Design Manifold of Translational Parallel Manipulators

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Abstract

In this work, a representation method of translational parallel manipulators of three degrees of freedom is presented. The representation is based on an initial configuration. Topology and geometry are represented by joint twists and topology codes. The necessary constraints on topology and geometry are deduced to obtain the desired mobility. Using the representation method with the necessary constraints, a design manifold of TPMs of 3 DOF is formulated from which three example of TPM are reported.

Résumé

Cette article présente une méthode de représentation des manipulators parallèles en translation (MPT) à trois degrés de liberté. Cette représentation est basée sur une configuration initiale. La topologie et la géométrie sont représentées par le torseur de vitesse de chaque articulation et un code de topologie. Les contraintes sur la topologie et la géométrie sont ensuite déduites afin d'obtenir la mobilité requise. En utilisant cette méthode et les contraintes nécessaires, un manifold de MTP à trois degrés de liberté est formulé pour lequel trois exemples de MTP sont rapportés.

1 Introduction

In general, parallel manipulators (PMs) have a much higher payload, stiffness, speed and accuracy compared to serial manipulators (SMs) of equivalent size. These complementary characteristics make them suitable in the situations where SMs cannot satisfy the application requirements [1].

Among PMs of 3 degrees of freedom (DOF), translational PMs (TPMs) have intensively been studied. The early designs of TPMs are probably: the Delta [2], the Y-Star [3], the UPU [4], the PM with only translational DOF [5], the PUU and RUU [6], and the Orthoglide [7]. More recent, TPMs are those generated based on Screw Theory [8] and a Cartesian PM [9]. Topologies of orientational 3 DOF PMs have been proposed [10][11][12]. PMs with 3 mixed DOF in translation and orientation have also been proposed [13, 14]. Kinematic modeling for TPMs, e.g. UPU [15], Y-Star [16] and Delta PM [17], are specific to only one topology. So, given a kinematic model, kinematic design is limited to one topology. This makes the implementation of optimization algorithms, genetic algorithms for instance, very difficult when we want to take topologies as design variables. In the following sections, a representation method is presented, questions concerning mobilities are discussed and a design manifold is formulated.

2 Representation of PMs

For the existing representation methods of PMs, a PM is always represented by its kinematical topology and geometric parameters defining the end effector (EE), the base and all other links. When implementing certain optimization algorithms, stochastic ones for instance, using these methods, choosing an initial configuration is quite problematic. First of all, for each PM, there are a number of assembly modes. Then for each assembly mode, there are a number of aspects in its workspace. The number of assembly modes itself may not be realistically dealt with when the algorithms take both topological and geometric information as input variables. Then searching a non-singular initial configuration for a chosen assembly mode is rather time consuming due to the closed form nature of the kinematic chains. It is also a problem to know whether the EE, the base and the links defined by parameters generated in the course of optimization can be assembled and whether the assembled PM can possibly have the desired mobility.

To address the above problems, an initial configuration is introduced. It is the parameters defining topology and geometry of all links with an initial configuration together that represent the PM. The EE of PMs of 3 DOF is linked to the base by three kinematic chains. So a PM can be interpreted as three serial manipulators (SMs) sharing a single EE. As far as the i^{th} serial kinematic chain (SKC) is concerned, the twist $\mathbf{t}$ of the EE is a function of its joint variables $\mathbf{q}_i = [\; q_{i1} \;\; q_{i2} \;\; \cdots \;\; q_{in} \;]^T$

$$\mathbf{t} = \mathcal{F}_i(\mathbf{q}_i)$$

The closing of these SKCs exerts motion constraints on the EE. These constraints can be represented by the following equations.

$$\mathcal{F}_1(\boldsymbol{q}_1) = \mathcal{F}_2(\boldsymbol{q}_2) \tag{1}$$

$$\mathcal{F}_2(\boldsymbol{q}_2) = \mathcal{F}_3(\boldsymbol{q}_3) \tag{2}$$

Eq.(1) and (2) represent twelve scalar equations. So the DOF of the mechanism are $3 \times n - 12$. For PMs to possess 3 DOF, the number of joint variables is obviously 5.

