TOWARDS THE APPROPRIATE SYNTHESIS OF THE FOUR-BAR LINKAGE

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ABSTRACT

Uncertainties are an inherent element in all mechanisms, arising from the manufacturing and assembly process or even from the operation of the device. In terms of synthesis routines for mechanisms, uncertainties are generally neglected since they are difficult to account for. In this work, the concept of appropriate design is utilized to develop routines which can more easily account for uncertainties in the geometrical parameters. These routine have been developed for linkages, specifically the four-bar linkage, and are capable of synthesizing the complete set of design solutions, referred to as allowable regions, for a set of desired coupler curve characteristics. The description of the desired coupler curve may contain any number of precision points and/or trajectories. Several problems are solved in this work, including obtaining a representation of the coupler curve corresponding to a set of design parameters containing uncertainties, and synthesizing the appropriate designs for multiple descriptions of desired coupler curves. The results are quite promising and show great potential for using the appropriate design methodology for linkage synthesis.

Keywords: Uncertainties, interval analysis, dimensional synthesis, coupler curve.

CONCEPTION APPROPRIÉE DES MÉCANISMES DE QUATRE BARRES

RÉSUMÉ

Les incertitudes sont inhérentes à tous les mécanismes, résultant des tolérances de fabrication et d'assemblage ou à cause du fonctionnement même du mécanisme. Les incertitudes sont généralement négligées dans les outils de conceptions des mécanismes, car elles sont difficiles à intégrer. Dans ce travail, le concept de conception appropriée est utilisé pour développer des modules qui peuvent plus facilement prendre en compte les incertitudes dans les paramètres géométriques. Ces modules ont été développés pour les mécanismes, en particulier les mécanismes plans à quatre barres, et sont capables de synthétiser l'ensemble complet des solutions de conception, appelées régions admissibles, pour un ensemble de caractéristiques de courbes du point de bielle souhaitées. La description de la courbe du point de bielle souhaitée peut contenir un certain nombre de points de précision et / ou des trajectoires. Plusieurs problèmes sont résolus dans ce travail, y compris l'obtention d'une représentation de la courbe du point de bielle correspondant à un ensemble de paramètres de conception contenant des incertitudes et la conception appropriée pour des descriptions multiples des courbes du point de bielle souhaitées. Les résultats sont très prometteurs et présentent un grand potentiel pourutiliser la méthodologie de conception appropriée pour la synthèse des mécanismes.

Mots-clés : Incertitudes, analyse par intervalles, conception dimensionelle, courbe du point de bielle.

INTRODUCTION

The four-bar linkage is a planar mechanism consisting of four rigid members: the frame, input link, output link, and coupler link. These members are connected by four revolute pairs forming a closed-loop kinematic chain with 1-degree-of-freedom. A point on the coupler link, known as the *coupler point*, traces a path as the input link is rotated; this is referred to as the *coupler curve*. Desirable coupler curves may be generated when the geometric parameters of the linkage are properly selected. The process of selecting these parameters, based on some desired coupler curve characteristics, is known as dimensional synthesis. *Exact synthesis* concerns precisely replicating the desired coupler curve characteristics, whereas *approximate synthesis* considers replicating the desired coupler curve characteristics with an allowable amount of error.

An issue with conventional synthesis techniques is accounting for the uncertainties inherent in mechanisms. Unless the mechanical errors are accounted for in the synthesis routine, the desired performance may not be physically possible in a synthesized design. The issue of uncertainties plays a significant role in the actual performance of a mechanism, making it no longer possible to obtain an exact solution for the coupler point. Instead, the coupler point can only be determined to lie within some bounds, where the bounds are functions of the uncertainties in the mechanism. Exact synthesis is not applicable to mechanisms with uncertainties and conventional approximate synthesis techniques are not able to deal with uncertainties.

It would be beneficial to account for the uncertainties during synthesis, such that the desired performance of a mechanism is achieved. Moreover, it is useful to provide a set of allowable designs which satisfy the desired criteria. A designer is then free to select the design which best suit their needs. To this end, the concept of *appropriate design* is applied to the synthesis of four-bar linkages.

Appropriate design was introduced for the synthesis of parallel manipulators by Merlet and Daney [1]. The appropriate design methodology involves computing *allowed regions* which satisfy the desired characteristics of the mechanism. All of the design solutions inside the allowed regions are guaranteed to achieve the desired characteristics while being robust with respect to uncertainties. The general idea is that the end-user can select any solution from inside the allowed regions and obtain performance which meets or exceeds the desired characteristics of the mechanism. Thus, applying the appropriate design methodology for the synthesis of four-bar linkage gives designers a very powerful tool.

