Kinematic analysis of 5-DOF parallel mechanisms (3T2R) with prismatic actuators based on identical limbs

M. Tale Masouleh[1], C. M. Gosselin[2]

[1] *Département de génie mécanique, Université Laval, Québec, QC, Canada, G1K 7P4, mehdi.tale-masouleh.1@ulaval.ca*

[2] *Département de génie mécanique, Université Laval, Québec, QC, Canada, G1K 7P4,gosselin@gmc.ulaval.ca*

Abstract: This paper presents two types of five-degree-of-freedom parallel mechanism generating 3T2R motion with linear inputs. The kinematic geometry of the mechanism is presented and several important kinematic issues including the inverse kinematic problem, the velocity equations and the constant orientation workspace are investigated. The main contribution of this work is the determination of the workspace using Bohemian domes. This approach provides a 3D solid model that can be built in any CAD system.

Keywords: kinematic, parallel mechanism, workspace, Bohemian domes.

Analyse cinématique de deux mécanismes parallèles ayant des pattes identiques à 5 ddl avec actionneurs linéaires prismatiques

Résumé: Cet article présente deux types de mécanismes parallèles à 5 - ddl produisant des mouvements de type 3T2R avec des entrées linéaires. La géométrie du mécanisme est présentée et plusieurs questions cinématiques importantes (incluant le problème géométrique inverse, les équations de vitesse et l'espace atteignable d'orientation constante) sont étudiées. La contribution principale de ce travail est la détermination de l'espace atteignable en utilisant les dômes bohémiens. Cette approche fournit un modèle 3D qui peut être établi dans n'importe quel systéme de DAO.

Mots clé: cinématique, mécanismes parallèles, espace atteignable, dômes bohémiens.

1 INTRODUCTION

Although 6-DOF parallel mechanisms can be used as versatile robots and machine tools, their complexity remains a major obstacle to their industrial applications. In comparison, limited-DOF parallel mechanisms have simpler mechanical structure, lower manufacturing cost and simpler control algorithms.
This paper investigates the kinematics of 5-DOF parallel mechanisms generating 3 translational and 2 rotational DOF(3T2R) with five identical limbs. To this end, two mechanisms are presented which arose from the type synthesis performed in [1] .

2 KINEMATIC MODELING

The type synthesis performed in [1] for parallel mechanisms with identical limbs reveals that the 5-DOF parallel mechanisms performing 3T2R motion with one prismatic actuator are composed of legs of the following types: PRRRR, RPRRR, RRPRR, RRRPR where R stands for a revolute joint and P for a prismatic actuator and where some geometric constraints on the axes should be satisfied. From a practical point of view, in parallel mechanisms, the actuated joint should be as near as possible to the base in order to decrease the inertia of the mechanism. Thus, the first two of the above legs will be kept and are presented in Fig. 1.

2.1 5-PRRRR architecture

This mechanism belongs to a family called *n*-pteron where here, with $n = 5$, we have a *pentapteron*. The fundamental characteristic of the *n*-pteron robots is their linear orthogonal actuators with identical limb structure like the *Tripteron*($n = 3$) [2] and *Quadrupteron*($n = 4$) [3]. Figure 2(a) provides a CAD model of the pentapteron. From Fig. (1(a)), the constrained rotational DOF is in the direction parallel to $\mathbf{e}_{1i} \times \mathbf{e}_{2i}$, where $\mathbf{e}_{1i}$ and $\mathbf{e}_{2i}$ are unit vectors respectively along the axes of the first (second) and third(fourth) revolute joint of the i^{th} leg. Here and throughout this paper, it is assumed that $i = 1, \ldots, 5$. It should be noted that for simplicity the second and third revolute joints are built with intersecting and perpendicular axes and can thus be assimilated to a U joint, leading to a PRUR architecture ($\Gamma = 0$) where Γ_i is the absolute value of the cosine of the angle between the first revolute joint, $\mathbf{e}_{1i}$, and the direction of the P joint. If the prismatic joint is directed in such a way that $\mathbf{e}_1 \parallel \boldsymbol{\rho}_i$, ($\Gamma = 1$), the limb architecture will be a CUR.

