A dependent-screw suppression approach to the singularity analysis of a 7-DOF redundant manipulator: Canadarm2

Xianwen Kong
Département de Génie Mécanique
Université Laval,
Québec, Québec, Canada, G1K 7P4
xwkong@gmc.ulaval.ca

Clément M. Gosselin*
Département de Génie Mécanique
Université Laval,
Québec, Québec, Canada, G1K 7P4
gosselin@gmc.ulaval.ca

Abstract

A dependent-screw suppression approach is proposed for the singularity analysis of 7-DOF (degree-of-freedom) redundant manipulators. This approach is applied to the singularity analysis of the Canadarm2. Five families of singular configurations are identified for the Canadarm2. The singular configurations obtained are identical to those obtained using the reciprocity-based method. Unlike the results presented previously, there are no denominators in the equations describing the singular configurations, and there are also no intersections between any two of the five families of singular configurations described using the equations in this paper.

Résumé

On propose une approche de suppression des visseurs dépendants pour l'analyse des singularités des manipulateurs redondants à 7 degrés de liberté. Cette approche est appliquée à l'analyse des singularités du Canadarm2. Cinq familles de configurations singulières sont identifiées pour le Canadarm2. Les configurations singulières obtenues sont identiques à celles obtenues en utilisant la méthode basée sur la réciprocité. À la différence des résultats présentés précédemment, il n'y a aucun dénominateur dans les équations décrivant les configurations singulières, et il n'y a également aucune intersection entre les cinq familles de configurations singulières décrites en utilisant les équations de cet article.

1 Introduction

Several 7-DOF (degree-of-freedom) redundant manipulators, such as the Canadarm2[1] and anthropomorphic arms [1, 2], have been proposed. Singularity analysis of 7-DOF redundant manipulators is an important issue in the design and control of 7-DOF redundant manipulators and has thus received much attention from many researchers [1–12].

In a singular (or velocity-degenerate) configuration, a redundant manipulator loses at least one DOF, or in other words, its 6×7 Jacobian matrix, $[\mathbf{J}]$, becomes rank deficient [13]. Up to now, several approaches have been proposed for the determination of the singular configurations of redundant manipulators:

(1) Determinant-based approach [3]. Using this approach, the singular configurations are determined using

$$|[J]^T[J]| = 0 \tag{1}$$

where the superscript, ^{T}, represents the transpose of a matrix. The notation, $|\cdot|$, denotes the determinant of a matrix. However, the expression obtained is usually too complex to be used to find symbolic solutions to the singularity analysis.

*Corresponding author.

[1]For the pictures and vedioclips of Canadarm2, please visit the websites of Canadian Space Agency, MD Robotics or NASA.

(2) Cofactor or sub-determinant approach [1, 6, 7]. Using this approach, the singular configurations are determined using

$$\begin{cases} |[J]_{\underline{1}}| = 0 \\ |[J]_{\underline{2}}| = 0 \\ \cdots \\ |[J]_{\underline{7}}| = 0 \end{cases} \tag{2}$$

where $[J]_{\underline{i}}$ denote the matrix obtained from $[J]$ by removing its i-th column. As pointed out in [10], this approach is difficult to use in the singularity analysis of the Canadarm2.

(3) Decomposition approach [4–6]. This method is based on the Gram-Schmidt type decomposition and has been used to the singularity analysis of redundant manipulators with a spherical wrist.

(4) Reciprocity-based approach [2, 7–10]. This approach is based on the fact that in a singular configuration, at least one screw is reciprocal to all the joint screws [14]. The singular configuration of a 7-DOF manipulator can be obtained either geometrically [2] or algebraically [7–10]. The reciprocity-based approach has been used to identify the singular configurations of several types of redundant manipulators [7–10].

(5) The singular vector approach [11, 12]. This approach is a reformulation of the reciprocity-based approach [8-10] using linear algebra terms instead of the reciprocal screw.

In fact, it is not easy to find a general expression of a reciprocal screw for a group of six linearly dependent screws. For example, an expression of a reciprocal screw for a set of six linearly dependent screws under the condition of $s_4 = 0$ was given at row 1 of Table 3 in [10]. It can be verified that this expression of the reciprocal screw is invalid if $s_6 = 0$ and c_5=0 since all the six elements vanish. Meanwhile, it is noticed that an approach that does not use the concept of reciprocal screw [15] is generally more concise than the reciprocal screw based approach [16] in revealing the geometric characteristics of the singular configurations of a class of parallel manipulators. Inspired by the above fact, this paper aims at simplifying the singularity analysis of 7-DOF redundant manipulators. The singularity analysis of the Canadarm2 will be taken as an example to illustrate the proposed approach. The example problem was partially solved in [11] using the singular vector approach and well solved in [10] using the reciprocity-based approach.