In view of a SKC, the twist $\mathbf{t}$ of the EE has the form of

$$\begin{aligned} \mathbf{t} &= \mathbf{J}_s \dot{\mathbf{q}} \\ \begin{bmatrix} \boldsymbol{\omega} \\ \dot{\mathbf{p}} \end{bmatrix} &= \underbrace{\begin{bmatrix} \mathbf{A} \\ \mathbf{B} \end{bmatrix}}_{6\times 5} \dot{\mathbf{q}} \end{aligned} \tag{3}$$

For the initial configuration, let

$$\mathbf{A} \equiv \begin{bmatrix} \boldsymbol{a}_1 & \boldsymbol{a}_2 & \boldsymbol{a}_3 & \boldsymbol{a}_4 & \boldsymbol{a}_5 \end{bmatrix}$$
$$\mathbf{B} \equiv \begin{bmatrix} \boldsymbol{b}_1 & \boldsymbol{b}_2 & \boldsymbol{b}_3 & \boldsymbol{b}_4 & \boldsymbol{b}_5 \end{bmatrix}$$

$[\ \boldsymbol{a}_i \ \ \boldsymbol{b}_i\ ]^T$ is the normalized i^{th} joint twist, i.e., the twist of the EE due to i^{th} joint only. Then the topology and geometry information can be obtained from the joint twists at this configuration. Introducing a second index to distinguish between the SKCs and $\hat{\mathbf{T}}_{ij}$ as the Lie Group matrix corresponding to $[\ \boldsymbol{a}_{ij} \ \ \boldsymbol{b}_{ij}\ ]^T$, the kinematic model is as following

$$\mathbf{T} = \mathbf{T}_0 e^{\hat{\mathbf{T}}_{11}q_{11}+\hat{\mathbf{T}}_{21}q_{21}+\hat{\mathbf{T}}_{31}q_{31}+\hat{\mathbf{T}}_{41}q_{41}+\hat{\mathbf{T}}_{51}q_{51}} \tag{4}$$

$$\mathbf{T} = \mathbf{T}_0 e^{\hat{\mathbf{T}}_{12}q_{12}+\hat{\mathbf{T}}_{22}q_{22}+\hat{\mathbf{T}}_{32}q_{32}+\hat{\mathbf{T}}_{42}q_{42}+\hat{\mathbf{T}}_{52}q_{52}} \tag{5}$$

$$\mathbf{T} = \mathbf{T}_0 e^{\hat{\mathbf{T}}_{13}q_{13}+\hat{\mathbf{T}}_{23}q_{23}+\hat{\mathbf{T}}_{33}q_{33}+\hat{\mathbf{T}}_{43}q_{43}+\hat{\mathbf{T}}_{53}q_{53}} \tag{6}$$

where $\mathbf{T}$ is the homogeneous coordinate of the EE and $\mathbf{T}_0$ is its position at initial configuration. Based on this representation, the design manifold can be formulated. But first, we will analyze the necessary conditions for the represented PMs to possess the required mobility.

3 Constraint Analysis

For $m \times n$ matrix $\mathbf{A}$, the following equations always hold [18].

$$\mathbf{A} = \mathbf{U} \underbrace{\begin{bmatrix} \sigma_1 & 0 & \cdots & 0 & \cdots & 0 \\ 0 & \sigma_2 & \cdots & 0 & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & \cdots & \sigma_r & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & \cdots & 0 & \cdots & 0 \end{bmatrix}}_{m\times n} \mathbf{V}^T$$

$$\mathbf{U} = \underbrace{\begin{bmatrix} \mathbf{u}_1 & \mathbf{u}_2 & \cdots & \mathbf{u}_m \end{bmatrix}}_{m\times m}$$

$$\mathbf{V} = \underbrace{\begin{bmatrix} \mathbf{v}_1 & \mathbf{v}_2 & \cdots & \mathbf{v}_n \end{bmatrix}}_{n\times n}$$

$$Null(\mathbf{A}) = span\{\ \mathbf{v}_{r+1}\ ,\ \mathbf{v}_{r+2}\ ,\ \cdots\ ,\ \mathbf{v}_n\ \} \tag{7}$$

$$Rank(\mathbf{A}) = r \tag{8}$$

where σ_i is the i^{th} diagonal element and

$$\sigma_1 \geq \sigma_2 \geq \cdots \geq \sigma_r > 0$$
$$\sigma_{r+1} = \sigma_{r+2} = \cdots = \sigma_m = 0$$