The workspace of each limb for a constant orientation of the platform(*vertex space*) is a so-called *Bohemian dome* and hence the workspace of the pentapteron is the intersection of five Bohemian domes extruded along their prismatic actuators. This quadratic surface can be generated by moving a circle that remains parallel to a plane along a planar curve that is perpendicular to the same plane where $\mathbf{v}_{2i}$ and $\mathbf{v}_{1i}$ are vectors respectively perpendicular to the plane and the curve. The formulation of the inverse kinematic problem (IKP) is rather complex and skipping mathematical details for $\Gamma = 1$ we have:

$$\begin{aligned}\rho_i =& y - \mathbf{n}_i^T \mathbf{j} + \\ & (-1)^{a_i} \sqrt{l_{2i}^2 - \left(\mathcal{N}_{yi} + (-1)^{b_i+1} \sqrt{\mathcal{M}_{yi}}\right)^2}\end{aligned} \tag{1}$$

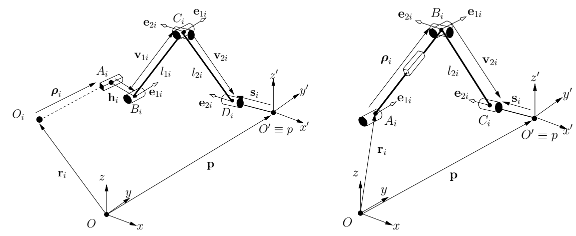
where

$$\mathbf{n}_i = \mathbf{r}_i + \mathbf{h}_i - \mathbf{s}_i \tag{2}$$

$$\mathcal{N}_{yi} = \sin\theta\ \left(x - \left(\mathbf{n}_i^T \mathbf{i}\right)\right) + \cos\theta\ \left(z - \left(\mathbf{n}_i^T \mathbf{k}\right)\right) \tag{3}$$

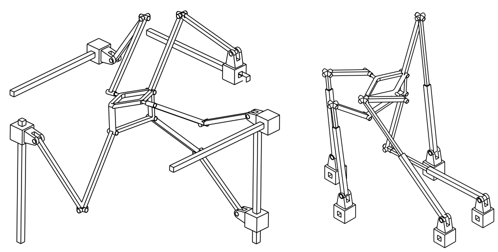
$$\mathcal{M}_{yi} = l_{1i}^2 - \left(\cos\theta\left(x - \left(\mathbf{n}_i^T \mathbf{i}\right)\right) - \sin\theta\left(z - \left(\mathbf{n}_i^T \mathbf{k}\right)\right)\right)^2 \tag{4}$$

$a_i = \pm 1$ and $b_i = \pm 1$ represent the different working modes of the mechanism. For each limb, four solutions exist, thereby leading to $4^5 = 1024$ assembly modes for the pentapteron. Angles θ and ϕ are



(a) (b)

Figure 1: Schematic representation of the proposed kinematic legs, (a) CUR, (b) RPUR.



(a) (b)

Figure 2: Schematic representation of the proposed kinematic legs, (a) CUR, (b) RPUR.

the two permitted rotational DOF and $\mathbf{p} = [x,\ y,\ z]^T$ is the Cartesian position of one point of the mobile platform. Finally, $\mathbf{i}$, $\mathbf{j}$ and $\mathbf{k}$ are unit vectors along the x, y and z axes of the fixed frame.

2.2 5-RPRRR architecture

This architecture can be considered as a 5-RPUR for the reason given above. The advantage of this mechanism, (Fig. (2(b))), is the uniqueness of kinematic arrangement for the legs. This was not true for pentapteron for which two cases were possible, $\Gamma = 0$ and $\Gamma = 1$. This advantage leads to a simpler IKP and a larger workspace. Similarly to the pentapteron, the constant-orientation workspace of this mechanism can be found by using Bohemian dome interpretations. Since the direction of prismatic actuators is not fixed, thus the swept circles will be transformed into a surface, $\mathcal{P}_i$, and the curve will be a circle of radius $r_m = \frac{\rho_{max}+\rho_{min}}{2}$ as illustrated in Fig. (3). Finally, the volume generated should be rotated by $\theta + \frac{\pi}{2}$ about $\mathbf{e}_{1i}$. Skipping mathematical derivations, and considering $\mathbf{a}_i = \mathbf{s}_i + \mathbf{p} - \mathbf{r}_i$, the solution of the IKP will be:

$$\boldsymbol{\rho}_i = \begin{bmatrix} \mathbf{a}_i \cdot \mathbf{i} \pm (\mathbf{k}^T \mathbf{e}_{2i})\sqrt{\mathcal{K}_i} \\ 0 \\ \mathbf{a}_i \cdot \mathbf{k} \mp (\mathbf{i}^T \mathbf{e}_{2i})\sqrt{\mathcal{K}_i} \end{bmatrix}, \quad \mathcal{K}_i = l_{2i}^2 - (\mathbf{a}_i^T \mathbf{e}_{1i})^2 \tag{5}$$

