An Approach to Modelling and Analysis of Unilaterally Constrained Mechanical Systems

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ABSTRACT

A novel approach to the dynamic analysis of constrained systems based on the d'Alembert-Lagrange principle was outlined in [1]. In this approach, the space of virtual displacements and generalized velocities is decomposed into admissible and constrained motion subspaces in a physically meaningful way. Vectors in these subspaces are homogeneous in physical units; therefore, a meaningful Euclidean norm exists that is invariant under coordinate transformations. These two subspaces are mutually orthogonal and are spanned by base vectors characterizing the admissible and constrained directions of motion. The formulation is developed using the original non-minimum set of generalized coordinates. It was shown that this method is valid for both holonomic and non-holonomic constraints, and can be used to analyze redundantly constrained systems as well.

In this paper, we propose to apply this approach to the dynamic analysis of unilaterally constrained multibody systems (e.g., a robotic manipulator performing a contact task with a stiff environment). We aim to demonstrate how the forces associated with unilateral constraints affect the dynamics of the whole multibody system during the contact transition phase. In fact, the unilateral nature of the constraints leads to a system with a time-varying number of degrees of freedom. This means that no unique set of independent generalized coordinates exists [2]. In addition, during the periods of contact, the total set of bilateral and unilateral constraints can become redundant. The proposed approach is able to handle this set of constraints. This method will make it possible to separate the dynamics associated with the unilaterally constrained directions without the introduction of a new set of independent variables. This decomposition can significantly facilitate the analysis of contact transitions. After performing the decomposition, the singular perturbation technique may be employed in the space of constrained motion to reduce the complexity of the dynamic equations [3]. This will make the dynamic equations associated with the unilateral constraints easier to investigate. This method will also remove the need of a precise identification of the stiffness of the environment that is usually very difficult to estimate.

The dynamics of the transition phase is also closely related to the interaction control of multibody systems [4]. The control objectives are normally defined in terms of the original non-minimum set of generalized coordinates. As was discussed above, in this approach, the dynamic equations characterizing admissible and constrained motions are also expressed in that set. This opens up the possibility to define the desired force "trajectories" in the constrained directions and the desired position trajectories in the admissible directions as a set of new virtual constraints. It was demonstrated in [1] that the above decomposition

technique can also be employed to stabilize the constraints and filter out the constraint violations. This can make it possible to eliminate the violations to the control constraints (these violations are the position/force tracking errors). This provides an essential element to build advanced control algorithms for the contact transition phase. The approach proposed here can have significant potential in providing a deeper understanding of the dynamic behaviour of unilaterally constrained systems.

References

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