Dexterous workspace of a general geometry 3-PRRR kinematically redundant planar parallel manipulator

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Abstract

In this work, the dexterous workspace of a general 3-PRRR[1] six actuated-joint-degrees-of-freedom (ADOF) kinematically redundant planar parallel manipulator is determined. The 3-PRRR manipulator is an adaptation of the 3-RRR manipulator with a prismatic joint added to each leg as a redundant actuator. Obtaining the dexterous workspace by discretizing a large area around the manipulator and determining if each point is in the workspace is relatively simple though inefficient. This work proposes a geometrical method to determine the dexterous workspace of a 3-PRRR planar parallel manipulator. With this method, an exact solution is obtained as opposed to an approximation given by discreet methods. The geometrical method uses the four-bar mechanism analogy to determine the dexterous workspace.

Keywords: dexterous workspace, kinematic redundancy, planar parallel manipulator.

Espace dextre d'un manipulateur 3-PRRR parallèle plan avec redondance cinématique

Résumé

Ce travail consiste à déterminer l'espace dextre d'un manipulateur parallèle plan avec redondance cinématique (3-PRRR). Ce manipulateur est adapté du manipulateur 3-RRR avec l'ajout d'une articulation prismatique actionnée à chaque patte. Obtenir l'espace dextre en discrétisant une surface englobant le manipulateur est relativement simple, mais inefficace. Ce travail propose une méthode géométrique pour obtenir l'espace dextre du manipulateur 3-PRRR. Avec cette méthode, une solution exacte de l'espace dextre peut être obtenue contrairement à une solution approximative obtenue avec les méthodes discrètes. Cette méthode se base sur l'analogie des mécanismes à quatre barres pour déterminer l'espace dextre.

Mots-clé: espace de travail dextre, redondance cinématique, manipulateur parallèle plan.

[1] The notation 3-PRRR indicates that the manipulator consists of three serial kinematic chains that connect the base to the end-effector. Each chain is composed of two actuated (therefore underlined) joints and two passive joints. P indicates a prismatic joint while R indicates a revolute joint.

1 INTRODUCTION

The shape and size of the dexterous workspace, sometimes referred to as the primary workspace, is a useful criterion to compare dimensions of a given architecture or even to compare different architectures. The dexterous workspace can be defined as the area where a manipulator is able to reach with any orientation. In other words, for planar manipulators, any position where the end-effector (tool) can rotate 2π rad is part of the dexterous workspace.

Since the dexterous workspace is an important design criterion, much research has been conducted in this field. A relatively simple planar parallel manipulator, the 3-RRR, has been studied by several researchers. Gosselin and Angeles [1] determined several design criteria for a symmetric 3-RRR, one of which being the dexterous workspace which was defined by two concentric circles for each leg. The dexterous workspace of the manipulator consists of the area of the intersection of the concentric circles of the three legs. Williams and Reinholtz [2] developed a geometrical method to determine and optimise the dexterous workspace once again defined by two concentric circles for each leg. Kumar [3] used screw theory to determine the controllably dexterous workspace which is the dexterous workspace devoid of any discontinuities. Pennock and Kassner [4] also treated the controllably dexterous workspace for a general geometry manipulator instead of their symmetrical counterparts. Zhaohui and Zhonghe [5] introduced a third concentric circle using four-bar linkage analogy defining a second dexterous workspace near the base of each leg. The manipulator was a symmetric 3-RRR manipulator.

Ebrahimi, Carretero and Boudreau [6] introduced kinematic redundancy to the 3-RRR manipulator as well as other manipulators to counterbalance certain drawbacks of parallel manipulators. The goal was to reduce singularities and increase the workspace of these manipulators. The dexterous workspace was obtained by discretizing an area around the robot and determining if each point was in the workspace. However, this method is inefficient and is only an approximation where the precision is dictated by the number of points used. In this work, a geometrical method of establishing the dexterous workspace of a general geometry 3-PRRR manipulator is developed. The term workspace will always denote the dexterous workspace in what follows.

2 FOUR-BAR MECHANISM

The studied architecture is a parallel planar manipulator with a total of 6-ADOF and is shown in Figure 1. The length of each link is defined by L_{ij} which is the length of the i^{th} link of the j^{th} leg. Since the workspace of a parallel manipulator is the intersection of the workspace of each leg, it is simpler to establish the workspace of each leg individually and determine their intersection afterwards.

