A Novel Approach to Dynamics Analysis of Multiple-Point Impacts Involving Multibody Systems

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1 Introduction

During the past years, dynamics analysis of interactions involving multibody systems, and the related energy considerations have drawn the attention of many researchers [1]. In this work, we discuss a novel approach to the dynamics analysis of multiple-point impact scenarios in multibody systems, based on a physically-meaningful decomposition of the tangent space. Moreover, a generalized definition of the energetic coefficient of restitution is introduced which can be of potential in dynamics analysis of impact in multibody systems.

2 Decomposition of Contact Dynamics Characteristics

Collisions in multibody systems are generally characterized by unilateral constraints. Here, we limit the scope of our study to the analysis of holonomic scleronomic systems subject to a set of m independent unilateral constraints. For now, we assume that the normal impulsive forces developed during impact for all the active contact pairs are compressive (non-vanishing). Note that the said m unilateral constraints can also be interpreted as m independent vectors in the tangent space, representing the constrained directions. The set of these vectors defines a subspace in the tangent space, that can be called the *constrained-motion space* [2]. The orthogonal complement of this space forms another subspace of the tangent space, which shall be termed the *admissible-motion space*. Considering the potential inhomogenuity of the generalized velocity vector in physical units, two orthogonal projection operators can be defined, characterizing the mentioned subspaces in the tangent space [2]. Using these operators, the generalized velocity vector of the system can be decomposed into the components belonging to the constrained- and admissible-motion spaces, which further will allow us to decouple the kinetic energy of the whole system into the parts associated with the said spaces, which can be called T_c and T_a respectively. Based on the above decomposition, it can be shown that the energy dissipation caused by virtue of the effects associated with normal contact forces reduces only the constrained part of the kinetic energy T_c, while T_a is left unchanged [3].

Note that, cases might occur where the normal impulses of some of the active contacts vanish due to effect of the interactions at other contact points [1]. In order to make the forgoing decomposition able to address such general cases, we should specify which of the closed contacts have compressive normal impulses. Let us consider the vector of generalized momenta of the system at the pre-impact instant.

Each component of this vector represents the generalized momentum associated with one of the unilateral contacts. Note that the normal impulse associated with the i-th contact pair will not vanish if its corresponding generalized momentum is strictly less than zero. Thus, we can identify and remove the rows of the Jacobian matrix which are associated with the contact pairs which do not have compressive normal impulses. The decomposition of the tangent space and the derivation of the corresponding projection operators can be done using the modified Jacobian. The above technique turns out to be useful in the analysis of simultaneous multiple-impact problems, particularly in cases where the dynamics behavior of the system at some of the impact points is affected by the interactions in other contact pairs. These results can also be used to introduce a new interpretation of the energetic coefficient of restitution. In our analysis, we define this coefficient as the square root of the ratio of the post- and pre-impact values of T_c [3]. This new definition is originally given for impacts in multibody systems and is of potential to capture the effects of not only local, but also global energy-dissipating mechanisms (e.g. losses due to wave propagation). We believe that this definition using the above decomposition establishes a novel interpretation of the energetic coefficient of restitution, which can be of considerable use in the dynamics analysis of interactions in multibody systems.

3 Conclusion

In this paper, we discussed a novel approach to dynamics analysis of impact in multibody systems. This approach relies on a physically-meaningful decomposition of the dynamics characteristics of impact associated with the spaces of admissible and constrained motions. Based on such a decomposition, a novel interpretation of the energetic coefficient of restitution was proposed. The above approach, along with this new interpretation, can provide us with a new analysis tool for the dynamics analysis of contact scenarios in complex mechanical systems.

References

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