Workspace Investigation of Translational Planar Wire-actuated Parallel Manipulators

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Abstract

The objective of this paper is to investigate the layout for the translational planar wire–actuated parallel manipulators to generate the largest possible workspace. The workspace of these manipulators is a function of the winding direction of wires around the pulleys, the relative position of the pulleys with respect to each other, the radius of the pulleys, and the direction of the external force. A discrete method based on the calculation of the null space of the Jacobian matrix, taking into account the minimum and maximum allowable wire tensions, is used for workspace analysis. The simulation results show how the aforementioned parameters affect the workspace of these manipulators.

Keywords: translational planar wire-actuated parallel manipulators, workspace analysis, null space of the Jacobian matrix

1 INTRODUCTION

Wire-actuated parallel manipulators (WAPMs) are a type of parallel manipulators in which the mobile platform is connected to the base platform through the wires. The position and orientation of the mobile platform are changed by increasing or decreasing the length of wires while keeping the wire tensions positive. An illustration of WAPMs can be seen in Figure 1.

The advantages of WAPMs over existing parallel manipulators are high payload-to-weight ratio, low inertia, large workspace, simple dynamic model, easy transportation, and light weight. However, the major drawback of WAPMs is that the wires can only pull the mobile platform and cannot push on it.

Since the wires can only apply tensile force, actuation redundancy is required to apply/resist force and moment, i.e. more wires than the degree-of-freedom (DOF) of manipulator are required to avoid the configurations wherein certain external wrenches require compressive force in one or more wires [1].

The workspace of WAPMs is a set of positions where the center of the mass of the mobile platform can be located as long as all wires remain in tension for a given mobile platform orientation, also referred to as the constant orientation workspace. Many studies, using the discrete and analytical methods, for determining the workspace of WAPMs have been conducted. In discrete methods based on wire tension, the entire task space is discretized. The condition of positive wire tension is checked point by point in a predefined region to find the static workspace for each position and orientation of the mobile platform. The null space of the Jacobian matrix of the manipulators based on the formulation of wire tension was used in [2-5] for the discrete solution of the workspace. Using this method the minimum and maximum allowable wire tensions can be taken into account. In [6, 7] convex hull theory was utilized for the discrete solution to the force-closure workspace of wire-actuated parallel manipulators.

Analytical solutions to define the borders of the workspace of the planar WAPMs were investigated in [8-11]. Although the analytical methods are less computational compared to the discrete methods, the minimum and maximum allowable wire tensions for the workspace of these manipulators cannot be included.

In [12], the largest workspace and the global condition number of the Jacobian matrix of the 6–6 wire-actuated manipulators were analyzed for different geometries, sizes and orientations of the mobile platform. In [13], the largest dexterous workspace of planar wire-actuated parallel manipulators in terms of the number of wires, external load and orientation of the mobile platform was investigated. The effect of variations in design parameters, such as the geometry of the wire attachment points on the mobile platform; the diameter of the pulley; the size, mass, and orientation of the mobile platform; and the minimum and maximum allowable wire tensions, on the workspace of the 3-DOF planar WAPMs was investigated in [14]. The workspace of the wire-actuated parallel manipulators, considering the uncertainty in the wire connections, was studied in [15]. The geometric representations of the uncertainties in the wire attachment points to the base and to the mobile platform were developed. It was presented that a small difference in the diameters of the wire and peg hole at the wire connections could lead to a significant change in the workspace when compared to the results with no uncertainty.

In this paper, the workspace of the 2-DOF translational planar manipulators comprising a point mass as a mobile platform driven by wires is investigated. A method to determine the static workspace of the manipulators based on the calculation of positive wire tension derived from the force balance equation is presented. The entire predefined region is discretized and for each position of the mobile platform, the condition of positive wire tension is checked to find the static workspace of the manipulator. The main application of the translational planar WAPMs is in axisymmetric tasks that require neither rotational motions nor moment applied by the mobile platform. Example tasks include planar spray painting, spot and seam welding, some planar assembly tasks, and translational/force-only haptic interfaces.

