configurations of the planar mechanism with a spring.



(a)

(b)

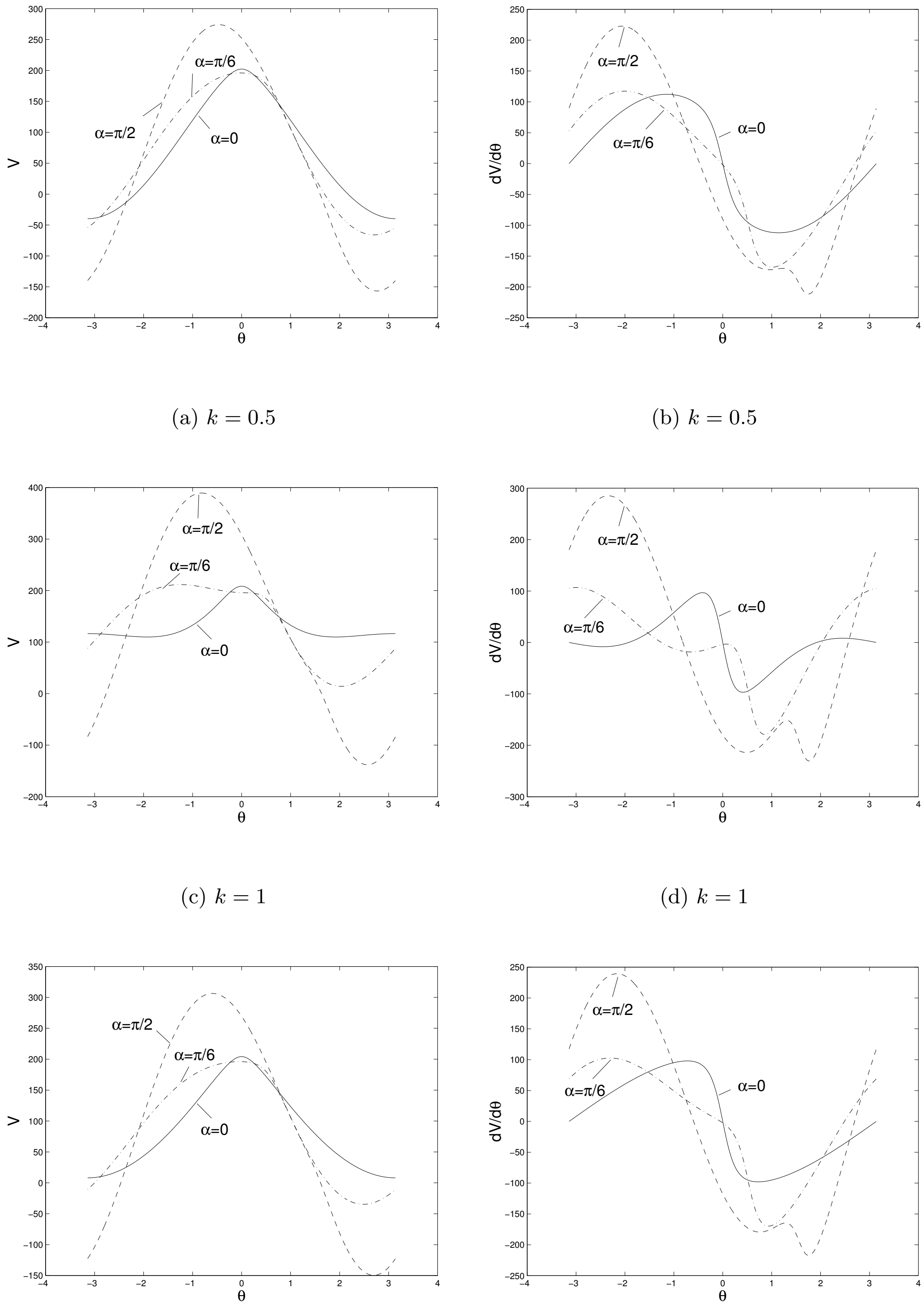
Figure 3: The curves of the functions V and $\frac{dV}{d\theta}$ for the planar one-link mechanism with a spring.

