Static Balancing with a Torsional Elastic Bar

Philippe Tremblay and Clément Gosselin

Département de Génie mécanique, Université Laval, Québec, QC, Canada, G1K 7P4,
philippe.tremblay.5@ulaval.ca, gosselin@gmc.ulaval.ca

Abstract

The general problem of static balancing with a torsion bar is first defined for the case of a one-dof mechanism. Equations representing the balancing and the elastic deformation criteria are then obtained. These equations are adapted for the introduction of a reduction system in the mechanism and then solved for a general case.

Keywords: static balancing, torsion bar

Équilibrage statique avec une barre en torsion

Résumé

On définit d'abord le problème général de l'équilibrage statique avec une barre en torsion d'un mécanisme à un degré de liberté. Les équations représentant cet équilibre statique et le critère de déformation élastique de la barre sont ensuite développées. Elles sont adaptées pour l'introduction d'un système de réduction dans le mécanisme, puis résolues pour un cas général.

Mots clés: équilibre statique, barre en torsion

1 INTRODUCTION

The notion of static balancing is well documented in the literature. It is generally achieved through the use of counterweights or springs. However, these methods are not very well suited for applications where the payload and the vertical amplitude of movement are both large. The use of counterweights requires either the addition of significant mass to the system or a larger working space. The addition of a large mass significantly increases the inertia of the system, which is undesirable if it is to be in movement itself, while the necessity of more space is often impossible in several applications. Also, because of the large variations of potential energy involved and of the physical limitations of typical springs, their use is not suitable for these applications. In this case, the use of a torsion bar is much more relevant, especially that of a round bar.

2 GENERAL PROBLEM

Consider the one-degree-of-freedom mechansim of Fig. 1 that can be used to raise or lower a mass m. The mass is linked to a rotating joint by a massless arm of length l and the torsion bar is rigidly linked to the joint. K' is a section property representing the torsional moment of inertia of the bar and θ represents the position of the arm.

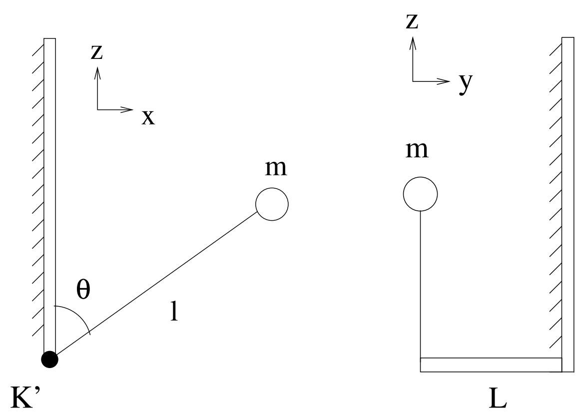

Figure 1: One-dof mechanism

If the bar has a length L and is at rest for $\theta = 0$, the relation between its angular position (θ) and the torque it exerts on the joint (T) is

$$T = \frac{K'G\theta}{L} \tag{1}$$

where G is the shear modulus of the bar material. In order for the load to be statically balanced, the torque produced by the rotation of the bar must be equal to the torque that the payload produces about the pivot. That is,

$$\frac{K'G}{L}\theta = mgl\sin\theta \tag{2}$$

where g is the gravitational acceleration. Unfortunately, the gravity torque is proportional to $\sin\theta$ whereas the bar torque is directly proportional to θ. Consequently, perfect static balancing will be impossible to acheive in this case. Fig. 2 illustrates the situation for a typical case, where the difference between the two curves represents the residual torque. However, by choosing carefully the values of initial torque in the bar (torque at $\theta = 0$) and of equivalent stiffness ($K'G/L$), the residual torques obtained are sufficiently low for this method to be considered viable. It is also desired that the balancing be adjustable for different loads, which can be achieved by varying the effective length of the bar.

In addition to balancing the torque of the load, it is imperative that the deformation in the bar remains elastic since the

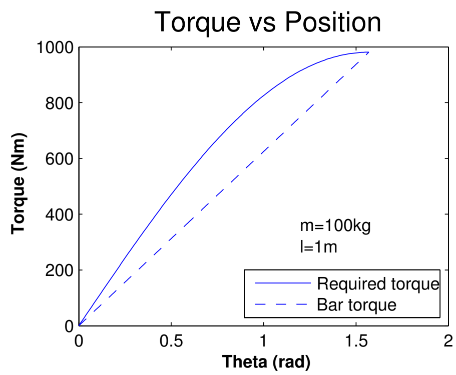

Figure 2: Torque at the pivot

movement will be repeated. The maximal stress in the bar is a function of its section. In the case of a round bar,

$$\tau_{max} = \frac{Td}{2J} \tag{3}$$

where d is the diameter of the bar and J the polar moment of inertia of the cross section. For this application, the bar is only subjected to torsional stress. Using the Von Mises criteria to determine failure, the equivalent stress must be lower or equal than the yield strength S_{yt} of the bar material:

$$\sigma_e = \sqrt{3\tau_{max}^2} = 1.73\tau_{max} \leq S_{yt} \tag{4}$$

3 Mathematical Representation

It is impossible to achieve perfect static balancing with the simple use of a torsion bar. The combination of a reasonable maximal bar length, say $0.5\ m$, combined with a desired angular range of 90 degrees, will produce severe plastic deformation. This problem can be avoided with the use of a reduction system between the bar and the joint of the mechanism. If r is the reduction ratio and subscripts b and j respectively refer to the bar and the joint, the reduction system produces the following relations:

$$T_b = rT_j \tag{5}$$

$$\theta_j = r\theta_b \tag{6}$$

The elastic behaviour of the bar can then be modeled by

$$T_b = \frac{K'G}{L}\theta_b = \frac{K'G}{Lr}\theta_j \tag{7}$$

To acheive static balancing:

$$T_b = rT_j \tag{8}$$

$$\frac{K'G}{Lr}\theta_j = rmgl\sin\theta_j \tag{9}$$

Solving (9) for the reduction ratio,

$$r^2 = \frac{K'G}{Lmgl}\frac{\theta_j}{\sin\theta_j} \tag{10}$$

Equation (10) needs to be satisfied in order to obtain static balancing. By combining equations (3) and (4), a condition for the maximal allowable torque in the bar, τ_{ma}, is formulated in order to keep the deformation in the elastic domain.

$$\tau_{max} \leq \tau_{ma} = \frac{S_{yt}}{1.73} \tag{11}$$

$$\frac{T_{b\ max}\ d}{2J} \leq \frac{S_{yt}}{1.73} \tag{12}$$

From equation (10) and inequality (12) and considering the use of a round bar for which $K' = J = \pi d^4/32$:

$$\frac{r^2}{d^4} = \frac{\pi}{32} \frac{G}{L_{max} m_{min} g l} \frac{\theta_j}{\sin\theta_j} \tag{13}$$

$$\frac{r}{d^3} \leq \frac{S_{yt}}{1.73} \frac{\pi}{16 m_{max} g l} \tag{14}$$

In equation (13), L was replaced by L_{max} to account for the desired value of L_{max}, and m was replaced by m_{min} because it is the minimal load that will require the maximum bar length to be balanced. The maximal torque in the bar will occur for the maximal payload (or m_{max}), so equation (15) was substituted into (12) to obtain (14):

$$T_{b\ max} = r T_{j\ max} = r m_{max} g l \tag{15}$$

In these equations, all the parameters except r and d are either material properties or constants that can be known or approximated from the physics of a desired problem. When this is the case, we are left with a system of 2 unknowns and 2 relations that can be solved to obtain values of r and d providing static balancing. It is important to remember that we only have partial balancing. The minimization of the residual torque will not be treated here. Instead, we neglect the ratio $(\theta_j/\sin\theta_j)$ of equation (13) and leave everything else as stated. Therefore, we obtain a feasible partially balanced system.

4 Results

If the material is properly chosen, the resolution of the aforementioned system gives values of r and d which balance the system and maintain elastic deformation in the bar. Such materials include aluminium alloys Al 7075 or titanium alloys Ti 64. The system to solve can be represented by a graph such as the one of Fig. 3, where the gray area represents the elastic deformation zone, which is delimited by inequality (14), and the curve, representing equation (13), corresponds to the different combinations of r and d that would balance the system. All points located on that curve and in the gray area are then acceptable solutions to the problem. But since it is desirable to have a bar of minimal mass and a reduction ratio as low as possible, the optimal choice is the point of intersection between the two. For example, consider a system for which $l = 1\ m$, $L_{max} = 0.5\ m$, $m_{min} = 25\ kg$, $m_{max} = 100\ kg$, $g = 9.81\ m/s^2$ and with a bar of titanium alloy with properties $G = 42.9\ GPa$, $S_{yt} = 1120\ MPa$ and $\rho = 4430\ kg/m^3$. The graphical solution is shown in Fig. 3.

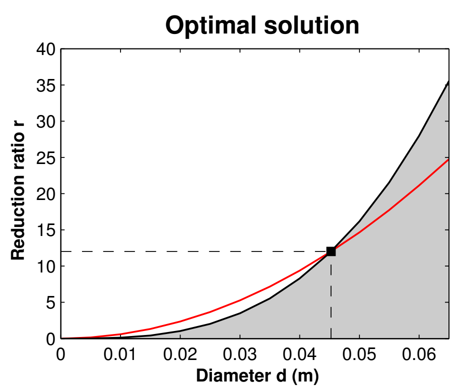

Figure 3: Optimal design of the torsion bar

The optimal solution is $r = 12.02$, $d = 4.53\ cm$ and $m_b = 3.57\ kg$.The mass of the bar is low and hence does not add a lot of inertia to the system. The reduction ratio is also quite low and would be easy to obtain. This solution is therefore viable. It is also interesting to note the variation of results as a function of the choice of the bar length. With all other parameters constant, the mass of the required bar will always be the same for different values of L_{max}. This has been demonstrated mathematically and shows that energy absorption is a mass property for a given material. A variation of L_{max} will also create an important variation of r, but a small variation of d.

5 Conclusion

The general problem of static balancing with a torsion bar was defined for the case of a one-dof mechanism. Equations representing the balancing and the elastic deformation criteria were then obtained. By solving these equations for a general case, good results were obtained, thus validating the interest in this method.

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