This paper is organized as follows. In Section 1, a brief review of the current methods for the singularity analysis of 7-DOF redundant manipulators is presented. In Section 2, a dependent-screw suppression approach is proposed. The singular configurations of the Canadarm2 are identified in Sections 3. Finally, conclusions are drawn.

2 A dependent-screw suppression approach

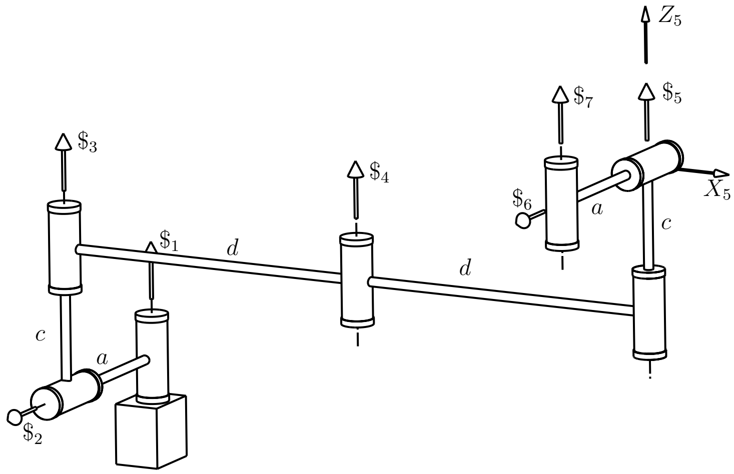
In a singular configuration, the 6×7 matrix of screws, $[\$]=[\$_1\ \$_2\ \$_3\ \$_4\ \$_5\ \$_6\ \$_7]$, of a 7-DOF redundant manipulator becomes rank deficient [13]. Here, $\$_i$ denote the screw of joint i. For convenience, a set of six of the seven screws is called the basic screws. The other screw is called the redundant screw. The submatrix corresponding to the set of basic screws is called the basic submatrix. The singularity analysis of 7-DOF redundant manipulators can be performed using the following two steps.

Step 1 Find the rank deficiency conditions of one basic submatrix. These conditions can be obtained using

$$|[\$]_{\underline{i}}| = 0 \tag{3}$$

where, $[\$]_{\underline{i}}$ represents a basic submatrix which is obtained from $[\$]$ by removing the screw $\$_i$.

To simplify the derivation of the singularity conditions of redundant manipulators, the six basic screws should satisfy the following conditions: (a) they should not be inherently linearly dependent [8], (b) the joints corresponding with the six basic screws should include three successive joints which are equivalent



(a) Zero-displacement configuration.

No.	α_{j-1}	a_{j-1}	d_j	θ_j
1	0	0	0	θ_1
2	$\pi/2$	0	a	θ_2
3	$-\pi/2$	0	c	θ_3
4	0	d	0	θ_4
5	0	d	c	θ_5
6	$\pi/2$	0	a	θ_6
7	$-\pi/2$	0	0	θ_7

(b) Link parameters.

Figure 1: Kinematic representation of the Canadarm2.

to a planar or spherical joint, if any, and (c) the corresponding joints should constitute a serial kinematic chain, if possible, under the first two conditions.

Step 2 Find the additional conditions for the rank deficiency of the screw-matrix for each rank deficiency condition obtained in Step 1. Under each of the rank deficiency conditions obtained in Step 1, the basic submatrix is rank deficient. In other words, the six basic screws are linearly dependent. For most of the 7-DOF redundant manipulators proposed in the literature, at least one basic screw can always be represented as a linear combination of at least one of the other basic screws. For convenience, such a basic screw is called a dependent screw. Thus, the rank deficiency of the screw-matrix is equivalent to the rank deficiency of the screw-matrix with the dependent screw,$\$_k$, suppressed, which is called the suppressed matrix.