The determination of the workspace of each leg can be simplified even further by first considering an RRR manipulator, then adding the effect of the prismatic joint. A leg of RRR configuration is shown in Figure 2. For a given position of the end-effector (X_2, Y_2), when one considers this position to be fixed, the leg is equivalent to the familiar four-bar mechanism. If the point (X_2, Y_2) is in the workspace, the link L_3 (platform) must be able to rotate 2π rad. The workspace of each leg is the area encompassing all the points where this is possible. In what follows, links L_1, L_2 and L_3 will be considered as fixed length links (FL links) and L_0 will be considered as a variable length link (VL link).

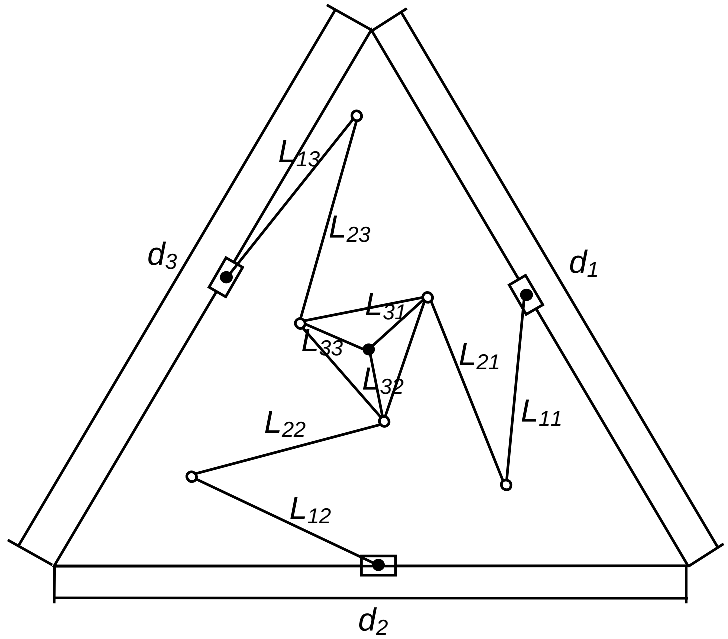

Figure 1: Kinematically redundant 6 ADOF parallel manipulator (3-PRRR) [6]

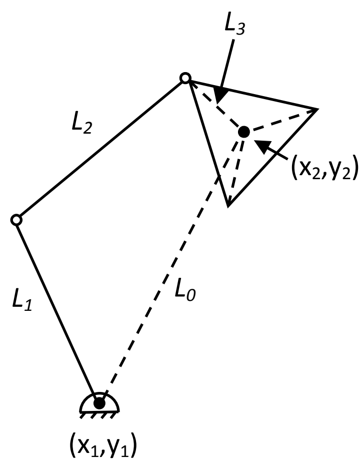

Figure 2: One leg without the prismatic joint (RRR)

The four-bar mechanism analogy is valid for any orientation of L_0. As the end-effector changes position, the base length (L_0) of the four-bar mechanism changes and is thus a variable. Each value of L_0 where L_3 can rotate 2π rad thus generates a circle of workspace. Establishing the area of the workspace is then reduced to a matter of determining the limits on the values of L_0 that permit a full revolution of L_3.

The well-known Grashof's criterion is given for reference in Table 1 [7] where S is the length of the shortest link and L is that of the longest link. M_1 and M_2 are the lengths of the two remaining links. Figure 3 illustrates each category.

Table 1: Categories of four-bar mechanisms

Category	Criterion	Shortest link	Name of category	Figure
1	$L + S < M_1 + M_2$	L_0	Double crank	3(a)
2	$L + S < M_1 + M_2$	L_1 or L_3	Crank-rocker	3(b)
3	$L + S < M_1 + M_2$	L_2	Double rocker	3(c)
4	$L + S = M_1 + M_2$	Any	Change point	3(d)
5	$L + S > M_1 + M_2$	Any	Triple rocker	3(e)

