Suppression of Lateral Vibration of an Axially-Moving Cantilever Beam with a Piezoelectric Actuator

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Abstract

A pair of piezoelectric (PZT) plates are bonded on an axially moving cantilever beam to form a control actuator. The model governing lateral vibration of the beam is presented. A direct velocity feedback is employed. Spillover instability of the closed-loop system is investigated. Three control strategies are proposed.

1. Introduction

A typical example of an axially moving cantilever beam is flexible robotic manipulators with a prismatic joint. Other similar systems include extendable appendages, elevator cables, satellite tethers, and extruding work pieces. Dynamics of lateral vibration of an axially moving cantilever beam has been well understood [1]. An interesting behavior of such a system is "unstable shortening" [2]. Control of vibration of such systems poses some challenges. Normally traditional actuators have to be installed at a stationary position, which may interfere traveling of the media. Piezoelectric (PZT) actuators provide a promising alternative because they can be easily applied to a traveling media such as beam or band with little added mass. Many studies have been reported on the use of PZT actuators to control vibration of a stationary beam [3]. However, there have been few studies on the application of the PZT technology to a moving beam. A Master thesis research has been conducted at Lakehead University to investigate vibration suppression of an axially moving cantilever beam using a PZT actuator. This paper gives a brief report of the study.

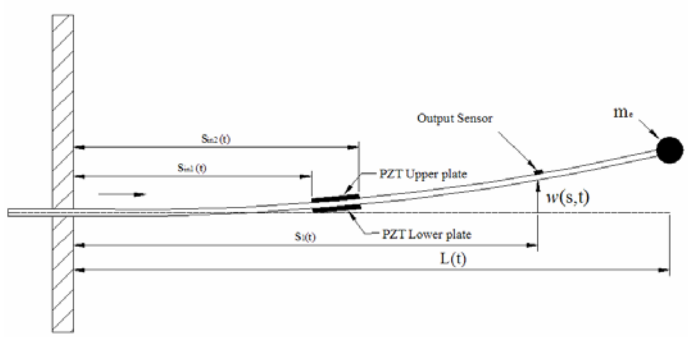

Figure 1. An axially moving cantilever beam

2. Modeling

Figure 1 shows a schematic of an axially traveling beam attached by an end mass m_e. The lateral deflection of the beam is denoted as $w(s,t)$ where s and t represent the axial location and time, respectively. A pair of PZT patches is placed over the region $s_{in1} = L - r_{in1}$ to $s_{in2} = L - r_{in2}$. A sensor is attached at $s_1 = L - r_1$. Note that r_{in1}, r_{in2}, and r_1 are constant. The lateral deflection is expressed as a sum of the first three modal components

$$w(s,t) = \sum_{i=1}^{3} \phi_i(s) q_i(t) = \Phi q \tag{1}$$

where $\Phi = [\phi_1 \quad \phi_2 \quad \phi_3]$ and $q = [q_1 \quad q_2 \quad q_3]^T$, $\phi_i = 1/\sqrt{L}\psi_i(\alpha)$ with $\psi_i(\alpha)$ as the ith mode shape of a stationary cantilever beam and $\alpha = s/L$. A lengthy derivation results in the equation of motion for the system

$$M(t)\ddot{q}(t) + D(t)\dot{q}(t) + K(t)q(t) = B_1(t)V(t) \tag{2}$$

where $M(t)$, $D(t)$, and $K(t)$ are functions of L, $\dot{L}$, and $\ddot{L}$, $V(t)$ is the voltage applied to the PZT plates, $B_1(t)$ is given as

$$B_1(t) = g_p \frac{1}{L}\frac{1}{\sqrt{L}}\left[\Psi'(\alpha_{in2}) - \Psi'(\alpha_{in1})\right]^T \quad (3)$$

where g_p is a constant related to the properties of the PZT material.