Based on the above reasoning, these additional conditions can be found by first determining the dependent screw and then calculating the rank deficiency condition of the suppressed matrix using

$$|[\$]_{\underline{k}}| = 0 \tag{4}$$

The proposed approach will be used to perform the singularity analysis of the Canadarm2.

3 Singularity analysis of the Canadarm2

The Canadarm2 is shown schematically in Fig. 1. Its Denavit-Hartenberg parameters are shown in Fig. 1(b), where the notations used are the ones in [10] modified by taking into consideration the symmetry of the canadarm2.

3.1 The screw-matrix and the basic submatrix

It is noted that joints 3, 4 and 5 have parallel axes and are thus equivalent to a planar joint [Fig. 1(a)]. According to the conditions on the selection of basic screws, the joint screws of either joints 1–6 or joints 2–7 can be selected as basic screws. Since the linear dependency of screws are coordinate-frame independent, any coordinate-frame located on links 3, 4 or 5 can be selected to ensure the simplest possible expressions for the joint screws.

As in [10], the joint screws of joints 2–7 are selected as basic screws for the Canadarm2, and the frame O_5-$X_5Y_5Z_5$ is chosen as the reference frame. Then, the screw-matrix and the basic submatrix for the Canadarm2 are expressed respectively as:

$$
\begin{aligned}
[\$] &= [\ \$_1 \quad \$_2 \quad \$_3 \quad \$_4 \quad \$_5 \quad \$_6 \quad \$_7\ ] \\
&= \begin{bmatrix}
s_2\, c_{345} & -s_{345} & 0 & 0 & 0 & 0 & -s_6 \\
-s_2\, s_{345} & -c_{345} & 0 & 0 & 0 & -1 & 0 \\
c_2 & 0 & 1 & 1 & 1 & 0 & c_6 \\
(ds_{45} + ac_{345} + ds_5)\, c_2 - 2\, cs_2\, s_{345} & -2\, cc_{345} & ds_5 + ds_{45} & ds_5 & 0 & 0 & -ac_6 \\
-\left(-dc_{45} + as_{345} - dc_5\right) c_2 - 2\, cs_2\, c_{345} & 2\, cs_{345} & dc_5 + dc_{45} & dc_5 & 0 & 0 & 0 \\
-s_2\ (-ds_3 + a - ds_{34}) & dc_{34} + dc_3 & 0 & 0 & 0 & 0 & -as_6
\end{bmatrix}
\end{aligned}
\tag{5}
$$

where $s_i = \sin\theta_i$, $c_i = \cos\theta_i$.

$$
\begin{aligned}
[\$]_{\underline{1}} &= [\ \$_2 \quad \$_3 \quad \$_4 \quad \$_5 \quad \$_6 \quad \$_7\ ] \\
&= \begin{bmatrix}
-s_{345} & 0 & 0 & 0 & 0 & -s_6 \\
-c_{345} & 0 & 0 & 0 & -1 & 0 \\
0 & 1 & 1 & 1 & 0 & c_6 \\
-2\, cc_{345} & ds_5 + ds_{45} & ds_5 & 0 & 0 & -ac_6 \\
2\, cs_{345} & dc_5 + dc_{45} & dc_5 & 0 & 0 & 0 \\
dc_{34} + dc_3 & 0 & 0 & 0 & 0 & -as_6
\end{bmatrix}
\end{aligned}
\tag{6}
$$

3.2 Step 1: Find the rank deficiency condition for a basic submatrix

The rank deficiency condition [Eq. (3)] of the basic submatrix [Eq. (6)] is obtained as

$$d^2 s_4 s_6 (as_{345} + dc_{34} + dc_3) = 0 \tag{7}$$

This leads to the following three cases: Case 1

$$s_4 = 0 \tag{8}$$

Case 2

$$\begin{cases} s_4 \neq 0 \\ s_6 = 0 \end{cases} \tag{9}$$

Case 3

$$\begin{cases} s_4 \neq 0 \\ s_6 \neq 0 \\ as_{345} + dc_{34} + dc_3 = 0 \end{cases} \tag{10}$$

It is pointed out that Eqs. (8)–(10) cover all the solutions to Eq. (7), although they involve inequalities, as opposed to the usual conditions found in the literature. As will be seen in Section 3.3, the form used here will facilitate the derivation of the additional conditions for the rank deficiency of the screw-matrix.

