ADAPTIVE CONTROL OF MULTIPLE MOBILE MANIPULATORS TRANSPORTING A RIGID OBJECT

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ABSTRACT

This paper presents a nonlinear control scheme for multiple mobile manipulators robots (MMR) moving an object in coordination. The dynamic parameters of the handled object and the mobile manipulators are estimated online by using the virtual decomposition approach. The control law is designed based on an appropriate choice of the Lyapunov function candidate. The proposed control design ensures that the workspace position error converges to zero. Numerical simulations are carried out for two mobile manipulators transporting an object to show the effectiveness of the proposed controller.

Keywords*:* Mobile manipulator; Virtual decomposition approach; Coordination; Nonlinear control.

COMMANDE ADAPTATIVE D'UN GROUPE DE MANIPULATEURS MOBILE TRANSPORTANT UN OBJECT RIGIDE

RÉSUMÉ

Cet article présente un système de commande non linéaire d'un groupe de robots manipulateurs mobiles (RMM) transportant un objet rigide en coordination. Les paramètres dynamiques de l'objet manipulé et les manipulateurs mobiles sont estimés en ligne en utilisant l'approche de décomposition virtuelle. La loi de commande est conçue sur la base d'un choix approprié de la fonction candidate de Lyapunov. Le contrôleur proposé assure une convergence vers zéro de l'erreur en position dans l'espace de travail. Une simulation numérique d'un groupe de deux manipulateurs mobiles transportant un objet montre l'efficacité du contrôleur proposé.

Mots clés : manipulateur mobile; l'approche de décomposition virtuelle; coordination; commande non linéaire

1. INTRODUCTION

The need for robots capable of locomotion and manipulation led to the design of mobile manipulator robot (MMR) platforms. Typical examples of MMR are the arms of satellites, the underwater exploration in the seabed, the vehicles extra-planetary exploration. The most popular manipulators mobile, more or less automated, are the cranes mounted on trucks.

Some operations requiring the handling of heavy objects become very difficult for single mobile manipulator. These operations require the involvement of multiple mobile manipulators working in cooperation. However, this makes the robotic system very complex as its control design increases significantly in terms of complexity. The problem of controlling the mechanical system forming a closed kinematic chain mechanism lies in the fact that these types of system impose a set of kinematic constraints on the coordination of the position and velocity of the mobiles manipulator. Therefore, there is reduction of degrees of freedom for the entire system. Also, the object internal forces produced by all mobile manipulators must be controlled. Few works were proposed to solve the control problem of these robotic systems, having high degrees of freedom and tightly interconnected because all manipulators are in contact with the object.

1.1 Previous works

So far the majority of research works have focused on three big mechanisms of coordination's, the decentralized control, the leader-follower control approach and the motion planning.

In the first approach, the position and the internal force of the object are controlled in a certain direction of the workspace. Khatib [1] proposes an extension of a method developed for manipulators with fixed base to holonomic mobile manipulator robots, with a new command for decentralized cooperation tasks. In [2-4], the authors proposed a control algorithm using geometric constraints between the contact points, and the point representing the object that reduces the effect of sensor noise. Then they extended and implemented this algorithm to multiple omnidirectional mobile robots handling a single object in coordination. Another study proposed by Hirata and Kume in [5] is applied to a group of holonomic mobile manipulators transporting an object in coordination without using torque / force sensor.

In the second approach, the leader-follower approach is used for the coordination of multiple mobile manipulators. In this approach, a single or group of MMR are designated as leaders trying to follow a desired trajectory while the other group members follow the leaders. This control approach was addressed in [6], [7], [8]. In [9] the authors introduced the notion of virtual leader in which, every follower considers the rest of the team (leader and other followers) as the virtual leader.

Finally, few studies have addressed the Motion planning approach which is another fundamental problem in robotics especially in multi robots system, where more than one robot have to perform a task of transporting an object in cooperation in a known or unknown environment. Among related works are those presented in [10, 11]. Other structure for planning optimal trajectories was introduced in [12] for two mobile manipulators pushing a common object to a desired location. The authors in [13] proposed a control method for multiple mobile manipulators holding a common object. The measures of kinematic and dynamic manipulability are given taking into account collision avoidance. However the dynamics of the object is ignored. In [14] a method for trajectory planning of mobile manipulators groups in cooperation, taking into consideration the dynamic characteristics of mobile manipulators and the object to be grasped was proposed. The dynamics is composed of equations of motion of mobile manipulators, movement of the object, non-holonomic constraints of mobile platforms, and geometric constraints between the end-effectors and the object. In [15, 16] a planning approach based on genetic algorithms is proposed.

1.2 Main contribution

All previous studies based on Lagrangian or Newton/Euler approaches require knowledge of the exact parameters of the system. In fact, this is difficult in practice and the resulting model is usually uncertain.