The objective of the current paper is to present the layout for translational planar WAPMs to generate the largest possible workspace. Accordingly, the workspace area of the translational planar WAPMs is a function of the winding direction (clockwise or counter-clockwise) of wires around the pulleys, the position of the pulleys, the radius of the pulleys, and the direction of the external force applying on the mobile platform. The organization of this paper is as follows: In Section 2 the translational planar WAPMs are described in details. In Section 3 the static force balance equations are derived to analyze the workspace of the manipulators. The simulation results of the workspace analysis are reported in Section 4. Finally, conclusions appear in Section 5.

2 MANIPULATOR DESCRIPTION

The two translational planar WAPMs studied in this paper are shown in Figure 1. The manipulator consists of three wires connecting to the base through pulleys at one end and attaching to a single point on the mobile platform at the other end. To accurately control the position of the mobile platform, a pulley is placed between the motor and the mobile platform. The wire release point on the pulley is referred to as the anchor position. Once the position of the mobile platform is changed, the anchor position varies.

For a 2-DOFtranslational planar manipulator with three wires, there is one degree of actuation redundancy. Figure 1 shows two different configurations of the 2-DOF translational WAPMs based on the configuration of wires with respect to their pulleys; anchor position. In Figure 1(a) all wires are wound around their pulleys in the same direction (clockwise). In Figure 1(b), wires 1 and 3 are wound in the same direction (clockwise) and wire 2 is wound in opposite direction (counter-clockwise).

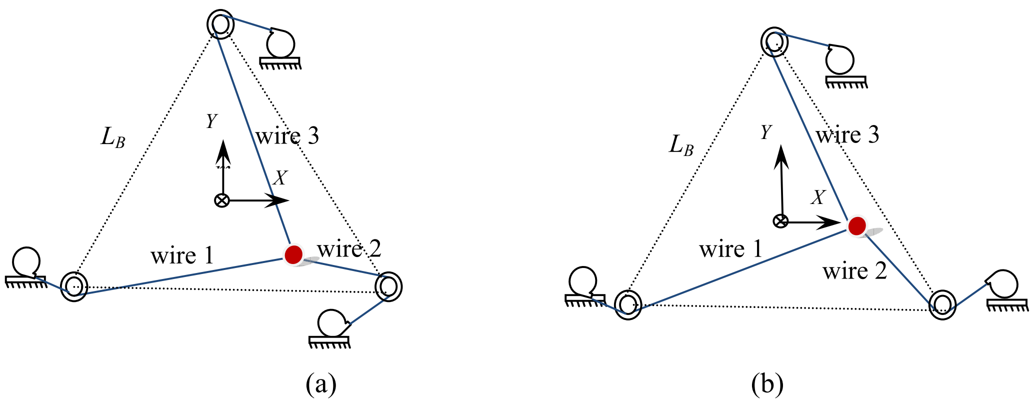


Figure 1. 2-DOF translational planar wire-actuated parallel manipulators.

The base platform supports the motion of mobile platform in the XY plane. Since the wires can only apply tensile force on the mobile platform, more wires than the degree of freedom of manipulator are required to fully control the position of the mobile platform and avoid configurations in which the wires could become slack. This is a requirement only if there is no gravity, e.g., when the manipulator moves in horizontal plane [16]; otherwise two wires may be sufficient.

3 STATIC MODELLING AND WORKSPACE ANALYSIS

The workspace of WAPMs is a set of positions where the center of geometry (centroid) of the mobile platform can be located provided all wires are kept in positive tension. In this paper a method for determining the workspace wherein all possible forces can be applied on the mobile platform with positive wire tensions is presented.