3.1 Energy function curve and the equilibrium position of the one-link planar mechanism with a spring

In order to determine the number of real equilibrium positions of the system, one can compute numerical solutions. Substituting the following parameters: $\alpha = \pi/4$, $g = 9.8\ m/s^2$, $m = 2\ kg$, $k = 0.5\ N/cm$, $c = 10\ cm$, $l = 20\ cm$, $h = 15\ cm$, $s^o = 10\ cm$ into Eq. (14), four real solutions for θ can be obtained. However, only two of them satisfy the original equation, i.e., Eq. (10). They are $\theta = -19.1°$ and $\theta = 151.7°$. Fig. 2 shows the equilibrium configurations of this mechanism corresponding to these two solutions. To verify whether the equilibrium positions of the system obtained above are stable, we have to examine the sign of the second derivatives of V with respect to θ. When $\theta = -19.1°$, $\frac{d^2V}{d\theta^2} = -228.4 < 0$, while with $\theta = 151.7°$, $\frac{d^2V}{d\theta^2} = 144.8 > 0$. Hence, we can say that this mechanism has only one stable equilibrium configuration when $\theta = 151.7°$ for the above given parameters. By plotting the curves of the total potential energy function and its first derivative with respect to θ, we can directly obtain the same results, determined by means of the algebraic method. The three different cases $k = 0.5\ N/cm$, $k = 1\ N/cm$ and $k = 0.6533\ N/cm$, i.e., $a_1 > a_2$, $a_1 < a_2$ and $a_1 = a_2$ are considered. The corresponding curves of the functions V and $\frac{dV}{d\theta}$ with $\theta \in [-\pi\ \pi]$ are shown in Fig. 3. From Fig. 3a, it is clear that the function V has a local maximum and a local minimum, within a period 2π, corresponding to each case. When V is a minimum this system is in a stable equilibrium. When V is a maximum this system is in a unstable equilibrium. In addition, it can be seen that the extreme value of V increases with an increase in k. From Fig. 3b, the conditions for the equilibrium position of the system, i.e., the angle θ can also be directly obtained. Furthermore, it can be seen that the value of θ reduces with an increase in k.

3.2 Variation of parameter α

Since the extreme values of V or the points for which $\frac{dV}{d\theta} = 0$ play a very important role in the determination of the equilibrium configurations of the system we will now study its variation under changes in the geometry of the mechanism for different orientations of the spring. The plots of the values of V and $\frac{dV}{d\theta}$ as functions of θ for different values of parameter α are shown in Figs. 4a and 4b for given values of the other parameters in the case $k = 0.5$. Similarly, Figs. 4c – 4f show the



(a) $k = 0.5$

(b) $k = 0.5$

(c) $k = 1$

(d) $k = 1$

(e) $k = 0.653$

(f) $k = 0.653$

Figure 4: Variation of V and $\frac{dV}{d\theta}$ with respect to θ for different values of α.

plots of V and $\frac{dV}{d\theta}$ when $k = 1$ and $k = 0.6533$, respectively. From Figs. 4a, 4c and 4e, the same trend is observed. That is, in the three cases the absolute extreme value of V increases with an increase in α. Furthermore, except for the case in which $\alpha = 0$, the energy function V has only one local maximum and minimum value during the 2π period.

However, from Figs. 4c and 4d, it is seen that there is a special case when $k = 1$ and $\alpha = 0$. In this case, this mechanism has four equilibrium positions. The corresponding angles θ are 0, π, $110.2°$ and $-110.2°$. When $\theta = \pm 110.2°$ this system is in stable equilibrium positions, as shown in Fig. 5. In fact, when $\alpha = 0$ Eq. (10) can be rewritten as

$$(-a_1 + a_2 - \frac{a_3}{s}) \sin\theta = 0 \tag{15}$$

which leads to two distinct cases which correspond to the different equilibrium positions of the system. These cases are obtained with

$$s = \frac{a_3}{(-a_1 + a_2)} \quad \text{and} \quad \sin\theta = 0 \tag{16}$$

In the first condition of Eq. (16), since s and a_3 are always positive, we impose the condition $a_2 > a_1$, i.e., $k > mgc/hl$. When this condition is satisfied, i.e., $k = 1$, two solutions for θ are obtained, i.e., $\theta = \pm 110.2°$ which correspond to the two stable equilibrium positions. The second condition of Eq. (16) also leads to two solutions for θ which are 0 and π. They correspond to the two unstable equilibrium positions in which the fixed link and moving link are collinear along the vertical direction. Therefore, for the planar mechanism with a spring, four equilibrium configurations are obtained if and only if $\alpha = 0$ and $k > mgc/hl$. In other words, when the fixed link attached to the end of the spring is located along the vertical axis and the spring is stiff enough, the four equilibrium configurations of the system can be obtained. Otherwise, the system has only two equilibrium configurations.