U and **V** are orthogonal matrices. From eq. (3), for TPM, we have

$$\mathbf{0} = \mathbf{A}\dot{\mathbf{q}} \tag{9}$$

That is, $\dot{\boldsymbol{q}}$ lies in the null space of matrix **A**. From eq.(7), it follows

$$\dot{\mathbf{q}} = \begin{bmatrix} \mathbf{v}_{r+1} & \mathbf{v}_{r+2} & \cdots & \mathbf{v}_n \end{bmatrix} \begin{bmatrix} c_1 \\ c_2 \\ \vdots \\ c_{n-r} \end{bmatrix}$$

where

$$\begin{bmatrix} c_1 & c_2 & \cdots & c_{n-r} \end{bmatrix}^T \subset \mathbb{R}^{(n-r)\times 1}$$
$$\| \begin{bmatrix} c_1 & c_2 & \cdots & c_{n-r} \end{bmatrix}^T \|_2 \neq 0$$

Therefore, the motion of the EE in translation is

$$\dot{\boldsymbol{p}} = \mathbf{B} \begin{bmatrix} \mathbf{v}_{r+1} & \mathbf{v}_{r+2} & \cdots & \mathbf{v}_n \end{bmatrix} \begin{bmatrix} c_1 \\ c_2 \\ \vdots \\ c_{n-r} \end{bmatrix} \tag{10}$$

From eq. (8) and eq. (10), the necessary conditions for a PM to have 3 DOF in translation are only

$$n - r \geq 3 \Longleftrightarrow Rank(\mathbf{A}) \leq n - 3 = 2 \tag{11}$$
$$Rank(\mathbf{B}) = 3 \tag{12}$$

By the same way, to ensure a PM has 3 DOF in orientation, the necessary conditions are

$$Rank(\mathbf{A}) = 3 \tag{13}$$
$$Rank(\mathbf{B}) \leq n - 3 = 2 \tag{14}$$

For cases other than those above, PMs may have 3 DOF, but the orientation and translation of PMs are always coupled.

4 Design Manifold

Each SKC of TPM is a mechanical system in which rigid bodies, called links, are coupled one after another by lower kinematic pairs (also called joints). There are six such pairs, namely, revolute (R), prismatic (P), cylindrical (C), helical (H), planar (E) and spherical (S). In this work, the C joint is represented by a combination of a R joint and a P joint; the E joint by a combination of two P joints; and the S joint by a combination of three intersecting R joints. This way, we are only have to deal with R, P and H joints. Having established a coordinate system, a joint is represented by its normalized twist, i.e., the normalized twist of the EE generated only by this joint. We designate $[\ f_{ij}\ \ g_{ij}\ ]^T$ as the joint type code of the i^{th} joint of the j^{th} SKC of TMP and $[\ 1\ \ 0\ ]^T$ for R joint, $[\ 0\ \ 1\ ]^T$ for P joint and $[\ 1\ \ d\ ]^T$ for H joint with pitch d. The i^{th} normalized joint twist of the j^{th} SKC is

$$\begin{bmatrix} f_{ij}\mathbf{e}_{ij} \\ f_{ij}\mathbf{m}_{ij} + g_{ij}\mathbf{e}_{ij} \end{bmatrix} = \begin{bmatrix} \mathbf{e}_{ij} & 0 \\ \mathbf{m}_{ij} & \mathbf{e}_{ij} \end{bmatrix} \begin{bmatrix} f_{ij} \\ g_{ij} \end{bmatrix}$$

where

$$\mathbf{G}_{ij} \equiv \begin{bmatrix} \mathbf{e}_{ij} & 0 \\ \mathbf{m}_{ij} & \mathbf{e}_{ij} \end{bmatrix}$$

represents the joint axis location with unit vector $\mathbf{e}_{ij}$ representing the orientation of the axis and vector $\mathbf{m}_{ij}$ as the moment of $\mathbf{e}_{ij}$ on the axis relative to the origin of coordinate system.