3.3 Step 2: Find the additional conditions for the rank deficiency of the screw-matrix

For each of the rank deficiency conditions [Eqs. (8)-(10)] of the basic submatrix, the additional conditions for the rank deficiency of the screw-matrix can be found by first determining the dependent screw and then calculating the rank deficiency condition of the suppressed matrix.

3.3.1 Case 1

In this case, Eq. (8) is satisfied. Substituting Eq. (8) into Eq. (6), we have

$$\begin{aligned}[\$]_{\underline{1}} &= [\ \$_2 \quad \$_3 \quad \$_4 \quad \$_5 \quad \$_6 \quad \$_7\ ] \\ &= \begin{bmatrix} -s_{35}\, c_4 & 0 & 0 & 0 & 0 & -s_6 \\ -c_{35}\, c_4 & 0 & 0 & 0 & -1 & 0 \\ 0 & 1 & 1 & 1 & 0 & c_6 \\ -2\, cc_{35}\, c_4 & ds_5 + dc_4\, s_5 & ds_5 & 0 & 0 & -ac_6 \\ 2\, cs_{35}\, c_4 & dc_5 + dc_4\, c_5 & dc_5 & 0 & 0 & 0 \\ dc_3\, c_4 + dc_3 & 0 & 0 & 0 & 0 & -as_6 \end{bmatrix}\end{aligned} \tag{11}$$

From Eq. (11), we can find that $\$_3$, $\$_4$ and $\$_5$ are linearly dependent. Since $\$_4$ and $\$_5$ are linearly independent, $\$_3$ is a dependent screw since it can always be expressed as a linear combination of $\$_4$ and $\$_5$. Removing $\$_3$ from [\$], we obtain a suppressed matrix as

$$\begin{aligned}[\$]_{\underline{3}} &= [\ \$_1 \quad \$_2 \quad \$_4 \quad \$_5 \quad \$_6 \quad \$_7\ ] \\ &= \begin{bmatrix} s_2\, c_{35}\, c_4 & -s_{35}\, c_4 & 0 & 0 & 0 & -s_6 \\ -s_2\, s_{35}\, c_4 & -c_{35}\, c_4 & 0 & 0 & -1 & 0 \\ c_2 & 0 & 1 & 1 & 0 & c_6 \\ c_2\, dc_4\, s_5 + c_2\, ac_{35}\, c_4 + c_2\, ds_5 - 2\, cs_2\, s_{35}\, c_4 & -2\, cc_{35}\, c_4 & ds_5 & 0 & 0 & -ac_6 \\ c_2\, dc_4\, c_5 - c_2\, as_{35}\, c_4 + c_2\, dc_5 - 2\, cs_2\, c_{35}\, c_4 & 2\, cs_{35}\, c_4 & dc_5 & 0 & 0 & 0 \\ -s_2\, (-ds_3 + a - ds_3\, c_4) & dc_3\, c_4 + dc_3 & 0 & 0 & 0 & -as_6 \end{bmatrix}\end{aligned} \tag{12}$$

Using Eqs. (4) and (12), we obtain

$$|[\$]_{\underline{3}}| = |\ \$_1 \quad \$_2 \quad \$_4 \quad \$_5 \quad \$_6 \quad \$_7\ | = 0 \tag{13}$$

The expansion of Eq. (13) gives the rank deficiency condition of the suppressed matrix as

$$\begin{aligned} &dc_4[(-c_2 s_6 ac_3 + 2cs_2 s_3 s_6 + s_2 c_{35} ac_5 c_6)(as_{35} c_4 + dc_3 c_4 + dc_3) \\ &\quad + s_2(-ac_{35} c_4 + ds_3 c_4 + ds_3 - a)(-2cc_3 s_6 + as_{35} c_5 c_6)] = 0 \end{aligned} \tag{14}$$

Equations (8) and (14) comprise one family of singularities of the Canadarm2 as follows

$$\begin{cases} s_4 = 0 \\ (-c_2 s_6 ac_3 + 2cs_2 s_3 s_6 + s_2 c_{35} ac_5 c_6)(as_{35} c_4 + dc_3 c_4 + dc_3) \\ + s_2(-ac_{35} c_4 + ds_3 c_4 + ds_3 - a)(-2cc_3 s_6 + as_{35} c_5 c_6) = 0 \end{cases} \tag{15}$$