4 Equilibrium Analysis of a 2-DOF Planar Parallel Mechanism with a Spring

In order to design a mechanism with multiple equilibrium configurations, a 2-DOF parallelogram mechanism with one spring is proposed, as represented in Fig. 6. This five-bar mechanism consists

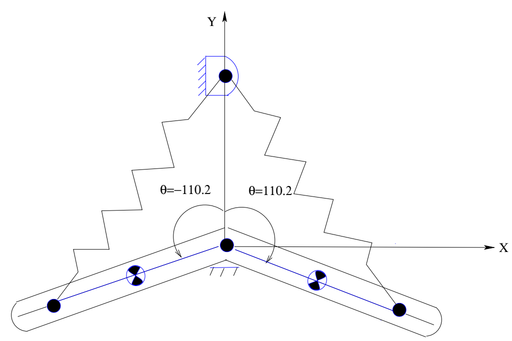


Figure 5: Two stable equilibrium positions of of the mechanism when $\alpha = 0$ and $k = 1$.

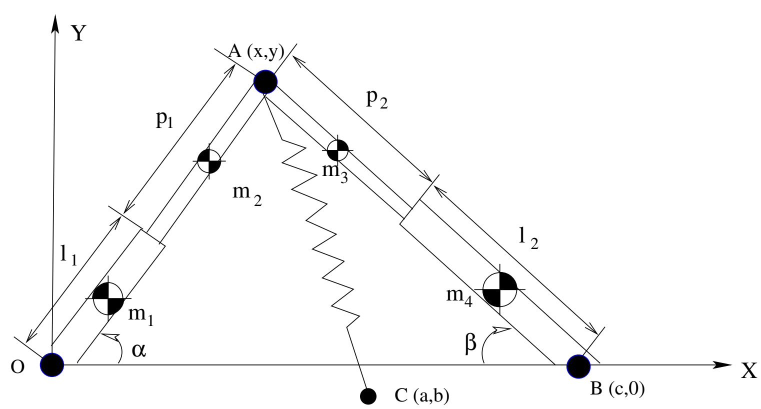


Figure 6: The 2-DOF planar parallel mechanism with a spring.

of two prismatic joints, three revolute joints and one spring. For simplicity, the angles α and β which are defined as the angles between the first moving element $l_i, i = 1, 2$ and the X-axis are chosen as the two input variables of the system and the undeformed length of the spring is assumed to be zero. From the geometry of the mechanism, we have

$$x = \frac{\cos\alpha \sin\beta}{\sin(\alpha+\beta)} c \tag{17}$$

$$y = \frac{\sin\alpha \sin\beta}{\sin(\alpha+\beta)} c \tag{18}$$

and the y component y_i of the position vector of the center of mass of the ith element is

$$y_1 = \frac{1}{2} l_1 \sin\alpha \tag{19}$$

$$y_2 = \frac{1}{2}(y + l_1 \sin\alpha) \tag{20}$$

$$y_3 = \frac{1}{2}(y + l_2 \sin\beta) \tag{21}$$

$$y_4 = \frac{1}{2} l_2 \sin\beta \tag{22}$$

Since the system uses a zero free-length spring the total potential energy V can be written as

$$V = m_1 g y_1 + m_2 g y_2 + m_3 g y_3 + m_4 g y_4 + \frac{1}{2} k s^2 \tag{23}$$

where $s = \sqrt{(x-a)^2 + (y-b)^2}$.

The first partial derivatives of V with respect to α and β are

$$\frac{\partial V}{\partial \alpha} = \frac{A^3 t_1 \cos\alpha + A t_2 \sin^2\beta + kc \sin\beta[(a\cos\beta - b\sin\beta)A - c\sin\beta\cos(\alpha+\beta)]}{A^3} \tag{24}$$

$$\frac{\partial V}{\partial \beta} = \frac{A^3 t_3 \cos\beta + A t_2 \sin^2\alpha + kc \sin\alpha[-(a\cos\alpha + b\sin\alpha)A + c\sin\beta]}{A^3} \tag{25}$$

where

$$A = \sin(\alpha+\beta) \tag{26}$$

$$t_1 = (m1 + m2) g l_1/2 \tag{27}$$

$$t_2 = (m2 + m3) g c/2 \tag{28}$$

$$t_3 = (m3 + m4) g l_2/2 \tag{29}$$

When the numerators of Eqs. (24) and (25) vanish, we can get the conditions for equilibrium of the mechanism. Using the method of numerical solutions and choosing the following parameters:

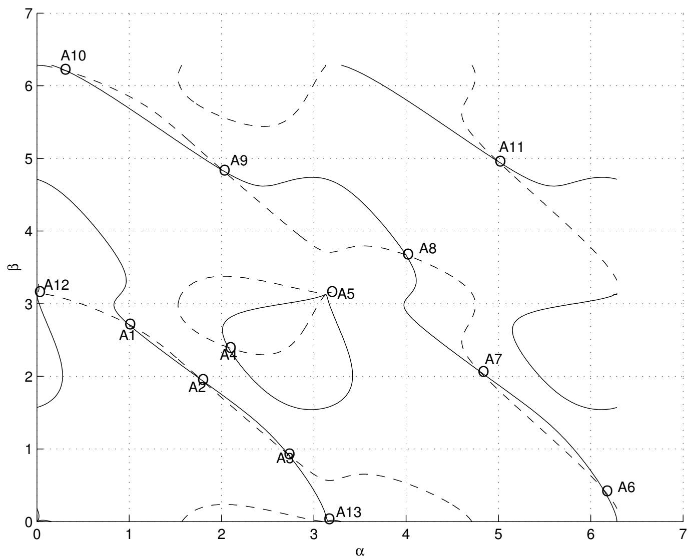
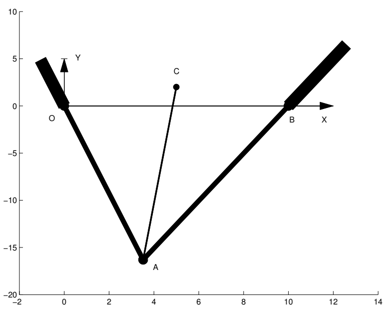


Figure 7: Curves of the equilibrium of the 2-DOF planar parallel mechanism with a spring.

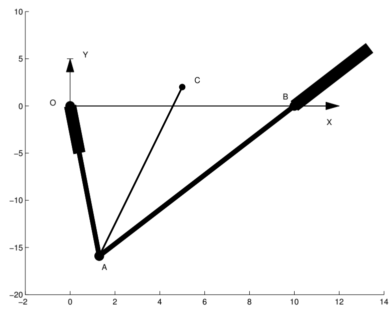
$g = 9.8\ m/s^2$, $m_1 = m_2 = m_3 = m_4 = 2\ kg$, $k = 1\ N/cm$, $a = 5\ cm$, $b = 2\ cm$, $l_1 = 5\ cm$, $l_2 = 7\ cm$, $c = 10\ cm$, two curves corresponding to the above equations $\frac{\partial V}{\partial \alpha} = 0$ and $\frac{\partial V}{\partial \beta} = 0$, are plotted in Fig. 7. In this figure, the continuous line represents $\frac{\partial V}{\partial \alpha} = 0$ while the dashed line represents $\frac{\partial V}{\partial \beta} = 0$. It is clear that there are thirteen intersection points $A_i, (i = 1, ..., 13)$, that is, these two equations have thirteen real solutions for α and β. Using the SIMULINK software the coordinates of these intersection points are obtained and are given in Table 1. By examining the sign of the second derivatives of V four stable equilibrium positions, i.e., cases 2,7,9,11 are obtained. Note that in the cases 5, 12, 13, the denominators of Eqs. (24), (25) become zero simultaneously, i.e., $A = 0$. In these cases, we have $y_1 = y_2 = y_3 = y_4 = y = 0$ and the expression for V becomes

$$V = \frac{1}{2}k(x - a)^2 \tag{30}$$

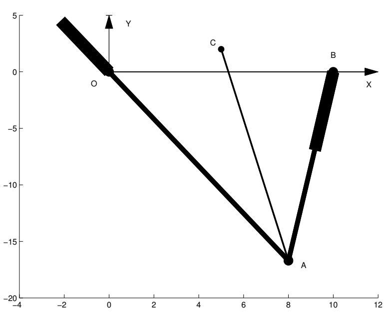
It is obvious that V has a minimum value when $x = a$ and the mechanism is in the stable equilibrium position since $\frac{\partial V^2}{\partial^2 x} = k > 0$. Fig. 8 illustrate these equilibrium positions of the mechanism. Therefore, this mechanism has seven stable equilibrium positions. In this example, since an ideal spring is used and the angles α and β are judiciously selected as the input variables, the complexity of the equilibrium problem of the mechanism reduced drastically and the number of the equilibrium positions increased to thirteen.