$$\mathbf{Q}_{ij} \equiv \begin{bmatrix} f_{ij} \\ g_{ij} \end{bmatrix}$$

is joint type code. The design manifold of TPMs is thus formulated as a function

$$\mathcal{M}_j(\{\mathbf{e}_{ij}\}, \{\mathbf{m}_{ij}\}, \{f_{ij}\}, \{g_{ij}\}) = \begin{bmatrix} f_{1j}\mathbf{e}_{1j} & f_{1j}\mathbf{m}_{1j} + g_{1j}\mathbf{e}_{1j} \\ f_{2j}\mathbf{e}_{2j} & f_{2j}\mathbf{m}_{2j} + g_{2j}\mathbf{e}_{2j} \\ f_{3j}\mathbf{e}_{3j} & f_{3j}\mathbf{m}_{3j} + g_{3j}\mathbf{e}_{3j} \\ f_{4j}\mathbf{e}_{4j} & f_{4j}\mathbf{m}_{4j} + g_{4j}\mathbf{e}_{4j} \\ f_{5j}\mathbf{e}_{5j} & f_{5j}\mathbf{m}_{5j} + g_{5j}\mathbf{e}_{5j} \end{bmatrix}^T \tag{15}$$

under the following constraints

$$Rank \begin{bmatrix} f_{1j}\mathbf{e}_{1j} \\ f_{2j}\mathbf{e}_{2j} \\ f_{3j}\mathbf{e}_{3j} \\ f_{4j}\mathbf{e}_{4j} \\ f_{5j}\mathbf{e}_{5j} \end{bmatrix}^T \leq 2 \tag{16}$$

$$Rank \begin{bmatrix} f_{1j}\mathbf{m}_{1j} + g_{1j}\mathbf{e}_{1j} \\ f_{2j}\mathbf{m}_{2j} + g_{2j}\mathbf{e}_{2j} \\ f_{3j}\mathbf{m}_{3j} + g_{3j}\mathbf{e}_{3j} \\ f_{4j}\mathbf{m}_{4j} + g_{4j}\mathbf{e}_{4j} \\ f_{5j}\mathbf{m}_{5j} + g_{5j}\mathbf{e}_{5j} \end{bmatrix}^T = 3 \tag{17}$$

Fig. 1 to Fig. 4 are examples of TPMs generated by the manifold.

5 Conclusion

An initial configuration of PMs can be represented by their normalized joint twists. PMs are thus represented by their normalized twists at its initial configuration. With this representation, the mobility can be readily analyzed and the kinematic model derived. The representation is actually a matrix; the design manifold is a generator of such matrices. The necessary conditions for PMs to have only translational degrees of freedom are deduced, a design manifold of TPMs is formulated. The sufficient conditions for PMs to have 3 DOF in translation only, the actuating scheme and the presence of redundant joints are to be dealt with in our future work.

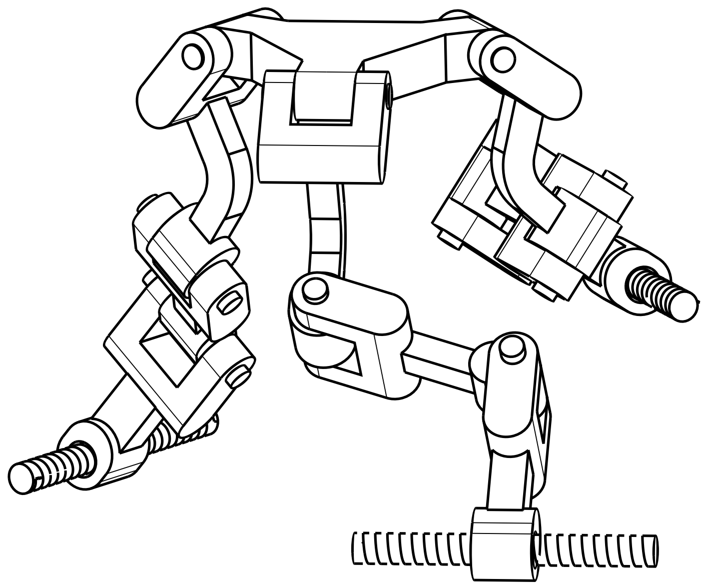

Figure 1: Example no. 1

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References

[1] Angeles, J., 1997, *Fundamentals of Robotic Mechanical Systems*, Springer-Verlag, New York, 510.