Another problem is that, the dynamics of the whole system is complicated. Any change in the structure of the group requires a new dynamics modelling (remove a faulty robot or add a robot to the system). Finally, for these types of systems with a large degree of freedom and tightly coupled, the parameters' adaptation using methods based on the full dynamics is very complicated due to the size of parameters' vector. To overcome these problems, we propose in this paper an adaptive decentralized approach based on an extension of the virtual decomposition control (VDC) methodology in [17], originally designed for fixed-base robotic systems with large degrees of freedom. This approach will be used in this paper to a group of non-holonomic mobile manipulators robot moving without sliding effect. The advantages of this approach are that we do not need to know the dynamics of the overall system; it makes the system very flexible and the calculation of the dynamic system, with respect to the changes in the system configuration, very easy. This approach makes also the adaptation of the physical parameters very simple and systematic. Differently from what was done in the previous works, our contributions in this paper are in order:

- To overcome the problem of adaptation and modelling of the systems using classical approaches, we use the VDC approach, which makes the system more flexible when it changes its configuration. In this case, adding or removing a failed robot from the system doesn't require recalculating the full dynamics of the system.
- VDC requires adjusting a high number of control gains for each mobile manipulator. For a given mobile manipulator, with *n* degrees of freedom, we have *n* gains matrices, each of size 6x6 and one gain matrix for the object. In this paper, this number is reduced to a single gain matrix of maximal size 6x6, which facilitates the adjustment of these gains parameters.
- VDC's stability of the whole system is proved from the stability of each subsystem. In this paper, the stability analysis and the control law are designed based on an appropriate choice of a candidate Lyapunov function of the entire system.

The rest of the paper is organized as follows. Section II presents the modeling of the system; Section III gives the problem control statement. Section IV explains the control design and simulation results are given in Section V. Finally a conclusion is given in Section VI.

2. MODELING OF SYSTEM

This section will briefly describe the kinematics and dynamics of the *i*-th MMR, the dynamic model for the handling object then will provide the dynamics of the entire system.

Fig 1 shows the *N* MMR handling a common rigid object, P_{ie} is the position / orientation vector of the *i*-th MMR effector and X_o is position / orientation vector of the object.

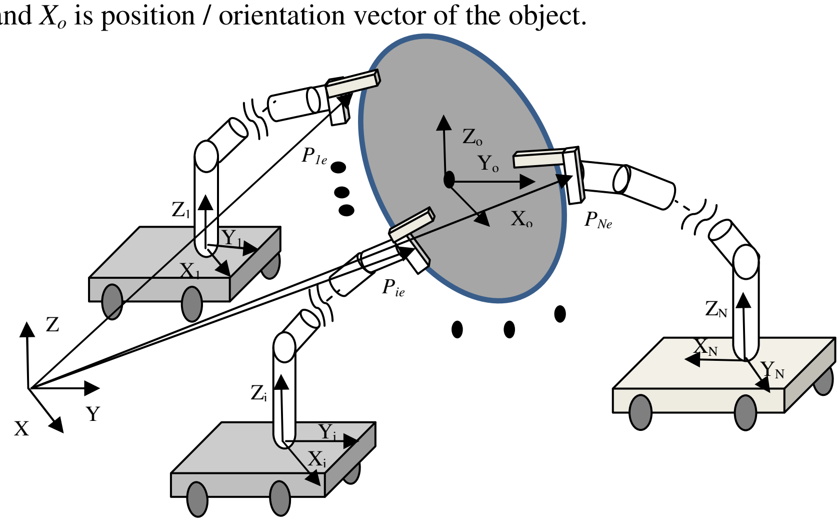

Fig. 1. Multiple MMRs handling a rigid object

2.1 Kinematics

The relationship between the effector velocity $\boldsymbol{V}_{ie} \in \mathbb{R}^6$ of the i-th mobile manipulator and the object velocity $\boldsymbol{V}_o \in \mathbb{R}^6$ is given by:

$$\boldsymbol{V}_{ie} = \boldsymbol{J}_{io}^T(\boldsymbol{X}_o)\boldsymbol{V}_o \tag{1}$$

where, $\boldsymbol{J}_{io} \in \mathbb{R}^{6\times 6}$ is the Jacobian matrix from the center of gravity of the objet to the i-th mobile manipulator end-effector.

2.2 The *i*-th mobile manipulator dynamics

The dynamic model of the *i-th* mobile manipulator without object is given in the joint space by the following equation:

$$\boldsymbol{M}_i(\boldsymbol{q}_i)\ddot{\boldsymbol{q}}_i + \boldsymbol{C}_i(\boldsymbol{q}_i,\dot{\boldsymbol{q}}_i)\dot{\boldsymbol{q}}_i + \boldsymbol{G}_i(\boldsymbol{q}_i) = \boldsymbol{U}_i \tag{2}$$

with, $\boldsymbol{M}_i \in R^{n\times n}$ the mass matrix, $\boldsymbol{C}_i \in \mathbb{R}^{n\times n}$ represents the Coriolis and centrifugal terms, $\boldsymbol{G}_i \in \mathbb{R}^n$ is the vector of gravity, and $\boldsymbol{U}_i \in \mathbb{R}^n$ is the input control vector.
In Cartesian workspace Eq. (2) is rewritten as follows:

$${}^i\boldsymbol{M}\dot{\boldsymbol{V}}_{ie} + {}^i\boldsymbol{C}\boldsymbol{V}_{ie} + {}^i\boldsymbol{G} = {}^i\boldsymbol{U} \tag{3}$$

Where, ${}^i\boldsymbol{M} = \boldsymbol{J}_{ie}^{+T}\boldsymbol{M}_i\boldsymbol{J}_{ie}^{+}$, ${}^i\boldsymbol{C} = \boldsymbol{J}_{ie}^{+T}\boldsymbol{M}_i\dot{\boldsymbol{J}}_{ie}^{+} + \boldsymbol{J}_{ie}^{+T}\boldsymbol{C}_i\boldsymbol{J}_{ie}^{+}$, ${}^i\boldsymbol{G} = \boldsymbol{J}_{ie}^{+T}\boldsymbol{G}_i$ and ${}^i\boldsymbol{U} = \boldsymbol{J}_{ie}^{+T}\boldsymbol{U}_i$. $\boldsymbol{J}_{ie} \in \mathbb{R}^{6\times n}$ is the Jacobian matrix of the MMR and $\boldsymbol{J}_{ie}^{+}$ is its Pseudo-inverse.