Figure 1 shows the fixed reference frame with an origin at the centroid of a triangular base. The attachment points of the wires to their corresponding pulleys are denoted by A_i with coordinates (a_{ix}, a_{iy}) in the fixed reference frame. The components of the position vector of the mobile platform in the fixed reference frame is expressed by $\mathbf{p} = [p_x, p_y]^T$. Figure 2 depicts the coordinates and parameters of a translational planar WAPM. The anchor position is defined as the point where the wire exits the pulley. Depending on the position of the mobile platform, the exact point at which the wire leaves the pulley can be calculated. Since translation along the Z-axis is neglected, the anchor position is modelled in two dimensions. The coordinates of the center of pulley i, D_i, are (d_{ix}, d_{iy}) with respect to the fixed reference frame and θ_i is the orientation of the position vector of the center of pulley with respect to the X-axis of the fixed reference frame. The position and radius, r, of each pulley are known, while the anchor position (a_{ix}, a_{iy}) must be calculated. The X- and Y- coordinates of the anchor position depend on the position of the mobile platform and the size of the pulley. Since the wire will exit tangent to the pulley, a right angle triangle can be formed with points A_i, P, D_i as reported in [15]. Using the Pythagorean theorem

$$\|\mathbf{h}_i\|^2 = r^2 + \|\mathbf{l}_i\|^2$$
$$(d_{ix} - p_x)^2 + (d_{iy} - p_y)^2 = r^2 + (a_{ix} - p_x)^2 + (a_{iy} - p_y)^2 \quad (1)$$

The anchor position is on the circumference of a circle with a known center and radius, i.e.,

$$(a_{ix} - d_{ix})^2 + (a_{iy} - d_{iy})^2 = r^2 \quad (2)$$

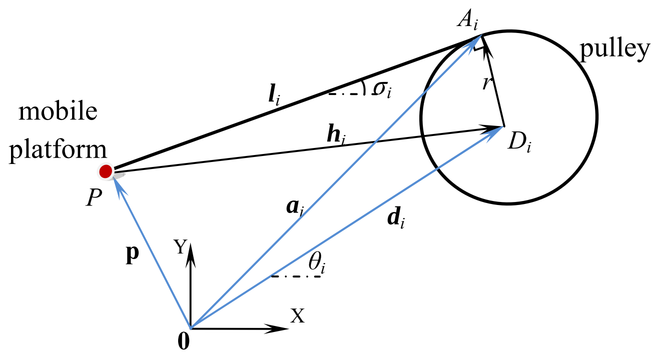


Figure 2. Coordinates, variables and parameters of the translational planar WAPM.

Solving equations (1) and (2) simultaneously leads to the coordinates of the anchor position, i.e., a_{ix} and a_{iy}. This set of equations has up to two real solutions. Depending on the clockwise or counter-clockwise winding of the wire around the pulley, the suitable solution is chosen. For example, in Figure 2 when the winding direction of the wire on the pulley is clockwise, one of the following possibilities can happen:

i. If $p_y = d_{iy}$, and $p_x > d_{ix}$ then the solution that has the smaller Y-coordinate for the anchor position is chosen.

ii. If $p_y = d_{iy}$, and $p_x < d_{ix}$, then the solution that has the greater Y-coordinate for the anchor position is chosen.
iii. If $p_y > d_{iy}$, then the solution with the greater X-coordinate for the anchor position is chosen.
iv. If $p_y < d_{iy}$, then the solution with the smaller X-coordinate for the anchor position is chosen [14].