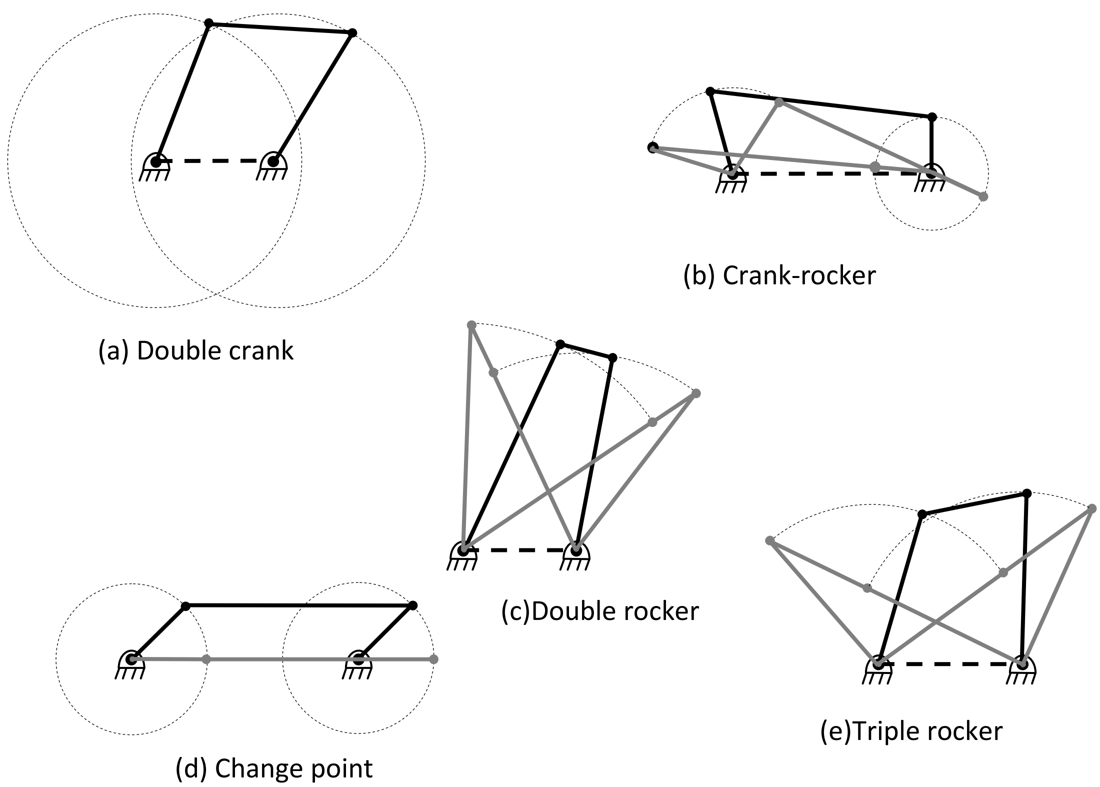

Figure 3: Categories of four-bar mechanisms (adapted from [7])

Of these categories, only three permit a complete revolution of L_3 and are of interest when determining the workspace of a leg: double crank, crank-rocker and change point. Since the lengths of the three FL links are known, the first step in determining the workspace of a leg is to establish the shortest of the three FL links. From this, several cases arise producing different types of workspaces (shown in Figure 4). The different classes shown are defined and explained in the following sections.

2.1 Case where L_3 is the shortest FL link

If L_3 is the shortest FL link, the mechanism can respect the conditions for all three categories of interest. Grashof's criterion for a crank rocker mechanism (category 2) is:

$$L + S < M_1 + M_2 \tag{1}$$

When L_0 is the longest link, Equation (1) becomes:

$$L_0 < L_1 + L_2 - L_3 \tag{2}$$

If Equation (2) is not met, the mechanism falls into the fifth category and no links are able to complete a revolution. Equation (2), therefore, defines the outer limit of the workspace. When the end-effector is moved towards the actuated revolute joint (X_1, Y_1), it is equivalent to the link L_0 becoming shorter until it becomes shorter than either M_1 or M_2 (in this case, L_1 or L_2) and Equation (1) becomes:

$$L_0 > L_3 + |L_1 - L_2| \tag{3}$$

The absolute value in Equation (3) makes it unimportant whether L_1 or L_2 is the longest. This equation defines a second concentric circle inside the first defined by Equation (2). Any value of L_0 bounded by Equations (2) and (3) is within the workspace. Thus, the area bounded by the two concentric circles with radii defined by these equations constitutes a part of the workspace.