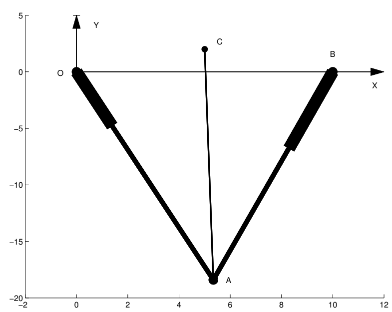
(a) case 2



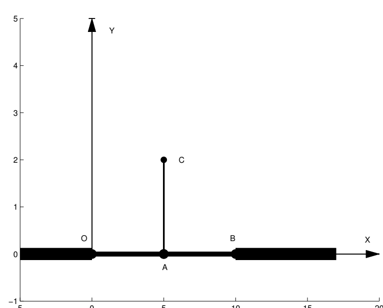
(b) case 7



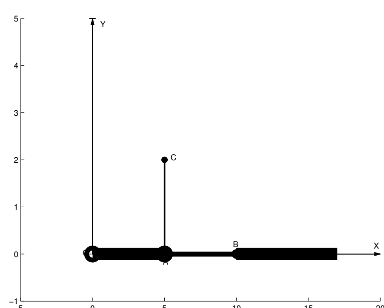
(c) case 9



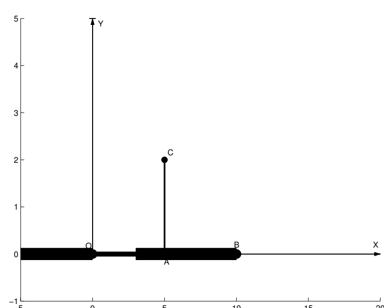
(d) case 11



(e) case 5



(f) case 12



(g) case 13

Figure 8: Equilibrium positions of the mechanism in seven cases.

Table 1: Equilibrium positions of the mechanism

Coordinates of A_i (rad)	Derivatives of V	Equilibrium Status
A_1 (0.9881, 2.72)	$D_1 > 0$; $D_2 < 0$	nonstable equilibrium
A_2 (1.784, 1.95)	$D_1 > 0$; $D_2 > 0$	stable equilibrium
A_3 (2.723, 0.9237)	$D_1 > 0$; $D_2 < 0$	nonstable equilibrium
A_4 (2.073, 2.398)	$D_1 < 0$; $D_2 < 0$	nonstable equilibrium
A_5 (3.142, 3.142)	singular formula	stable equilibrium
A_6 (6.133, 0.4497)	$D_1 > 0$; $D_2 < 0$	nonstable equilibrium
A_7 (4.795, 2.071)	$D_1 > 0$; $D_2 > 0$	stable equilibrium
A_8 (3.997, 3.672)	$D_1 > 0$; $D_2 < 0$	nonstable equilibrium
A_9 (2.018, 4.831)	$D_1 > 0$; $D_2 > 0$	stable equilibrium
A_{10} (0.32, 6.205)	$D_1 > 0$; $D_2 < 0$	nonstable equilibrium
A_{11} (4.995, 4.96)	$D_1 > 0$; $D_2 > 0$	stable equilibrium
A_{12} (0, 3.142)	singular formula	stable equilibrium
A_{13} (3.142, 0)	singular formula	stable equilibrium

5 Conclusion

In this paper, the equilibrium configurations of two spring mechanisms have been studied. The spring used in the first mechanism has non-zero free-length. This is different from previous works and has more practical relevance. However, in the last example, i.e., the 2-DOF planar parallel mechanism with prismatic actuators, an ideal spring has been used in order to simplify the equilibrium problem. For each spring mechanism, the conditions under which the system is in equilibrium have been obtained by using classical stability theory. In addition, the plots of the total potential energy and its first derivative functions have been used to illustrate the equilibrium configurations of the mechanism. Finally, by means of a numerical method, we have concluded that the planar 1-DOF simple mechanism has two equilibrium positions in which only one is stable. For the planar 2-DOF parallel mechanism, since an ideal spring is used thirteen equilibrium positions are obtained in which seven are stable. These results have significance in practical equilibrator design.

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