In this section, the workspace of the manipulators in Figure 1 is formulated using the static force balance equations. By subtracting the anchor positions from the position of the mobile platform, the vector of the magnitude and direction of each wire is formulated as follows

$$\mathbf{l}_i = \mathbf{a}_i - \mathbf{p} \quad , \quad i = 1,2,...,n \tag{3}$$

For static equilibrium, the sum of forces exerted on the mobile platform by the wires must equal the resultant external force applied on the mobile platform. Applying the static force balance results in

$$\sum_1^n \tau_i \cos(\sigma_i) = F_{\mathrm{ext_x}} \tag{4.1}$$

$$\sum_1^n \tau_i \sin(\sigma_i) = F_{\mathrm{ext_y}} \tag{4.2}$$

where τ_i is the tension of wire i, σ_i is the orientation of wire with respect to the X-axis of the fixed reference frame, $\cos(\sigma_i) = l_{\mathrm{ix}}/\|\mathbf{l}_\mathrm{i}\|$, $\sin(\sigma_i) = l_{\mathrm{iy}}/\|\mathbf{l}_\mathrm{i}\|$. The components of the external force acting on the mobile platform are F_{ext_x} and F_{ext_y} . For the translational planar WAPMs, the relationship between the $n \times 1$ vector of wire forces, $\boldsymbol{\tau} = [\tau_1 \; .... \; \tau_n]^T$, and the 2×1 vector of forces acting on the mobile platform $F = [F_{ext_x} \quad F_{ext_y}]^T$ can be written in the matrix notation

$$\begin{bmatrix} \cos(\sigma_1) & ... & \cos(\sigma_n) \\ \sin(\sigma_1) & ... & \sin(\sigma_n) \end{bmatrix} \begin{bmatrix} \tau_1 \\ \vdots \\ \tau_n \end{bmatrix} = \begin{bmatrix} F_{ext_x} \\ F_{ext_y} \end{bmatrix} \tag{5}$$

where the $2 \times n$ matrix in equation (5)is the transpose of the Jacobian matrix, $\mathbf{J}^T$.

The wire tensions τ_i need to be greater than or equal to zero for the mobile platform to be in the force-closure workspace

$$\boldsymbol{\tau} = \mathbf{J}^{\#T}\mathbf{F} + (\mathbf{I} - \mathbf{J}^{\#T}\mathbf{J}^T)\mathbf{k} \tag{6}$$

In equation (6), **I** is an $n \times n$ identity matrix, **k** is an arbitrary n-vector, the first term on the right-hand side is the particular solution and the second term is the homogeneous solution that maps **k** to the null space of the $2 \times n$ ($n > 2$) matrix $\mathbf{J}^T$. $\mathbf{J}^{\#T}$ is the Moore-Penrose inverse of $\mathbf{J}^T$

$$\mathbf{J}^{\#T} = \mathbf{J}(\mathbf{J}^T\mathbf{J})^{-1} \tag{7}$$

For the translational planar WAPMs of this paper, three wires are used to achieve the two degrees of freedom. Therefore, these manipulators have one degree of actuation redundancy, and equation (5) is underconstrained, which means that there are infinite solutions to the wire tension vector $\boldsymbol{\tau}$ to exert a given external force **F**. For these manipulators, an alternative to equation (6) can be expressed as below

$$\boldsymbol{\tau} = \mathbf{J}^{\#T}\mathbf{F} + \mathbf{N}\lambda \tag{8}$$

where the columns of the $n \times (n\text{-}2)$ matrix **N** span the null space of the transposed Jacobian matrix, $\mathbf{J}^T$, and λ is an arbitrary vector. For one degree of actuation redundancy, the null space is spanned by a single vector and λ has only one entry. The value of λ is chosen so that the positive wire tension $\boldsymbol{\tau}$ is achieved.

Therefore, testing each position for the mobile platform and choosing a λ, only if there is such a λ, that creates positive wire tension in equation (6) ensures that the position lies inside the workspace, this has been referred to as the null space method for formulating the workspace of wire-actuated parallel manipulators [15]. If there is no external force, the particular solution is a zero vector. To maintain all positive wire tensions, the matrix **N** must contain only positive entries or only negative ones. If the zero wire tension is considered as the minimum allowable wire tension, matrix **N** could contain zero entries as well.