3.3.2 Case 2

In this case, Eq. (9) is satisfied. Substituting the second equation of Eq. (9) into Eq. (11), we have

$$
\begin{aligned}
[\$]_{\underline{1}} &= [\ \$_2 \quad \$_3 \quad \$_4 \quad \$_5 \quad \$_6 \quad \$_7\ ] \\
&= \begin{bmatrix}
-s_{345} & 0 & 0 & 0 & 0 & 0 \\
-c_{345} & 0 & 0 & 0 & -1 & 0 \\
0 & 1 & 1 & 1 & 0 & c_6 \\
-2\,cc_{345} & ds_5 + ds_{45} & ds_5 & 0 & 0 & -ac_6 \\
2\,cs_{345} & dc_5 + dc_{45} & dc_5 & 0 & 0 & 0 \\
dc_{34} + dc_3 & 0 & 0 & 0 & 0 & 0
\end{bmatrix}
\end{aligned}
\tag{16}
$$

From Eq. (16), we can find that $\$_3$, $\$_4$, $\$_5$ and $\$_7$ are linearly dependent. Considering the first equation of Eq. (9), $\$_3$, $\$_4$, and $\$_5$ are found to be linearly independent. Thus, $\$_7$ is a dependent screw since it can always be expressed as a linear combination of $\$_3$, $\$_4$ and $\$_5$. Removing $\$_7$ from [\$], we obtain a suppressed matrix as

$$
\begin{aligned}
[\$]_{\underline{7}} &= [\ \$_1 \quad \$_2 \quad \$_3 \quad \$_4 \quad \$_5 \quad \$_6\ ] \\
&= \begin{bmatrix}
s_2\, c_{345} & -s_{345} & 0 & 0 & 0 & 0 \\
-s_2\, s_{345} & -c_{345} & 0 & 0 & 0 & -1 \\
c_2 & 0 & 1 & 1 & 1 & 0 \\
c_2\, ds_{45} + c_2\, ac_{345} + c_2\, ds_5 - 2\, cs_2\, s_{345} & -2\, cc_{345} & ds_5 + ds_{45} & ds_5 & 0 & 0 \\
c_2\, dc_{45} - c_2\, as_{345} + c_2\, dc_5 - 2\, cs_2\, c_{345} & 2\, cs_{345} & dc_5 + dc_{45} & dc_5 & 0 & 0 \\
-s_2\, (-ds_3 + a - ds_{34}) & dc_{34} + dc_3 & 0 & 0 & 0 & 0
\end{bmatrix}
\end{aligned}
\tag{17}
$$

Using Eqs. (4) and (17), we obtain

$$
|[\$]_{\underline{7}}| = |\ \$_1 \quad \$_2 \quad \$_3 \quad \$_4 \quad \$_5 \quad \$_6\ | = 0 \tag{18}
$$

The expansion of Eq. (18) gives the rank deficiency condition of the suppressed matrix as

$$
-d^2 s_2\, s_4 (as_{345} - dc_{45} - dc_5) = 0 \tag{19}
$$

From Eqs. (9) and (19), we obtain two families of singular configurations of the Canadarm2, which are represented respectively by the following two equations.

$$
\begin{cases}
s_4 \neq 0 \\
s_6 = 0 \\
as_{345} - dc_{45} - dc_5 = 0
\end{cases}
\tag{20}
$$

$$
\begin{cases}
s_4 \neq 0 \\
s_6 = 0 \\
s_2 = 0
\end{cases}
\tag{21}
$$

3.3.3 Case 3

In this case, Eq. (10) is satisfied. Considering the first two equations in Eq. (10), we obtain

$$\begin{aligned} &|[\ \$_3 \quad \$_4 \quad \$_5 \quad \$_6 \quad \$_7\ ]_{(1-5):}| \\ = &\begin{vmatrix} 0 & 0 & 0 & 0 & -s_6 \\ 0 & 0 & 0 & -1 & 0 \\ 1 & 1 & 1 & 0 & c_6 \\ ds_5 + ds_{45} & ds_5 & 0 & 0 & -ac_6 \\ dc_5 + dc_{45} & dc_5 & 0 & 0 & 0 \end{vmatrix} = -d^2 s_4 s_6 \neq 0 \end{aligned} \tag{22}$$

where the subscript "$_{(1-5):}$" denotes a submatrix composed of the rows 1 to 5 of a matrix. Thus, $\$_3$, $\$_4$, $\$_5$, $\$_6$ and $\$_7$ are linearly independent, and $\$_2$ is a dependent screw since it can always be represented as a linear combination of $\$_3$, $\$_4$, $\$_5$, $\$_6$ and $\$_7$. A suppressed matrix is then obtained by removing $\$_2$ from $[\$]$ as