[2] Clavel, R., 1985, *Device For Displacing and Positioning an Element in Space*, International patent, No. WO 87/03528.

[3] Hervé, J. M. and Sparacino, F., 1992, Star, "A New Concept in Robotics", *Third Int. Workshop on Advances in Robot Kinematics*, pp. 180–183.

[4] Tsai, L.-W., 1996, "Kinematics of a Three-DOF Platform With Extensible Limbs", *Recent Advances in Robot Kinematics*, J. Lenarcic and V. Parenti-Castelli (eds.), Kluwer Academic Publishers, pp. 401-410.

[5] Tsai, L.-W, Stamper, R. E. 1997, *A Parallel Manipulator with Only Translational Degrees of Freedom*, Technical Research Report, T.R. 97-72, The Institute for Systems Research, University of Maryland, U.S.A.

[6] Tsai, L.-W., 1999, "The Enumeration of a Class of Three-DOF Parallel Manipulators", *10th World Congress on the Theory of Machine and Mechanisms, Olulu, Finland*, June 20-24.

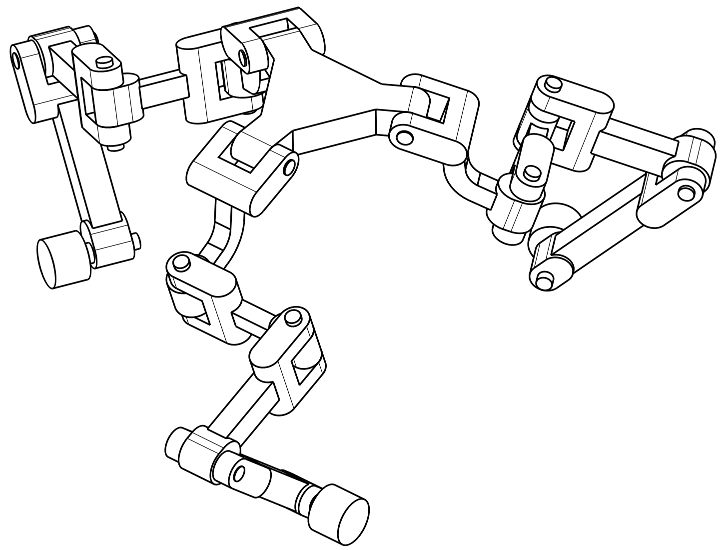

Figure 2: Example no. 2

[7] Wenger P. and Chablat, D., 2000, "Kinematic Analysis of a New Paralle Machine Tool : The Orthoglide", *Advances in Robot Kinematics*, edited by J. Lenarcic and M.M. Stanisic, published by Kluwer Academic Publishers, pp. 305-314.

[8] Kong, X., Gosselin, C.M., 2001, "Generation of Parallel Manipulators with Three Translational Degrees of Freedom Based on Screw Theory", *Symposium 2001 sur les mécanismes, les machines et la mécatronique de CCToMM*, June 1, 2001, the Canadian Space Agency, Saint-Hubert, Montréal, Québec, Canada

[9] Kim, H. S., and Tsai, L.-W., 2002, "Evaluation of a Cartesian Parallel Manipulator", *Advances in Robot Kinematics, Theory and Applications*, edited by J. Lenarcic and F. Thomas, published by Kluwer Academic Publishers, pp. 19-28.

[10] Gosselin, C. M., Sefrioui, J., and Richard, M. J., 1990, "On the direct kinematics of spherical three-degree-of-freedom parallel manipulators", *Transactions of the ASME, Journal of Mechanical Designs*, Vol. 116, No. 2, pp. 594-598

[11] Gosselin C., and Hamel J.F., 1994, "The agile eye: a high performance three-degree-of freedom camera-orienting device", *IEEE Int. Conference on Robotics and Automation*, San Diego, pp. 781-787.

[12] Karouia, M. and Hervé, J. M., 2002, "A Family of Novel Orientational 3-DOF Parallel Robots", *CISM-IFToMM RoManSy Symposia*, pp. 359-368, Udine, Italy, pp. 359-368.