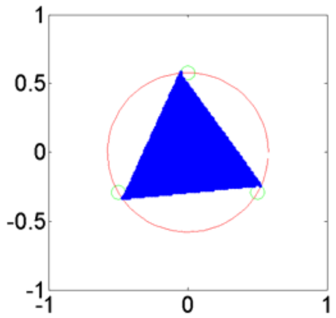
In the case that the external force is applied on the mobile platform and some entries of **N** are positive and some are negative, the range of λ can be identified so that it would create the positive tensions in wires. A range of λ that creates positive tension is found for each wire by determining the value of the corresponding λ that creates positive tension for that wire. The intersection of these ranges is the feasible range of λ that generates positive tension for all wires in the manipulator [4].

4 SIMULATION RESULTS

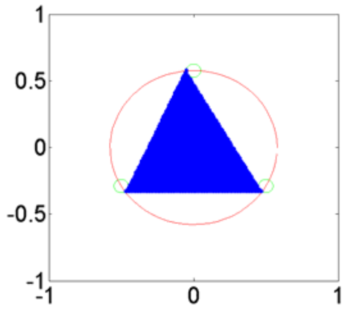
The translational planar WAPMs of Figure 1 are considered in this section. The positions of the center of pulleys, D_i, are changed on the perimeter of a circular frame, with a radius of $\sqrt{3}/3$ m, whose center lies at the origin of the fixed reference frame. The radius of the pulleys is r = 50 mm, the wire diameter is d_w = 5 mm, and the mass of the mobile platform is m = 1 kg. The area wherein the workspace is investigated is a 2m×2m square-shaped region. The minimum and maximum allowable wire tension is 0 N and 250 N, respectively. The small circles in the figures represent the pulleys.

In this section, the effect of design parameters on generating workspace is investigated. These parameters include the configuration of the manipulator based on the winding direction of wires around the pulleys, the position of the pulleys, the radius of the pulleys, and the direction of the external force.

To investigate which configuration generates the largest workspace, the three pulleys are considered to be in the same distance from each other on the circular frame so that the center of pulleys forms an equilateral triangle whose sides have a fixed length of1 m. Figure 3 depicts the workspace of the manipulators in Figure 1 when there is no external force applying on the mobile platform. The increments of 0.02 meters in the X and Y coordinates of the mobile platform position are used to generate the plots of this section. It should be noted that the resolution of the X and Y coordinates dictates the accuracy of calculating the workspace area. Calculating the workspace area of the two configurations shows that the workspace generated for the manipulator of Figure 1(a), where all wires are wound around the pulleys in the same direction, is larger than that of the manipulator in Figure 1(b).The workspace area of Figure 1(a) is 0.4189 m^2 and the workspace area of Figure 1(b) is 0.4160 m^2.The highlighted area in each plot is the collection of positions reachable by the mobile platform. At each point in Figure 3, the Jacobian matrix is calculated and inverted using equation (7) and a λ is identified to solve equation (8) such that the positive wire tensions do not exceed the maximum allowable tension. If there is no λ that makes each wire tension positive or if one/more wire tension exceed the maximum tension, then the position is not in the workspace.



(a) Workspace of manipulator in Figure 1(a)



(b) Workspace of manipulator in Figure 1(b)

Figure 3. Workspace of translational planar manipulators for two different configurations.

The variation of the workspace area for the manipulator in Figure 1(a) when the position of the pulleys is changed with respect to each other is depicted in Figure 4. The radius of pulleys is 50 mm, the radius of the circular frame is $\sqrt{3}/3$ m and the centre of pulley 1 is on the X-axis of the fixed reference frame with θ_1= 180˚. There is no external force applying on the mobile platform. The plot of Figure 4 shows that as the values of θ_2 and θ_3 increase, the area of the workspace increases until it reaches the maximum value. As can be seen in Figure 4, the area of the workspace in terms of different values of θ_2 and θ_3 has two maximum values. It should be noted that if the position of pulley 2 is switched with pulley 3, the size of the workspace does not change. The maximum workspace occurs when θ_2 is 115˚ and θ_3 is 240˚, and vice versa. Figure 5 depicts the largest possible workspace of the manipulator in Figure 1(a). The workspace area in Figure 5 is 0.5072 m^2 and the workspace area in Figure 3(a) is 0.4189 m^2.