$$\begin{aligned} [\$]_{\underline{2}} &= [\ \$_1 \quad \$_3 \quad \$_4 \quad \$_5 \quad \$_6 \quad \$_7\ ] \\ &= \begin{bmatrix} s_2\, c_{345} & 0 & 0 & 0 & 0 & -s_6 \\ -s_2\, s_{345} & 0 & 0 & 0 & -1 & 0 \\ c_2 & 1 & 1 & 1 & 0 & c_6 \\ (ds_{45} + ac_{345} + ds_5)\, c_2 - 2\, cs_2\, s_{345} & ds_5 + ds_{45} & ds_5 & 0 & 0 & -ac_6 \\ -(-dc_{45} + as_{345} - dc_5)\, c_2 - 2\, cs_2\, c_{345} & dc_5 + dc_{45} & dc_5 & 0 & 0 & 0 \\ -s_2\, (-ds_3 + a - ds_{34}) & 0 & 0 & 0 & 0 & -as_6 \end{bmatrix} \end{aligned} \tag{23}$$

Using Eqs. (4) and (23), we obtain

$$|[\$]_{\underline{2}}| = |\ \$_1 \quad \$_3 \quad \$_4 \quad \$_5 \quad \$_6 \quad \$_7\ | = 0 \tag{24}$$

The expansion of Eq. (24) gives the rank deficiency condition of the suppressed matrix as

$$s_2\, d^2 s_4 s_6\, (-ac_{345} + ds_3 - a + ds_{34}) = 0 \tag{25}$$

Combining Eqs. (10) and (25), we obtain the last two families of singular configurations of the Canadarm2, which are represented respectively by the following two equations.

$$\begin{cases} s_4 \neq 0 \\ s_6 \neq 0 \\ as_{345} + dc_{34} + dc_3 = 0 \\ -ac_{345} + ds_3 - a + ds_{34} = 0 \end{cases} \tag{26}$$

$$\begin{cases} s_4 \neq 0 \\ s_6 \neq 0 \\ as_{345} + dc_{34} + dc_3 = 0 \\ s_2 = 0 \end{cases} \tag{27}$$

3.4 Summary of the singular configurations

From the above analysis, five families of singular configurations of the Canadarm2 have been identified. These singular configurations are defined by Eqs. (15), (20), (21), (26) and (27) respectively. Unlike the results presented in [10], there are no denominators in the equations describing the singular configurations. In addition, there are no intersections between any two of the five families of singular configurations described using the equations in this paper.

It can be verified that (a) the equation obtained from Eq. (26) by replacing $s_6 \neq 0$ with $s_6 = 0$ represents a subset of the singular configurations described using Eq. (20), (b) the equation obtained from Eq. (27) by replacing $s_6 \neq 0$ with $s_6 = 0$ represents a subset of the singular configurations described using Eq. (21), and (c) either the equation obtained from Eq. (20) by replacing $s_4 \neq 0$ with $s_4 = 0$ or the equation obtained from Eq. (21) by replacing $s_4 \neq 0$ with $s_4 = 0$ represents a subset of the singular configurations described using Eq. (15). Thus, all the inequalities can be removed from Eqs. (20), (21), (26) and (27). The equations obtained in this way, together with Eq. (15), are indeed the singularity equations presented in [10]. This demonstrates that the singular configurations for the Canadarm2 obtained in this paper are the same as those presented in [10].

4 Conclusions

A dependent-screw suppression approach has been proposed for the singularity analysis of 7-DOF redundant manipulators. Singular configurations for the Canadarm2 have been identified. The singular configurations obtained for the Canadarm2 are identical to those revealed in [10]. The characteristics of the proposed approach are that (a) it is based on the concept of linear dependence instead of the concept of reciprocity, (b) there are no denominators in the equations describing each family of the singular configurations, (c) it is as efficient as the reciprocity-based method, and (d) there are no intersections between any two of the five families of singular configurations of Canadarm 2 obtained using the proposed approach.

Acknowledgments: The authors would like to acknowledge the financial support of the Natural Sciences and Engineering Research Council of Canada (NSERC) and of the Canada Research Chairs Program.

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