2.3 Object dynamic

The object handled by the N mobiles manipulators is rigid. To find the dynamic model that characterizes this manipulated object, the Newton Euler method is used. The equation of motion of the effort is given by:

$$\boldsymbol{M}_o(\boldsymbol{x}_o)\dot{\boldsymbol{V}}_o + \boldsymbol{C}_o(\boldsymbol{x}_o,\dot{\boldsymbol{x}}_o)\boldsymbol{V}_o + \boldsymbol{G}_o(\boldsymbol{x}_o) = \boldsymbol{F}_o \tag{4}$$

where $\boldsymbol{V}_o = [\boldsymbol{v}_o, \boldsymbol{w}_o]^T$, with $\boldsymbol{v}_o \in \mathbb{R}^3$ and $\boldsymbol{w}_o \in \mathbb{R}^3$ the linear and angular velocity, respectively. $\boldsymbol{F}_o \in \mathbb{R}^6$ is the vector of forces applied to the object, $\boldsymbol{M}_o \in \mathbb{R}^{6\times 6}$ is the mass matrix, $\boldsymbol{C}_o \in \mathbb{R}^{6\times 6}$ represents the centrifugal and Coriolis matrix and the $\boldsymbol{G}_o \in \mathbb{R}^6$ vector of gravity.

2.4 Total dynamic

From the general form of i-th mobile manipulator Eq. (3), the dynamic of N mobile manipulators without object is given by:

$$\boldsymbol{M}\dot{\boldsymbol{V}}_e + \boldsymbol{C}\boldsymbol{V}_e + \boldsymbol{G} = \boldsymbol{U} \tag{5}$$

with, $\boldsymbol{V}_e = \left[\boldsymbol{V}_{1e}^T, \boldsymbol{V}_{2e}^T, ..., \boldsymbol{V}_{Ne}^T\right]^T$, $\boldsymbol{M} = diag\left[{}^1\boldsymbol{M}, {}^2\boldsymbol{M}, ..., {}^N\boldsymbol{M}\right]$, $\boldsymbol{G} = \left[{}^i\boldsymbol{G}^T, {}^i\boldsymbol{G}^T, ..., {}^N\boldsymbol{G}^T\right]^T$ and $\boldsymbol{U} = \left[{}^1\boldsymbol{U}^T, {}^2\boldsymbol{U}^T, ..., {}^N\boldsymbol{U}^T\right]^T$.

In the presence of the object, the dynamics of the entire system is given by:

$$M\dot{V}_e + CV_e + G = U - F_e \tag{6}$$

with $F_e = \left[F_{1e}^T, F_{2e}^T, ..., F_{Ne}^T\right]^T \in \mathbb{R}^{6N}$. These effectors' forces are related to the object force by:

$$F_o = -J_o\left(x_o\right)F_e \tag{7}$$

Where $J_o \in \mathbb{R}^{6\times 6N}$ is the Jacobian matrix relating the two forces. Furthermore, the force of effector F_e is divided into two orthogonal components: the first contributes to the movement of the object and the second gives the internal force. This representation is given in [18], and has the following form:

$$F_e = -\left(J_o\left(x_o\right)\right)^+ F_o - F_I \tag{8}$$

where $J_o\left(x_o\right)^+$ is the pseudo inverse of $J_o\left(x_o\right)$ and $F_I \in \mathbb{R}^{6N}$ are the internal forces in the null space of $J_o\left(x_o\right)$.

3. Control Problem Statement

The control objective is to generate a set of torque inputs such that the error tracking position of the transported objet in the workspace converges asymptotically to zero. Formally speaking the control problem is to design the control input:

$$U = f\left(\dot{V}_e, V_e, X_o, V_o\right)$$

In such a way the following limits hold:

$$\lim_{t\to\infty}\left\|X_o - X_o^d\right\| = 0, \text{ and } \lim_{t\to\infty}\left\|V_o - V_o^d\right\| = 0 .$$

where $X_o^d \in \mathbb{R}^6, V_o^d \in \mathbb{R}^6$ are the desired position and velocity of the object generated in the workspace.

4. Control Problem Design

4.1 Methodology

The overall control system is designed in the following steps:

- The required velocity of the object $V_o^r \in \mathbb{R}^6$ as well as the velocities of the end-effectors in the workspace $V_{ie}^r \in \mathbb{R}^6$, are firstly computed then the required velocity ${}^iV_B^r \in \mathbb{R}^{6n}$ of the *n* body-fixed frames ${}^iB_j$ is calculated.
- The VDC approach is used to simplify the problem of the parameters adaptation of the complete systems where this problem is converted into a problem of estimating the parameters of each subsystem. From the velocities computed in the first step, calculate the estimated parameters.
- The control law of each mobile manipulator is designed.

4.2 Design

Step 1: The required velocity $V_o^r \in \mathbb{R}^6$ of the object is calculated based on the desired object velocity:

$$\boldsymbol{V}_o^r = \boldsymbol{V}_o^d + \lambda \boldsymbol{e}_o \tag{9}$$

where, $\boldsymbol{e}_o = \boldsymbol{X}_o^d - \boldsymbol{X}_o$ the position error vector and λ is a scalar constant.