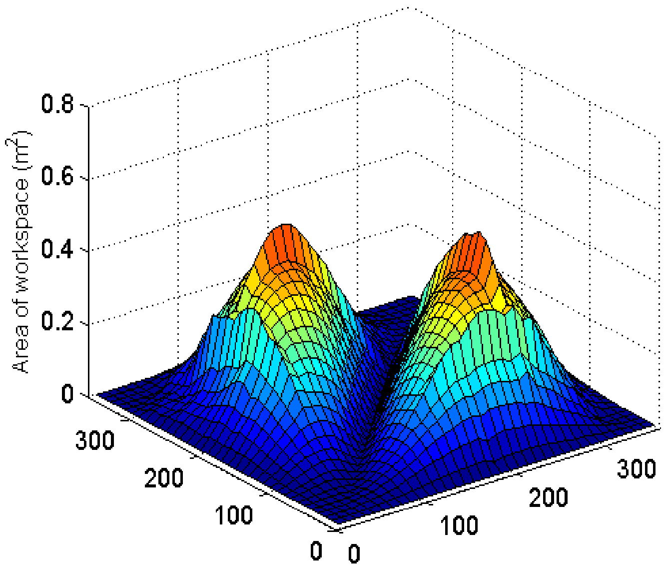


Figure 4. Variation of the workspace area for different values of θ_2 and θ_3 with no external force.

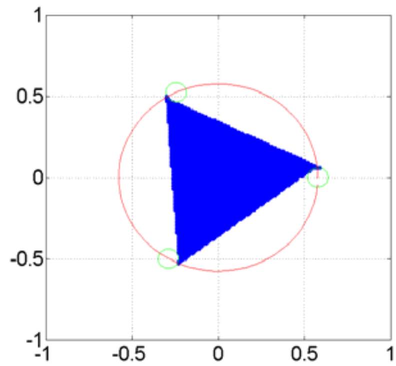


Figure 5. Largest workspace of manipulator in Figure 1(a) with no external force and pulley radius of 50 mm.

Another design parameter that could affect the workspace of the wire-actuated parallel manipulators is the radius of pulleys. It has been shown in [14] that as the radius of pulleys increases, the workspaces of the 3 DOF planar wire-actuated manipulators shrink. But the workspace of the manipulator in Figure 1(a) does not show such pattern. Having a point mass for the mobile platform, the number of wires, and the winding direction of wires around the pulleys may cause the irregular pattern depicted in Figure 6. Figure 6 illustrates the variation of the workspace area when the radius of pulleys changes from zero to 100 mm while the other parameters are kept constant. The largest workspace is obtained when the radius of pulleys is 58 mm.

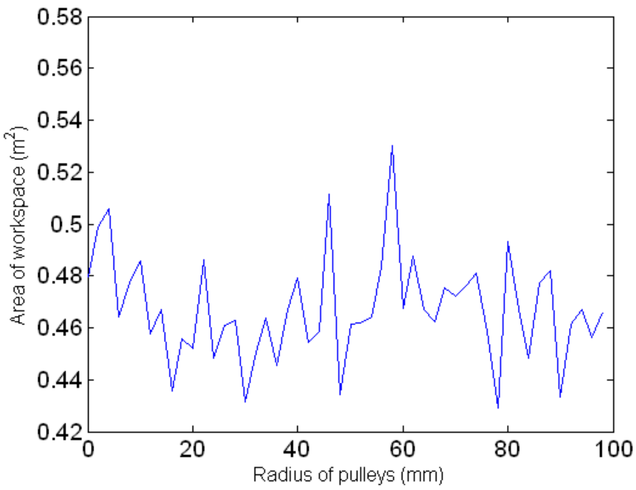


Figure 6. Variation of the workspace area in terms of the radius of pulleys.