In this work, the appropriate design methodology is utilized to synthesize the four-bar linkage designs which achieve desired coupler curve characteristics. In section 2, the design parameters of the four-bar linkage are described. In section 3, the classifications of four-bar linkages, as well as the concept of branches and circuits are introduced. In section 4, interval analysis and the three phases of an interval solving routine are introduced. A method for obtaining the coupler curve considering uncertainties is presented. In section 5, the task elements used to describe a desired coupler curve are introduced. Finally, in section 6, a synthesis routine is presented which uses the appropriate design methodology to compute allowed regions for several desired coupler curves.

LINKAGE DESCRIPTION

The four-bar linkage and its associated design parameters are provided in Figure 1. The fixed base location O_A is located at coordinates (u,v) with respect to the reference frame. The location O_B is located at coordinates (p,q) with respect to O_A or $(u+p,v+q)$ with respect to the reference frame. Link O_AA will always be assumed to be the input link with an input angle θ and length r. Link O_BB is the output link, which has an output angle ψ and length s. The coupler point is denoted $C=(C_x,C_y)$, where the coupler link is the triangle ABC with edge lengths a, b, and c. The parameters e and f describe the location of C relative to AB.

The equations describing the kinematics of the four-bar linkage can be formulated by solving a set of

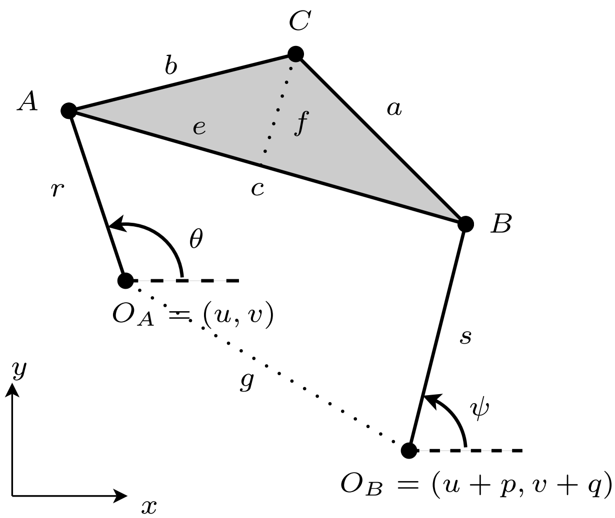


Fig. 1. Linkage description

distance equations (f_1 through f_5). The vertex $A = (A_x, A_y)$ of the coupler triangle lies on the circle centred at O_A with radius r. The vertex $B = (B_x, B_y)$ of the coupler triangle lies on the circle centred at O_B with radius s. The points A and B are separated by distance c. The points A and B are functions of the design parameters and input and output angles θ and ψ (f_6 through f_9). The edge lengths a and b of the coupler link are functions of the design parameters c, e, and f (f_{10} and f_{11}). Lastly, g is the distance between points O_A and O_B (f_{12}).

$$\begin{aligned}
f_1 &:= ||O_A A||^2 = (u - A_x)^2 + (v - A_y)^2 = r^2 \\
f_2 &:= ||O_B B||^2 = ((p+u) - B_x)^2 + ((q+v) - B_y)^2 = s^2 \\
f_3 &:= ||AB||^2 = (A_x - B_x)^2 + (A_y - B_y)^2 = c^2 \\
f_4 &:= ||AC||^2 = (A_x - C_x)^2 + (A_y - C_y)^2 = b^2 \\
f_5 &:= ||BC||^2 = (B_x - C_x)^2 + (B_y - C_y)^2 = a^2 \\
f_6 &:= A_x = u + r\cos(\theta) \\
f_7 &:= A_y = v + r\sin(\theta) \\
f_8 &:= B_x = (p+u) + s\cos(\psi) \\
f_9 &:= B_y = (q+v) + s\sin(\psi) \\
f_{10} &:= b = \sqrt{e^2 + f^2} \\
f_{11} &:= a = \sqrt{(c-e)^2 + f^2} \\
f_{12} &:= g = \sqrt{p^2 + q^2}
\end{aligned} \tag{1}$$

The equations in (1) may easily be simplified leaving a system of three equations with 11 parameters.