A double crank mechanism is also possible when L_0 is shortest, meaning there can be another workspace near the base of the leg. A leg with a workspace defined by three concentric circles is henceforth defined as Class 3 and is shown in Figure 4 (a). When L_0 is the shortest link, Equation (1) becomes:

$$L_0 < L_3 - |L_1 - L_2| \tag{4}$$

Not every four-bar mechanism has a second workspace inside the one described above. Since every link must have a positive valued length, a zero length for L_0 in Equation (4) represents a limit that must be satisfied for the inner workspace to exist:

$$0 < L_3 - |L_1 - L_2| \tag{5}$$

Furthermore, if this condition is not met, one of the FL links is longer than the sum of the other two and the end-effector is unable to get closer than $L_3 - |L_1 - L_2|$ from its base (X_1, Y_1) with any orientation. A leg of this type has only two concentric circles defining its workspace and is deemed a Class 2 as shown in Figure 4 (b).

2.2 Case where L_3 is not the shortest FL link

The only link able to complete a revolution in a mechanism of the crank-rocker category is the shortest, therefore, when L_3 is not the shortest FL link, the only category of interest is that of double crank since the crank-rocker category is impossible with L_3 as the crank. In this case, VL link L_0 must be the shortest link and Equation (1) becomes:

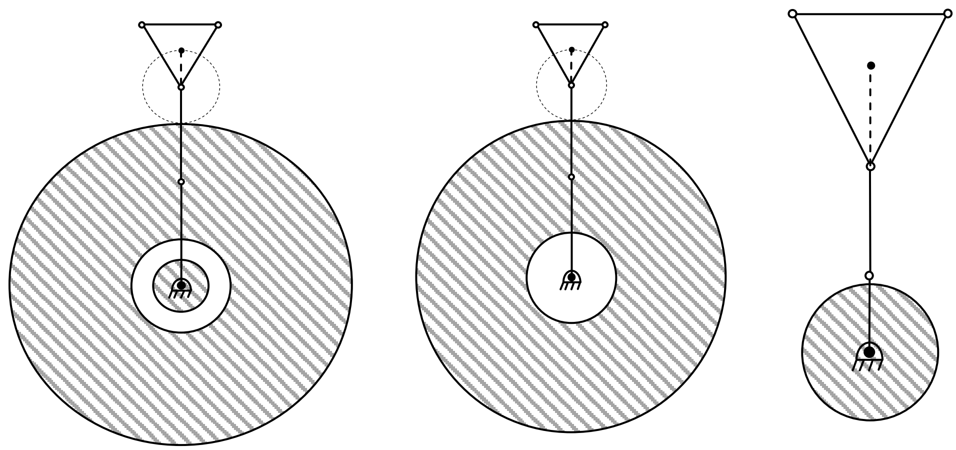

Figure 4: Classes of the workspace of each leg without the influence of the prismatic actuator

$$L_0 < M_1 + M_2 - L \tag{6}$$

where L, M_1 and M_2 are the same as in Equation (1) and Table 1. A leg of this configuration has a maximum of one circle, defining its workspace as a Class 1. See Figure 4 (c) for an example of its representation.

It is possible for a mechanism to be unable to satisfy Equation (6), in which case the leg and thus the entire manipulator has no workspace at all. This occurs if L_3 is not the shortest FL link *and* if the longest FL link is longer than the sum of the other two.

If the two shortest links have equal lengths, Equation (1) cannot be met unless VL link L_0 is shorter than the three FL links. Equation (6) thus also applies to the case when the two shortest FL links have equal lengths.

2.3 Transition from one category to another

As the end-effector moves and length L_0 varies, the mechanism falls into different categories. Figure 5 shows the transition from one category to another. This figure also shows the definition of the radius of each concentric circle defining the workspace, r_1, r_2 and r_3. These variables are used in Section 3. A Class 1 workspace is defined by r_1 alone, a Class 2 by r_2 and r_3 and a Class 3 by r_1, r_2 and r_3.