When an external force is acting on the mobile platform, the workspace generated is larger than when there is no external force. In addition, the direction of the external force applying on the mobile platform affects the workspace area. Figure 7 shows the variation of workspace when the direction of the external force with respect to the X-axis is changed from 0˚ to 360˚ while the other parameters are kept constant. For the plot of this figure, the external force is 9.81 N, the radius of pulleys is 58 mm, and the values ofθ_1, θ_2, and $\theta_3$are 0˚, 115˚, and 240˚, respectively. The simulation results show that when the direction of the external force is 128˚, then the workspace reaches its maximum value. When the external force is directed 88˚with respect to the X-axis, the workspace shrinks to its minimum value. In the case that the plane of motion is parallel to the field of gravity, the mass of the mobile platform will affect the size of the workspace [14].Figure 8 shows the workspaces of the manipulator in Figure 1(a) corresponding to the largest and smallest area of workspace.

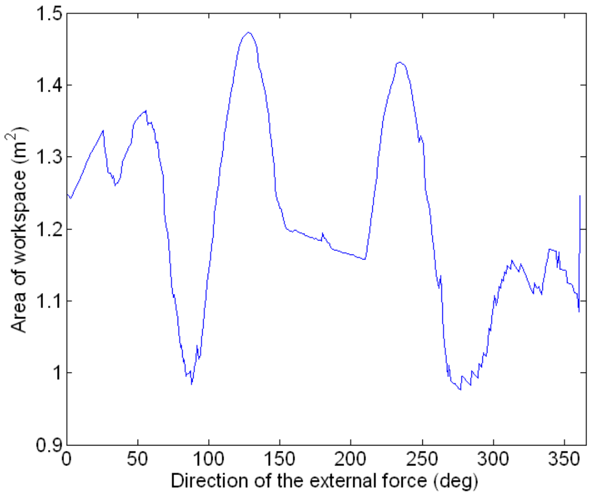
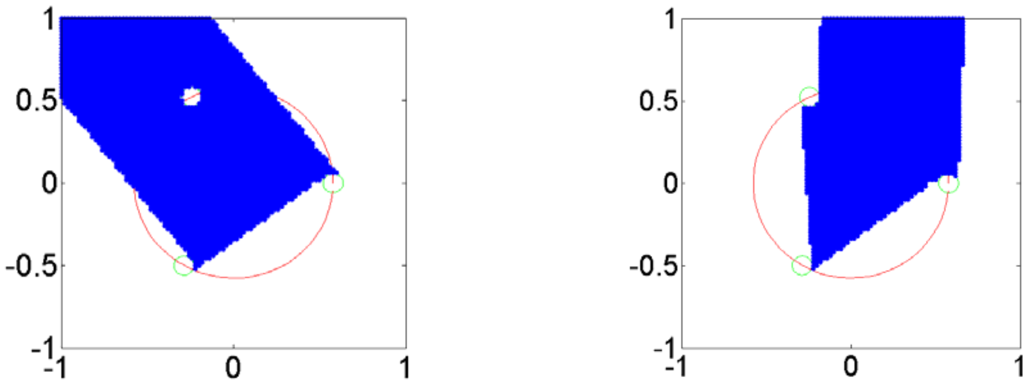


Figure 7. Workspace area as a function of the direction of the external force.



(a) (b)

Figure 8. Workspace of manipulator in Figure 1(a) when the external force generates (a) the largest workspace and (b) the smallest workspace.