The desired velocity of the end-effector of each mobile manipulator $\boldsymbol{V}_{ie}^d \in \mathbb{R}^6$ is computed based on the desired velocity of the object $\boldsymbol{V}_o^d \in \mathbb{R}^6$:

$$\boldsymbol{V}_{ie}^d = \boldsymbol{J}_{io}^T \boldsymbol{V}_o^d \tag{10}$$

The derivative of i-th end-effector's position error is given by:

$$\dot{\boldsymbol{e}}_i = \boldsymbol{V}_{ie}^d - \boldsymbol{V}_{ie} \tag{11}$$

Introducing the object velocity, Equation (11) be rewritten as:

$$\dot{\boldsymbol{e}}_i = \boldsymbol{J}_{io}^T \left(\boldsymbol{V}_o^d - \boldsymbol{V}_o \right) = \boldsymbol{J}_{io}^T \dot{\boldsymbol{e}}_o \tag{12}$$

Finally, the required velocity of the i-th end-effector is expressed by:

$$\boldsymbol{V}_{ie}^r = \boldsymbol{V}_{ie}^d + \lambda \boldsymbol{J}_{io}^T \boldsymbol{e}_o \tag{13}$$

And, from this velocity, the required velocity in different frame $\boldsymbol{B}$ is given by:

$$^i\boldsymbol{V}_B^r = \boldsymbol{J}_i \boldsymbol{V}_{ie}^r \tag{14}$$

with, $^i\boldsymbol{V}_B^r = [\,^i\boldsymbol{V}_{B_1}^{r^T} \ \dots \ ^N\boldsymbol{V}_{B_n}^{r^T}\,]^T \in \mathbb{R}^{6n}$ and $\boldsymbol{J}_i \in \mathbb{R}^{6n\times 6}$ the Jacobian matrix.

Step 2: In this step, the idea is to decompose virtually the robotic system into several objects and open chains. An object is a rigid body and an open chain consists of a series of rigid links connected one by one. This decomposition is illustrated in Figure 2.

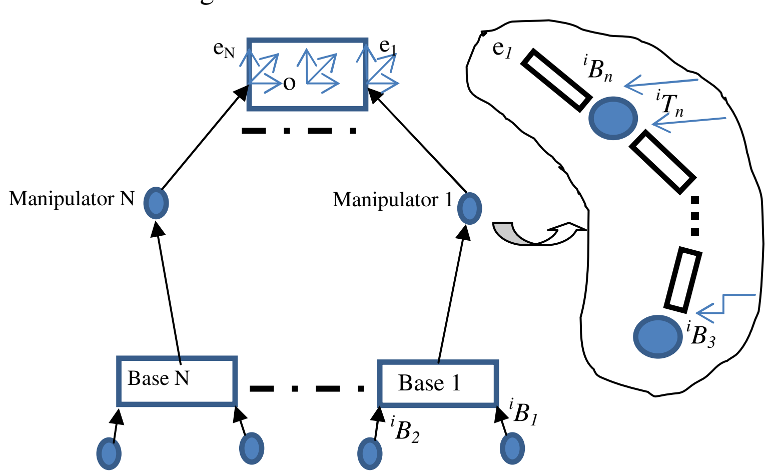

Fig. 2. Virtual decomposition of N mobile manipulators handling a rigid object

The transformation matrix of force/moment vectors from frame B to frame A is defined by:

$$ {}^{A}\boldsymbol{U}_{B} = \begin{bmatrix} {}^{A}\boldsymbol{R}_{B} & \boldsymbol{0}_{3\times3} \\ \boldsymbol{S}\left({}^{A}\boldsymbol{r}_{AB}\right)\times{}^{A}\boldsymbol{R}_{B} & {}^{A}\boldsymbol{R}_{B} \end{bmatrix} \tag{15} $$

where ${}^{A}\boldsymbol{R}_{B} \in \mathbb{R}^{3\times3}$ is the rotation matrix between frame A and B, and $\boldsymbol{S}\left({}^{A}\boldsymbol{r}_{AB}\right) \in \mathbb{R}^{3\times3}$ is a skew symmetric matrix built from the vector ${}^{A}\boldsymbol{r}_{AB} \in \mathbb{R}^{3}$ linking the origins of frame A and B, expressed in the coordinates of frame A.

The dynamics of the object based on its required velocity $\boldsymbol{V}_{o}^{r} \in \mathbb{R}^{6}$ is expressed in the linear form by the following equation:

$$ \boldsymbol{F}_{or}^{*} = \boldsymbol{M}_{o}\dot{\boldsymbol{V}}_{o}^{r} + \boldsymbol{C}_{o}\left(\boldsymbol{w}_{o}\right)\boldsymbol{V}_{o}^{r} + \boldsymbol{G}_{o} = \boldsymbol{Y}_{o}\boldsymbol{\theta}_{o} \tag{16} $$

with $\boldsymbol{F}_{or}^{*} \in \mathbb{R}^{6}$ the required object's force, $\boldsymbol{\theta}_{o} \in \mathbb{R}^{13}$ the parameter vector, $\boldsymbol{Y}_{o} \in \mathbb{R}^{6\times13}$ the dynamic regressor matrix. Since the physical parameters of object are unknown and need to be estimated, using the estimated parameters of $\boldsymbol{\theta}_{o}$ noted $\hat{\boldsymbol{\theta}}_{o}$, Eq. (16) can be written as:

$$ \boldsymbol{F}_{or}^{*} = \hat{\boldsymbol{M}}_{o}\dot{\boldsymbol{V}}_{o}^{r} + \hat{\boldsymbol{C}}_{o}\left(\boldsymbol{w}_{o}\right)\boldsymbol{V}_{o}^{r} + \hat{\boldsymbol{G}}_{o} = \boldsymbol{Y}_{o}\hat{\boldsymbol{\theta}}_{o} \tag{17} $$

where, $\dot{\hat{\boldsymbol{\theta}}}_{o} = -\boldsymbol{\rho}_{o}\boldsymbol{Y}_{o}^{T}\boldsymbol{s}_{o}$ is the adaptation function, and is chosen to ensure system stability, $\boldsymbol{s}_{o} = \boldsymbol{V}_{o}^{r} - \boldsymbol{V}_{o}$ and $\boldsymbol{\rho}_{o} \in \mathbb{R}^{13\times13}$ is diagonal positive matrix.

The dynamics of the jth rigid body of the ith manipulator is given in the linear form by the following equation:

$$ {}^{i}\boldsymbol{F}_{B_j}^{*r} = {}^{i}\boldsymbol{M}_{B_j}\,{}^{i}\dot{\boldsymbol{V}}_{B_j}^{r} + {}^{i}\boldsymbol{C}_{B_j}\left(\boldsymbol{w}_{B_j}^{r}\right){}^{i}\boldsymbol{V}_{B_j}^{r} + {}^{i}\boldsymbol{G}_{B_j} = {}^{i}\boldsymbol{Y}_{B_j}\,{}^{i}\boldsymbol{\theta}_{B_j} \tag{18} $$

with ${}^{i}\boldsymbol{M}_{B_j} \in \mathbb{R}^{6\times6}$ the matrix of inertial terms ${}^{i}\boldsymbol{C}_{B_j} \in \mathbb{R}^{6\times6}$ the matrix of centrifuge/coriolis term ${}^{i}\boldsymbol{G}_{B_j} \in \mathbb{R}^{6}$ the vector related to the gravity, ${}^{i}\boldsymbol{\theta}_{B_j} \in \mathbb{R}^{13}$ the parameter vector, and finally $\boldsymbol{Y}_{B_j} \in \mathbb{R}^{6\times13}$ is the dynamic regressor matrix.

Since the physical parameters of the i-th mobile manipulator are unknown and need to be estimated, then the vector ${}^{i}\hat{\boldsymbol{\theta}}_{B_j} \in \mathbb{R}^{13}$ is used and the dynamic Eq.(18) becomes:

$$ {}^{i}\boldsymbol{F}_{B_j}^{*r} = {}^{i}\hat{\boldsymbol{M}}_{B_j}\,{}^{i}\dot{\boldsymbol{V}}_{B_j}^{r} + {}^{i}\hat{\boldsymbol{C}}_{B_j}\left(\boldsymbol{w}_{B_j}^{r}\right){}^{i}\boldsymbol{V}_{B_j}^{r} + {}^{i}\hat{\boldsymbol{G}}_{B_j} = {}^{i}\boldsymbol{Y}_{B_j}\,{}^{i}\hat{\boldsymbol{\theta}}_{B_j} \tag{19} $$

where, ${}^{i}\dot{\hat{\boldsymbol{\theta}}}_{B_j} = -{}^{i}\boldsymbol{\rho}_{B_j}\,{}^{i}\boldsymbol{Y}_{B_j}^{T}\,{}^{i}\boldsymbol{s}_{B_j}$ is the adaptation function, and is chosen to ensure system stability, ${}^{i}\boldsymbol{s}_{B_j} = {}^{i}\boldsymbol{V}_{B_j}^{r} - {}^{i}\boldsymbol{V}_{B_j}$ and ${}^{i}\boldsymbol{\rho}_{B_j} \in \mathbb{R}^{13\times13}$ is diagonal positive matrix.

The vector of resulting forces / moments acting on the j-th rigid body is given by an iterative process. We begin by computing the vector of forces in the different cutting points:

$$ {}^{i}\boldsymbol{F}_{B_n} = {}^{i}\boldsymbol{F}_{B_n}^{*} $$

$$\begin{aligned} &{}^{i}\boldsymbol{F}^{r}_{B_{n-1}} = {}^{i}\boldsymbol{F}^{*r}_{B_{n-1}} + {}^{jB_{n-1}}\boldsymbol{U}_{B_n}\, {}^{i}\boldsymbol{F}^{*r}_{B_n} \\ &\vdots \\ &{}^{i}\boldsymbol{F}^{r}_{B_1} = {}^{i}\boldsymbol{F}^{*r}_{B_1} + {}^{B_1}\boldsymbol{U}_{B_2}\, {}^{i}\boldsymbol{F}^{*r}_{B_2} + \ldots + {}^{B_1}\boldsymbol{U}_{B_n}\, {}^{i}\boldsymbol{F}^{*r}_{B_n} \end{aligned} \quad (20)$$

From the Eq. (19) of the dynamics of rigid body and the forces in the different frame Eq. (20), we can express the dynamic model of *i*-th mobile manipulator by the following formula:

$$\boldsymbol{J}_i^T\, {}^{i}\boldsymbol{Y}\, {}^{i}\hat{\boldsymbol{\theta}} = {}^{i}\boldsymbol{U} \quad (21)$$

with ${}^{i}\boldsymbol{Y} = Blockdiag({}^{i}\boldsymbol{Y}_{B_1}, \ldots, {}^{i}\boldsymbol{Y}_{B_n}) \in \mathbb{R}^{13n\times 6n}$ regressor matrix, ${}^{i}\hat{\boldsymbol{\theta}} = [\,{}^{i}\hat{\boldsymbol{\theta}}^{T}_{B_1} \ldots {}^{i}\hat{\boldsymbol{\theta}}^{T}_{B_n}\,]^T \in \mathbb{R}^{13n}$ is a vector of the parameters vectors and $\boldsymbol{J}_i \in \mathbb{R}^{6n\times 6}$ is the Jacobian matrix.