3 Effect of the prismatic joint on the workspace

The effect of adding a redundant prismatic actuator is relatively simple though it introduces different types of workspaces. The limited range of motion of the prismatic actuators greatly influences the size and shape of the workspaces. In Figure 6, the five types of workspaces possible for the 3-PRRR are shown. Each type of workspace depends on the stroke length of the prismatic actuators. It is possible to see that the addition of the prismatic actuator has the effect of stretching the area and reducing the non-dexterous area. This is where the advantage of kinematic redundancy

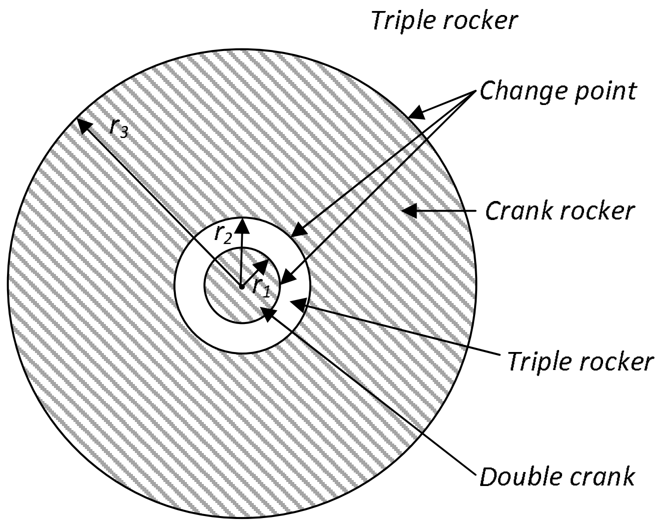

Figure 5: Transition from one category of four-bar mechanisms to another

becomes clear. If the prismatic joint has a large enough stroke, the non-dexterous parts of the workspace vanish completely.

In Figure 6, D is the stroke of the actuated prismatic joint. A workspace that does not contain holes (Type 1 shown in Figure 6 (a)) is possible regardless of the class of workspace of the individual leg before the addition of the redundant actuator (Figure 4). If the leg is of Class 1, the workspace, with the addition of kinematic redundancy, will be of Type 1 (Figure 6 (a)) with any value of D. If the leg is of Class 2 or 3, a prismatic actuator with a sufficiently large stroke can yield a Type 1 workspace for the leg. Equation (7) defines the condition that must be met in order for the hole in the workspace to disappear if the leg is of Class 2 while Equation (8) represents this same condition when a leg is of Class 3:

$$D \geq 2r_2 \tag{7}$$

$$D \geq 2\sqrt{r_2^2 - r_1^2} \tag{8}$$

where r_1 and r_2 are defined in Figure 5. For an identical value of r_2, a smaller stroke D is needed to obtain a Type 1 workspace when r_1 exists. As reflected by Equation (8), the non-dexterous area vanishes when the height of the non-dexterous area (distance between the base-proximal revolute joint and the highest point of the internal non-dexterous area) is shorter than the radius of the internal workspace (r_1).

A Type 2 workspace is possible when the leg is of Class 2 and when the condition in Equation (7) is not met (Figure 6 (b)).

The three remaining types of workspace are possible with Class 3 legs. Type 3 is encountered when the stroke D is too short for the outer and inner workspaces to connect. This type, shown in Figure 6 (c), occurs when the actuated prismatic joint has a stroke respecting the condition in Equation (9)