3. Velocity Feedback Control

If a velocity feedback is used, the voltage $V(t)$ is computed by

$$V(t) = -g_v \frac{Dw(s_1,t)}{Dt} = -g_v\left[\frac{\dot{L}}{L}\frac{1}{\sqrt{L}}\left[-\frac{1}{2}\Psi(\alpha_1) + (1-\alpha_1)\Psi'(\alpha_1)\right]q + \frac{1}{\sqrt{L}}\Psi(\alpha_1)\dot{q}\right] \quad (4)$$

Substituting $V(t)$ in Eq. (2) yields the equation of motion governing the dynamics of the closed-loop system

$$M(t)\ddot{q}(t) + \bar{D}(t)\dot{q}(t) + \bar{K}(t)q(t) = 0 \quad (5)$$

where $\bar{D}(t) = D(t) + B_1(t)g_v\frac{1}{\sqrt{L}}\Psi(\alpha_1)$, $\bar{K}(t) = K(t) + B_1(t)g_v\frac{\dot{L}}{L}\frac{1}{\sqrt{L}}\left[-\frac{1}{2}\Psi(\alpha_1) + (1-\alpha_1)\Psi'(\alpha_1)\right]$. Clearly, a velocity feedback modifies the damping and stiffness matrices. Although Lyapunov stability theory may offer some insight into the stability of the system defined by Eq. (5), an analytical solution is not possible in general. To understand the stability problem, the concept of "frozen" systems is adopted here. A "frozen" system is a stationary cantilever beam corresponding to any axial position at which the beam may stop. To further simplify the problem, the end mass is assumed to be zero. The parameter matrices of the "frozen" closed-loop system become:

$$M = \mu I_{3\times3},\ \bar{D} = g_p\frac{1}{L}\frac{1}{\sqrt{L}}\left[\Psi'(\alpha_{in2}) - \Psi'(\alpha_{in1})\right]^T g_v\frac{1}{\sqrt{L}}\Psi(\alpha_1),\ \bar{K} = \frac{EI}{L^4}\Lambda \quad (6)$$

where $I_{3\times3}$ is an identity matrix, μ and EI are the mass per length and the flexural rigidity of the beam, respectively and Λ is a diagonal matrix. Assume that the beam length varies between $L = .66$ m to $L = 1.09$ m and the length of the PZT patches are .036 m. To maximize the actuation, the PZT patches are applied over $r_{in1} = .66$ m to $r_{in2} = .624$ m. A computer simulation has been conducted to find the damping ratios $\zeta_1, \zeta_2, \zeta_3$ of the "frozen" system. With a collocated control where $r_1 = .642$ m, it is shown that the first damping ratio is always positive while the second damping ratio is negative, for $.66 \le L \le .82$ m, and the third damping ratio becomes negative, for $.75 \le L \le 1.09$ m. A negative damping ratio indicates an unstable system. Spillover instability is the phenomenon that the feedback system is stable for lower modes, but not for higher modes. It is also found that there is a correlation between the sign of ζ_i and the sign of $\bar{D}_{ii}$ where $\bar{D}_{ii}$ is the diagonal element of $\bar{D}$. Three control strategies have been proposed. 1. Gain-scheduling with two outputs. One sensor is collocated and the other non-collocated. An on-off gain scheduling scheme is designed for each of the outputs such that the damping ratios of the "frozen" system are kept positive for all the beam lengths. 2. Varying gains with two outputs. A relationship between two feedback gains and $\bar{D}_{ii}$ can be found. By prescribing a proper positive value to each of $\bar{D}_{ii}$, the variable gains can be found using a pseudo-inverse algorithm. 3. Varying gains with three outputs. With three outputs, the variable gains can be determined using a matrix inverse instead of a pseudo-inverse.

4. Conclusion

When PZT patches and sensor(s) are applied on the surface of an axially-moving cantilever beam, a collocated and constant gain velocity feedback cannot ensure the stability of the closed-loop system. Three control strategies have been proposed to overcome this problem.

Reference

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