Finally, we add to the dynamics of the manipulator the effect of the force at the contact point $\boldsymbol{F}_{ie} \in \mathbb{R}^6$ we obtain:

$${}^{i}\boldsymbol{U} - \boldsymbol{F}_{ie} = {}^{i}\boldsymbol{Q}\, {}^{i}\hat{\boldsymbol{\theta}} \quad (22)$$

with, ${}^{i}\boldsymbol{Q} = \boldsymbol{J}_i^T\, {}^{i}\boldsymbol{Y}$ and $\boldsymbol{F}_{ie}$ is computed from Eq. (8).

Step 3: the control law of the *i*-th mobile manipulators is given by:

$${}^{i}\boldsymbol{U} = {}^{i}\boldsymbol{Q}\, {}^{i}\widehat{\boldsymbol{\theta}} + \boldsymbol{F}^{d}_{ie} - {}^{i}\boldsymbol{\Gamma}\, {}^{i}\boldsymbol{s} \quad (23)$$

with, $\boldsymbol{s} = \boldsymbol{J}_o^T \boldsymbol{s}_o$ and ${}^{i}\dot{\widehat{\boldsymbol{\theta}}} = -{}^{i}\boldsymbol{\rho}\, {}^{i}\boldsymbol{Y}^{T}\, {}^{i}\boldsymbol{s}_{\boldsymbol{B}} = -{}^{i}\boldsymbol{\rho}\, {}^{i}\boldsymbol{Q}^{T}\, {}^{i}\boldsymbol{s}$ where ${}^{i}\boldsymbol{s}_{\boldsymbol{B}} = [\,{}^{i}\boldsymbol{s}^{T}_{B_1} \ldots {}^{i}\boldsymbol{s}^{T}_{B_n}\,]^T \in \mathbb{R}^{6n}$, ${}^{i}\boldsymbol{s} = \boldsymbol{V}^{r}_{ie} - \boldsymbol{V}_{ie}$ and ${}^{i}\boldsymbol{\Gamma} \in \mathbb{R}^{6\times 6}$ a diagonal positive gains matrix, we assume that the object moves in 6DoF . The control law of the *N* MMRs is represented by:

$$\boldsymbol{U} = \boldsymbol{Q}\,\widehat{\boldsymbol{\theta}} + \boldsymbol{F}^{d}_{e} - \boldsymbol{\Gamma}\,\boldsymbol{s} \quad (24)$$

with $\boldsymbol{Q} = diag({}^{1}\boldsymbol{Q},..{}^{i}\boldsymbol{Q},..{}^{N}\boldsymbol{Q})$, $\boldsymbol{\Gamma} = diag({}^{1}\boldsymbol{\Gamma},..{}^{i}\boldsymbol{\Gamma},\, {}^{N}\boldsymbol{\Gamma})$, $\widehat{\boldsymbol{\theta}} = [\,{}^{1}\widehat{\boldsymbol{\theta}}^{T} \,..{}^{i}\widehat{\boldsymbol{\theta}}^{T} \ldots {}^{N}\widehat{\boldsymbol{\theta}}^{T}\,]^T$, $\boldsymbol{F}^{d}_{e} = [\boldsymbol{F}^{d^T}_{1e} \,..\boldsymbol{F}^{d^T}_{ie} \ldots \boldsymbol{F}^{d^T}_{Ne}\,]^T$,
$\boldsymbol{U} = \left({}^{1}\boldsymbol{U}^{T}, \ldots, {}^{i}\boldsymbol{U}^{T}, \ldots.{}^{N}\boldsymbol{U}^{T}\right)^T$ and $\boldsymbol{s} = \left({}^{1}\boldsymbol{s}^{T}, \ldots, {}^{i}\boldsymbol{s}^{T}, \ldots.{}^{N}\boldsymbol{s}^{T}\right)^T$.

The main result of this paper is given in the following theorem.

Theorem 1:

Consider the MMR system dynamic Eq. (6) and the object dynamic Eq. (4), under the control design Eq. (23).

Then, the closed loop systems' states are bounded, in particular, the control objective is satisfied and the error tracking states are asymptotically stable in the sense that:

- $\lim_{t\to\infty}\left\|\boldsymbol{X}_o - \boldsymbol{X}^{d}_{o}\right\| = 0,$ and $\lim_{t\to\infty}\left\|\boldsymbol{V}_o - \boldsymbol{V}^{d}_{o}\right\| = 0$
- $\lim_{t\to\infty}\left\|\boldsymbol{P}_{ie} - \boldsymbol{P}^{d}_{ie}\right\| = 0,$ and $\lim_{t\to\infty}\left\|\boldsymbol{V}_{ie} - \boldsymbol{V}^{d}_{ie}\right\| = 0$

Proof:

Consider also this Lyapunov function candidate:

$$\boldsymbol{V} = \frac{\boldsymbol{1}}{\boldsymbol{2}}(\boldsymbol{s}_o^T \boldsymbol{M}_o \boldsymbol{s}_o + \boldsymbol{\Delta\theta}_o^T \boldsymbol{\rho}_o^{-1} \boldsymbol{\Delta\theta}_o + \boldsymbol{s}^T \boldsymbol{M s} + \boldsymbol{\Delta\theta}^T \boldsymbol{\rho}^{-1} \boldsymbol{\Delta\theta}) \tag{25}$$

where, $\boldsymbol{\Delta\theta}_o = \hat{\boldsymbol{\theta}}_o - \boldsymbol{\theta}_o$ is an estimate error vector of the object parameters and $\boldsymbol{\Delta\theta} = \hat{\boldsymbol{\theta}} - \boldsymbol{\theta}$ is an estimate error vector of the *N* mobile manipulators parameters.
The time derivative along the solutions of Eqs.(23-25) gives the following:

$$\dot{\boldsymbol{V}} = \boldsymbol{s}_o^T \boldsymbol{M}_o \dot{\boldsymbol{s}}_o + \frac{\boldsymbol{1}}{\boldsymbol{2}} \boldsymbol{s}_o^T \dot{\boldsymbol{M}}_o \boldsymbol{s}_o + \boldsymbol{\Delta}\dot{\boldsymbol{\theta}}_o^T \boldsymbol{\rho}_o^{-1} \boldsymbol{\Delta\theta}_o + \boldsymbol{s}^T \boldsymbol{M}\dot{\boldsymbol{s}} + \frac{\boldsymbol{1}}{\boldsymbol{2}} \boldsymbol{s}^T \dot{\boldsymbol{M}} \boldsymbol{s} + \boldsymbol{\Delta}\dot{\boldsymbol{\theta}}^T \boldsymbol{\rho}^{-1} \boldsymbol{\Delta\theta}$$

Further but simple calculation using the control law (24) yields:

$$\dot{\boldsymbol{V}} = -\boldsymbol{s}^T \boldsymbol{\Gamma s} \tag{26}$$

According to Barbalat's lemma, the system is stable. It is uniformly bounded. Thus, if we assume that the trajectory is uniformly bounded, according to Eq. (24) $\dot{s}$ is uniformly bounded. The second derivative of Lyapunov function is given by:

$$\ddot{\boldsymbol{V}} = -\boldsymbol{2s}^T \boldsymbol{\Gamma} \dot{\boldsymbol{s}} \tag{27}$$

$\ddot{V}$ is bounded, as s and $\dot{s}$ are bounded, which proves that $\dot{V}$ is uniformly continuous. It follows from Eq. (25), $\dot{V} \to 0$ as $t \to \infty$. This implies that s and s_o converges asymptotically to zero respectively, which in turn implies the asymptotic convergence of the tracking errors $\boldsymbol{e}_o$ and $\dot{\boldsymbol{e}}_o$ to be zero. We infer that the control objectives are fulfilled: $\boldsymbol{X}_o \to \boldsymbol{X}_o^d$ and $\boldsymbol{V}_o \to \boldsymbol{V}_o^d$.

5. SIMULATION RESULTS

Numerical simulations are carried out on two identical 2DoF MMRs handling a rigid object in coordination. The parameters of both MMR and the object are given in Table 1 [19].

Table 1. System parameters

	Masses (kg)	Moment of Inertia (kg.m^2)	Length (m)
Object	1	1	-
Articulation 1 (rotoid)	1	1	L_1=1
Articulation 2 (rotoid)	1	1	L_2=1
Platform	6	19	d=1 , r=0.5

The desired trajectory of the center of gravity of the object is generated in the Cartesian space. The starting point is $P_o(x_o, y_o, z_o, Beta) = (1.0, 1.1, 1.0, 0.0)$ and the arrival point is $P_f(x_o, y_o, z_o, Beta) = (1.0, 5.0, 1.2, 0.0)$ with a sinusoidal along the X-axis. The initial position is chosen as $X_o = (1.0, 1.1, 1, 0.0)$ and the velocity is chosen as $V_o = 0$. The control gains of the controller are chosen by the trial and error method to be ${}^i\Gamma = diag(200, 200, 200, 200)$, we assume that the object moves in 4DoF and $\lambda = 180$. The trajectory tracking is presented in Figure 3. The simulation results in the Cartesian space are presented in Figure 4 shows the desired and obtained trajectory along the X-axis, the Y-axis; the Z-axis and the variation of orientation along Z-axis. The trajectory tracking errors are presented in Figure 5.Under the proposed control law the tracking of the desired trajectory is achieved. The simulation results show the effectiveness of the proposed adaptive control in the presence of uncertain parameters.

Clearly, as shown in the simulation results, the trajectory tracking objective for the cooperating MMR carrying an object is successfully satisfied.

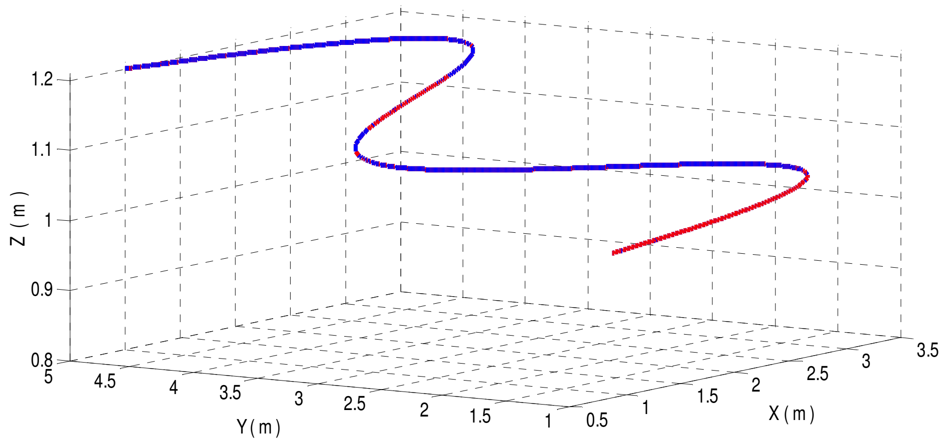

Fig. 3. The trajectory of the object.