[13] Carretero, J. A., Nahon, M. and Podhorodeski, R. P., 1998 "Workspace analysis of a 3-dof parallel mechanism" , *Proceedings of the 1998 IEEE/RSJ Intl. Conference on Intelligent Robots and System*, Victoria, B. C., Canada, October 1998.

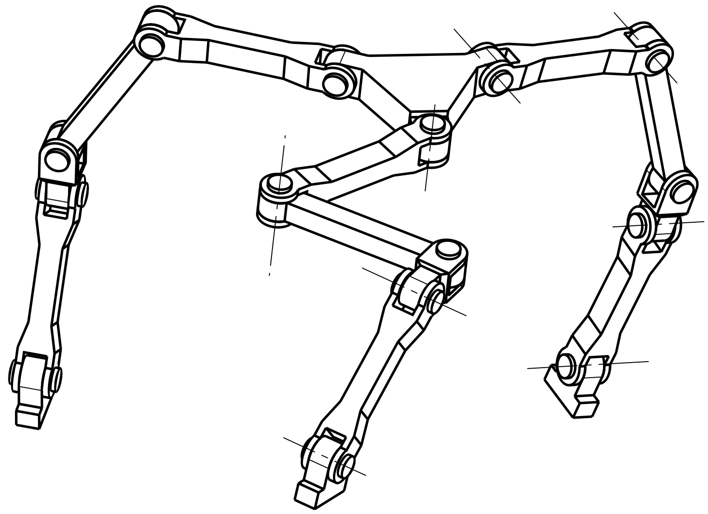

Figure 3: Example no. 3

[14] Liu, X., Wang, J., Gao, F. and Wang, L., 2001, "On the Analysis of a New Spatial Three-Degree-of-Freedom Parallel Manipulator" *IEEE TRANSACTION ON ROBOTICS AND AUTOMATION*, Vol. 17, No. 6 DECEMBER 2001.

[15] Tsai, L.-W., 2000, "Kinematics and Optimization of a Spatial 3-UPU Parallel Manipulator," *ASME Journal of Mechanical Design*, Vol. 122, pp. 439-446, December.

[16] Baron, L., 2001, "Workspace-Based Design of Parallel Manipulators of Star Topology with a Genetic Algorithm", *ASME 27th Design Automation Conference*, Pittsburg, September.

[17] Baron, L., Wang, X., and Cloutier, G., 2002, "The isotropic conditions of parallel manipulators of Delta topology", *Advances in Robot Kinematics, Theory and Applications*, edited by J. Lenarcic and F. Thomas, published by Kluwer Academic Publishers, pp. 357-367.

[18] Golub, G. H., Loan, C. F. V., 1991, *MATRIX COMPUTATIONS*, The Johns Hopkins University Press, Baltimore and London, 642.

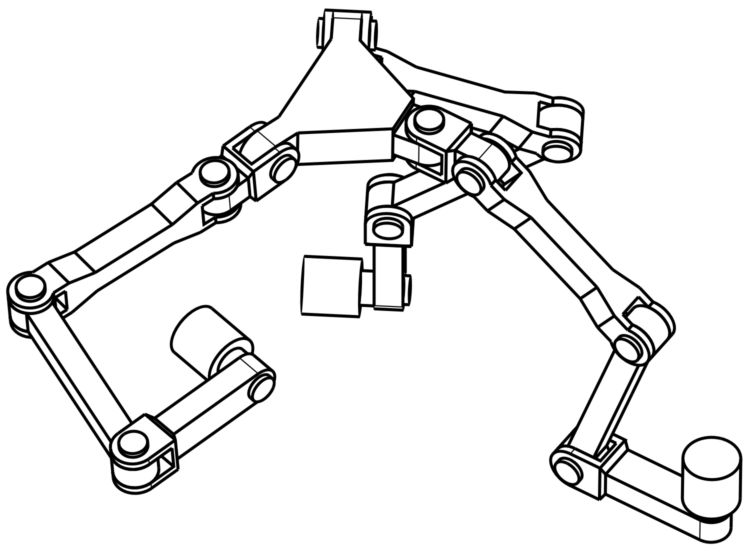

Figure 4: Example no. 4