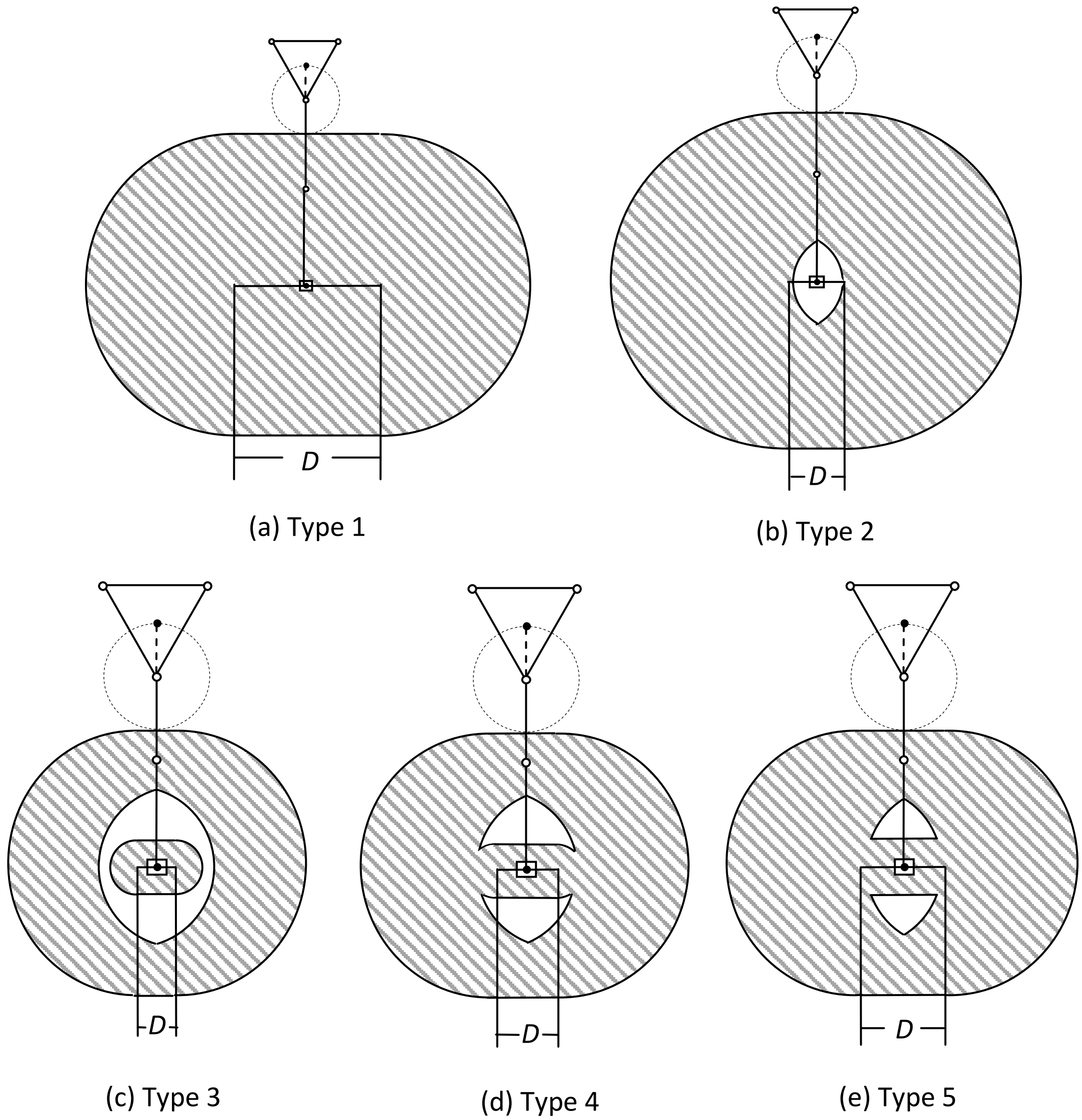

Figure 6: Types of workspaces of the individual legs of a kinematically redundant planar parallel manipulator (PRRR)

$$D < r_2 - r_1 \tag{9}$$

Types 4 and 5 differ by the number of arcs or lines needed to define the internal non-dexterous workspaces (ten for Type 4 and six for Type 5). These last types are possible if the condition in Equation (8) is not met. The differences between these types are caused by the stroke D of the actuated prismatic joint.

Type 4, shown in Figure 6 (d), is possible when:

$$r_2 - r_1 \leq D < \sqrt{r_2^2 - r_1^2} \tag{10}$$

Finally, the type in Figure 6 (e), Type 5, occurs when:

$$\sqrt{r_2^2 - r_1^2} \leq D < 2\sqrt{r_2^2 - r_1^2} \tag{11}$$

Distinguishing between Types 4 and 5 is important when determining the workspace in a general algorithm because the area is defined by a different number of lines and arcs. This is especially true when considering a manipulator with three legs of completely different dimensions.

4 FINAL WORKSPACE OF A 3-PRRR KINEMATICALLY REDUNDANT PARALLEL PLANAR MANIPULATOR

Once the workspace of each leg is established, the next step is to determine their intersection. Figure 7 shows an example of the final workspace of one of the legs and the total workspace of a 3-PRRR manipulator. In this example, each leg is built with identical dimensions but with slightly different strokes. L_1, L_2 and L_3 for each leg are equal to 4, 3 and 2 units of length respectively. The length of each side of the triangle defined by the coordinates (0,0), (1,2) and (2,0) as indicated in Figure 7 corresponds to the stroke of each prismatic joint. It is interesting to note the presence of an internal workspace in the form of an inverted triangle for this example shown in this figure.