In Figure 8, the largest possible workspace is obtained when the position of pulleys is fixed and an external force of 9.81 N is applied on the mobile platform. To investigate the variation of the workspace area as a function of the position of pulleys, the position of one of the pulleys is fixed and the other two pulleys are moved on the perimeter of the circular frame. When the orientation of the position vector of the center of fixed pulley is in the opposite direction of the external force, the workspace area increases regardless of the position of the other two pulleys.

The workspace area of the manipulator as a function of the position of pulleys 2 and 3 when the angular position of pulley 1, θ_1, is in the negative x direction and the external force of 9.81 N is in the positive X-direction is shown in Figure 9. Similar to the results shown in Figure 4, the plot of the workspace area has two maximum values. When the position of pulley 2 and pulley 3 is switched, the workspace has the same area as well. The maximum workspace occurs when $\theta_2$is 120° and $\theta_3$is 242° with respect to X-axis, and vice versa.

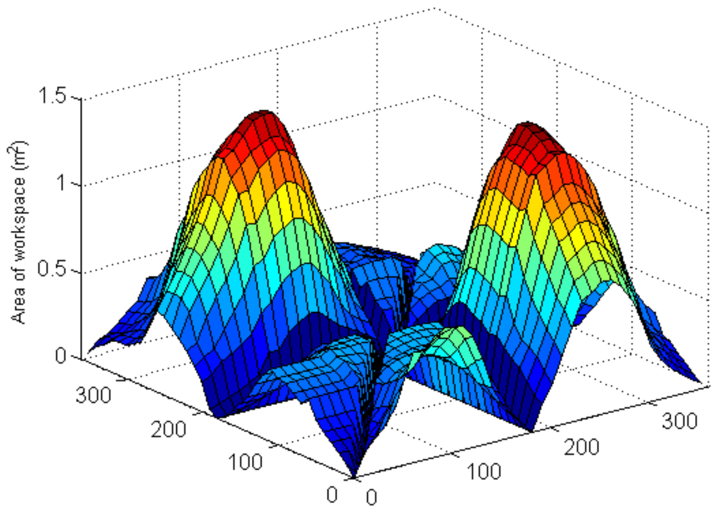


Figure 9. The graph of workspace area as a function of the position of pulleys 2 and 3.

Figure 10 depicts the largest possible workspace of the manipulator in Figure 1(a) when an external force of 9.81 N is applied on the mobile platform. The radius of the pulleys is 58 mm and the mass of the mobile platform is 1 kg.

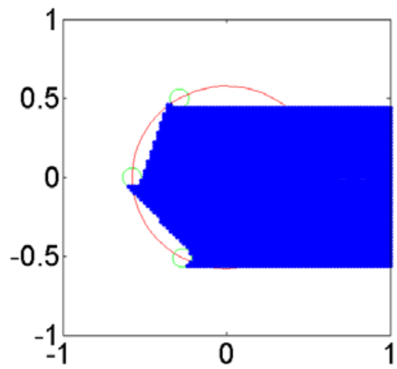


Figure 10. Workspace of manipulator in Figure 1(a) with an external force in X-direction.

5 CONCLUSION

In this paper, the layouts of the translational planar wire-actuated parallel manipulators that generate the largest possible workspace with and without an external force were investigated. The size of the .manipulator workspace is a function of the winding direction (clockwise or counter-clockwise) of wires around the pulleys, angular position of the pulleys, radius of the pulleys, and the direction of the external force. The force balance equations, taking into account the null space of the Jacobian matrix, were used for the workspace analysis. The condition of maintaining the positive wire tension was checked throughout the region to identify the static workspace of the manipulator. The results of the current paper could be used for fault tolerance purpose after a wire failure in a four-wire manipulator when it reduces to a three–wire manipulator with a point mass model for the mobile platform. One approach to studying fault tolerance of planar wire-actuated manipulators is to determine the layout in which the remaining wires of the manipulator could achieve the largest possible workspace after a failure occurs.

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