Comparing the IKP of the pentapteron and Eq. (5) it can be concluded that the workspace determination is less complicated for a 5- RPRRR and the unique inequality constraint associated to the IKP will be $\mathcal{K}_i \geq 0$. Also, from Eq. (5) it can be deduced that omitting the negative values for ρ_i, two different solutions can be predicted (working modes) for the IKP for a given pose yielding 32 different assembly modes for the mechanism as whole.

3 VELOCITY EQUATIONS

The velocity equations represent the linear mapping between the joint velocities and the Cartesian velocities. It can be shown that the velocity equation associated with the i^{th} leg of the above 5-DOF mechanisms can

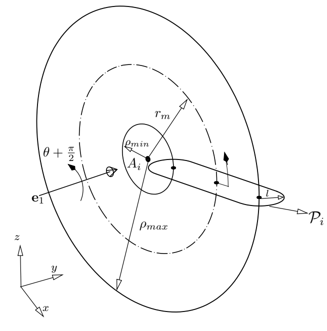


Figure 3: Vertex space for a 5-RPUR.

be written as:

$$\begin{aligned}\dot{\boldsymbol{\rho}}_i^T(\boldsymbol{\Pi}_i \times \boldsymbol{\Lambda}_i) &= \dot{\mathbf{p}}^T(\boldsymbol{\Pi}_i \times \boldsymbol{\Lambda}_i) + \boldsymbol{\omega}^T\mathbf{T}_i \\ \mathbf{T}_i &= \mathbf{s}_i \times (\boldsymbol{\Pi}_i \times \boldsymbol{\Lambda}_i) + \boldsymbol{\Lambda}_i\left(\boldsymbol{\Pi}_i^T\mathbf{v}_i\right)\end{aligned} \tag{6}$$

where for both cases we have $\boldsymbol{\Lambda}_i = \mathbf{e}_{2i} \times \mathbf{v}_{2i}$. For PRUR we have $\boldsymbol{\Pi}_i = \mathbf{e}_{1i} \times \mathbf{v}_{1i}$ where for RPUR this parameter is defined as follow: $\boldsymbol{\Pi}_i = \mathbf{e}_{1i} \times \boldsymbol{\rho}_i$. Since the mechanism has a constrained DOF, its angular velocity should be expressed with respect of the rate of change of the permitted rotational DOF, ω_x and ω_y. For the proposed mechanisms we have:

$$\boldsymbol{\omega} = [-\omega_x \sin\theta,\ \omega_y,\ \omega_x \cos\theta]^T \tag{7}$$

4 Conclusion

This paper introduced two new parallel mechanisms with identical limb structure that can be used in applications where a 3T2R motion is required. Bohemian domes appeared in the geometric interpretation of the vertex space of each limb and can be used in the determination of the constant-orientation workspace. It was concluded that the platform has a wide range of motion for the case of 5-RPRRR. It was demonstrated that both mechanisms respect a general formulation for velocity equations.

REFERENCES

[1] X. Kong and C.M. Gosselin, "Type synthesis of 5-DOF parallel manipulators Based on screw theory," *Journal of Robotic Systems*. 22(10), 535-547 (2005).

[2] X. Kong and C.M Gosselin, "Kinematics and singularity analysis of a novel type of 3-CRR 3-DOF translational parallel manipulator", *The International Journal of Robotics Research*, 21(9), 791-798 (2002).

[3] P.-L Richard, C.M. Gosselin and X. Kong, "Kinematic analysis and prototyping of a partially decoupled 4-DOF 3T1R paralle manipulator," *ASME International Design Engineering Technical Conferenced & Computers and Information in Engineering Conference* Philadelphia, Pennsylvania (2006) DETC2006-99570.