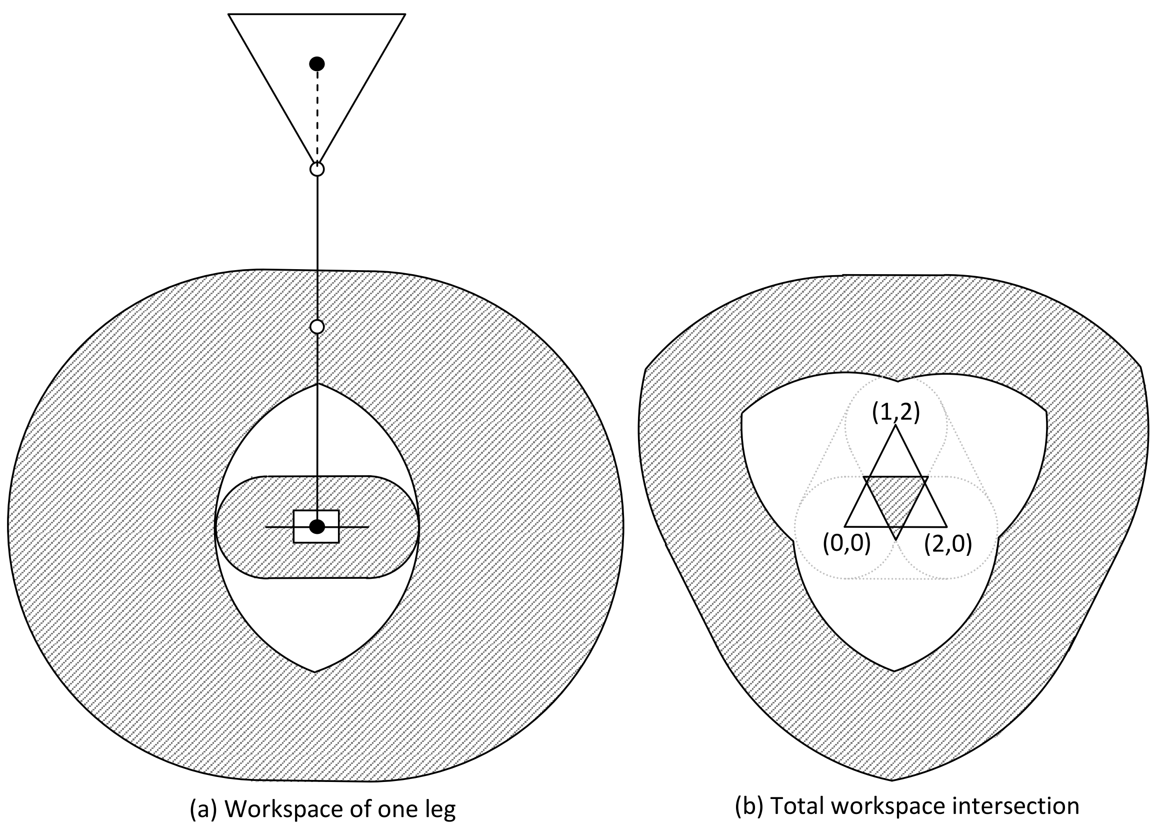

Figure 7: Workspace of one of the PRRR legs and the total workspace of a 3-PRRR kinematically redundant parallel planar manipulator

5 CONCLUSIONS

The workspace of each leg of a general geometry planar 3-RRR parallel manipulator has been established with the analogy of four-bar mechanisms. This was done by characterizing different categories of four-bar mechanisms and analyzing which categories permitted the end-effector to complete a full revolution for a given leg. A point is included in the workspace of a leg if the link representing the end effector is categorized as a crank at that point.

Using the workspace of each leg of the 3-RRR, the influence of the redundant prismatic actuation was then added. From this, five possible types of workspaces have been identified and the inherent complexity of the problem became apparent as shown in Figure 6.

The workspace of the manipulator consists of the intersection of the workspace of all three legs.

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