Dynamics Formulation and Modelling of Wheeled Rovers on Soft Soil

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Abstract

Wheeled rovers play an important role in planetary exploration. Given the difficulty in testing the rovers in situ, modelling and analysis of their performance is an important phase of the design process. Modelling of the rover itself is possible using multibody modelling techniques. However, simulating the rover's performance on the unstructured terrain of the planetary surface is an important part of rover development and is still a fairly undeveloped field.

A suitable model for contact between the wheels and ground must be found or developed. The simplest models might be visco-elastic or Coulomb friction, but much more appropriate for the case of rovers with unstructured terrain would be a contact model specifically designed for soft soil, such as those developed by Bekker [1] or Wong [2].

In order to incorporate any contact representation into a multibody rover model, the formulation must allow for the modelling and analysis of the interaction with the terrain. This means the formulation must make the positions and velocities of the wheel contact available to the contact model and allow the contact model to apply reaction forces to the rover. How these reaction forces are determined from the positions and velocities is determined by the structure of the terrain and the contact model.

The formulation presented here provides a simple way to determine the state and velocity of each wheel contact point, as well as apply the reaction forces to the wheel. It was used with four different contact models and two rovers to demonstrate the response of the rovers for a simple simulation. The formulation could lead to further developments, such as transforming the system from joint coordinates to coordinates corresponding to the contact directions of the wheels.

The equations of motion for the multibody system representing the rover can be written in the form

$$\mathbf{M}\dot{\mathbf{v}} + \mathbf{c} = \mathbf{f} + \mathbf{A}^{\mathrm{T}}\boldsymbol{\lambda} + \mathbf{A}_w^{\mathrm{T}}\boldsymbol{\lambda}_w \tag{1}$$

where $\mathbf{M}$ is the mass matrix, $\dot{\mathbf{v}}$ is the vector of accelerations, $\mathbf{f}$ is the force vector, $\mathbf{c}$ is the vector of Coriolis and centrifugal terms, $\mathbf{A}$ is the constraint Jacobian matrix, and $\boldsymbol{\lambda}$ is a vector of Lagrange multipliers corresponding to the joint constraints of the rover. The vector $\boldsymbol{\lambda}_w$ corresponds to the generalized reaction forces caused by the wheel/ground contact and $\mathbf{A}_w$ transforms these forces to the rover coordinates.

In the case of an ideal rolling contact between the wheels and the ground, the corresponding velocity-level constraint equations must be added to the system, which can be expressed in the form

$$\mathbf{A}_w \mathbf{v} = \mathbf{0}. \tag{2}$$

In this case, $\boldsymbol{\lambda}_w$ becomes the vector of Lagrange multipliers corresponding to the non-holonomic constraints at the ground. A more realistic situation for deformable terrain is when the perfect no-slip and no-penetration conditions are violated, i.e.

$$\mathbf{A}_w \mathbf{v} = \mathbf{u}_c \neq \mathbf{0} \tag{3}$$

where $\mathbf{u}_c$ is a vector of velocities corresponding to violations of the constraints in (2). In this case, constitutive relations must be defined that describe the behaviour of the contacts. In general, these relations can be used to determine the reaction forces, $\boldsymbol{\lambda}_w$, given the constraint violation, $\mathbf{u}_c$.

Four contact models were investigated and incorporated into the multibody equations using the preceding formulation. The first is an ideal rolling constraint. The second is a purely visco-elastic model, which considers each of the three directions of the wheel independently: longitudinal, transverse, and normal.

For the third model, a visco-elastic model was used for normal direction, while a Coulomb model was used for the longitudinal and transverse directions. The final contact model implemented and tested was the Bekker soft soil model, a semi-empirical contact model specifically designed to simulate wheels moving on soft soil and granular material [1].

In order to use the Bekker model in a general simulation, several issues should be mentioned. Firstly, as the model was developed for steady-state conditions, and a damping term had to be added to the vertical force component in order to prevent undamped oscillation in a dynamic simulation. Secondly, some of the forces determined by the Bekker model (such as compaction resistance) only exist when the wheel is moving, resulting in a discontinuity in force when the wheel is stopped. This was handled by using a continuous approximation for the force. Finally, defining a single contact point is not trivial when using a soft soil model, since contact is spread over a finite patch of the tire's surface.

Keywords: dynamics formulation, modelling, rover, soft soil.

REFERENCES

[1] M.G. Bekker. *Introduction to Terrain-Vehicle Systems*, The University of Michigan Press, 1969

[2] J.Y. Wong. *Terramechanics and Off-Road Vehicle Engineering, Second Edition: Terrain Behaviour, Off-Road Vehicle Performance and Design*. Elsevier, Amsterdam 2010