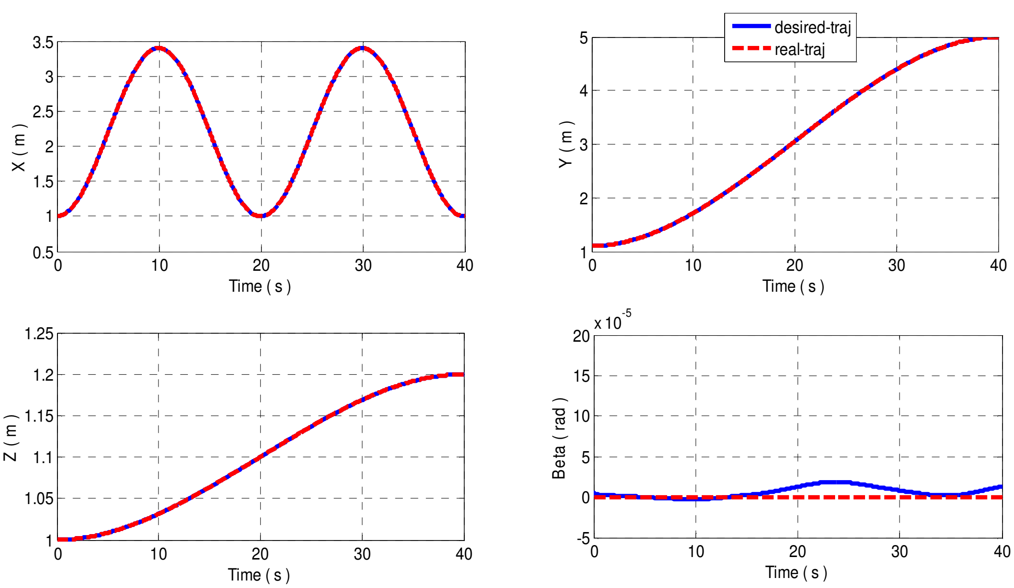

Fig. 4. The trajectories tracking in Cartesian space: X-axis, Y-axis, Z-axis and the orientation β_z .

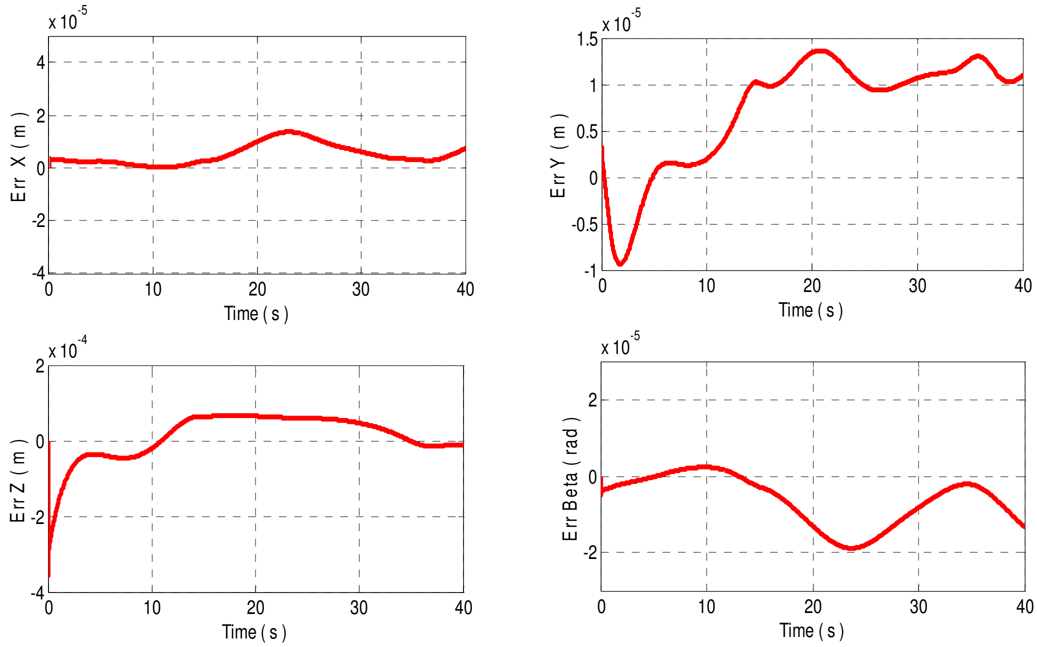

Fig. 5. Trajectory tracking errors.

6. CONCLUSION

In this paper a coordinated control scheme for a multiple manipulators mobiles transporting a rigid object in coordination has been presented. The desired trajectory of the object is generated in the workspace and the dynamic parameters of the handling object and the mobile manipulators are estimated online by using the virtual decomposition approach. The control law is designed based on an appropriate choice of Lyapunov function candidate and the asymptotic stability is proved. The proposed control design ensures that the workspace position error converge to zero asymptotically. The numerical simulation results showed the effectiveness of the proposed control.

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