$$\begin{aligned}
||AB||^2 &= (r\cos(\theta) - p - s\cos(\psi)))^2 + (r\sin(\theta) - q - s\sin(\psi))^2 = c^2 \\
||AC||^2 &= (u + r\cos(\theta) - C_x)^2 + (v + r\sin(\theta) - C_y)^2 = b^2 \\
||BC||^2 &= (p + u + s\cos(\psi) - C_x)^2 + (q + v + s\sin(\psi) - C_y)^2 = a^2
\end{aligned} \tag{2}$$

It will be shown in section 4 that it is preferable to model the system as (1) since the equations are simpler. This is because interval analysis methods will be more effective (interval analysis requires a trade-off between the number of equations and their complexity). The model of the linkage therefore contains the following design parameters $\mathbf{d}$

$$\mathbf{d} = (u, v, p, q, r, s, c, e, f)^T \tag{3}$$

CLASSIFICATIONS AND ASSEMBLIES

Four-bar linkages can be sorted into two categories, *Grashof-type* and *non-Grashof-type* linkages. Grashof linkages satisfy the the Grashof condition, which states that if the sum of the shortest and longest link is less than or equal to the sum of the remaining two links, then the shortest link can rotate fully with respect to a neighbouring link. Whereas, the links of non-Grashof linkages cannot rotate fully. Grashof-type linkages and non-Grashof-type linkages each have four different classifications which describe the ranges of the input and output angles. The term *crank* denotes a link which is able to rotate fully, while the term *rocker* denotes a link that cannot rotate fully. A *double-crank*/*double-rocker* is a linkage whereby both the input and output links are cranks/rockers. For non-Grashof linkages, the input and output links will rock through 0 or π. McCarthy and Song Soh [2] describe a routine for classifying a four-bar linkage based only on the length of the links (note that the reference for the input and output angles in Figure 1 are different from [2]). Three parameters are used to identify the classification, T_i for $i = 1, \ldots, 3$, and are evaluated as

$$\begin{aligned} T_1 &= g - r + c - s; \\ T_2 &= g - r - c + s; \\ T_3 &= -g - r + c + s; \end{aligned} \tag{4}$$

The classification of the linkage is then

sgn(T_1)	sgn(T_2)	sgn(T_3)	Classification	Type
+	+	+	crank-rocker	Grashof
+	-	-	rocker-crank	Grashof
-	-	+	double-crank	Grashof
-	+	-	double-rocker	Grashof
-	-	-	00-double-rocker	Non-Grashof
+	+	-	0π-double-rocker	Non-Grashof
+	-	+	$\pi 0$-double-rocker	Non-Grashof
-	+	+	$\pi\pi$-double-rocker	Non-Grashof

A folding linkage occurs when any one of the parameters T_i includes zero. Such a linkage is able to take on a configuration where points O_A, O_B, A and B lie on a line.

The description for circuits and branches is adopted from Chase and Mirth [3]. For a given assembly of a four-bar linkage, the coupler point will follow what is referred to as a *circuit*. In order to change the circuit being followed, the linkage would need to be disassembled and reassembled. The term *toggle position* can be used to describe positions which result in collinearity of the coupler and output links. At a toggle point, the linkage is able to change its *branch* (a branch is defined by a transmission angle, the angle between the coupler and output links, in the range of $(0, \pi)$ or $(-\pi, 0)$). For the purpose of synthesis, the linkage must ensure that the desired path is accomplished by a single circuit. This concept is referred to as an *assembly mode defect*. It may or may not be necessary to restrict the branch of the linkage, as some applications benefit from a change in branch.

INTERVAL ANALYSIS

Interval arithmetic and *interval analysis* procedures provide a means of performing reliable computations on computers so that uncertainties in the representation of the parameters are automatically taken into account. Parameters can be represented by intervals (*e.g.*, $[x] = [\underline{x}, \overline{x}]$), where interval computations provide guaranteed bounds on the solution over the domains of the parameters. The width of an interval is given by

$$\text{width}([x]) = \overline{x} - \underline{x} \tag{5}$$

Let $[\mathbf{x}]$ denote an interval vector. The interval evaluation of a function $f([\mathbf{x}])$ yields the inclusion function $[f]$, such that $f([\mathbf{x}])$ is contained inside of $[f]$.

$$f([\mathbf{x}]) = \{f(\mathbf{x}) \mid \mathbf{x} \in [\mathbf{x}]\} \subseteq [f] \tag{6}$$

The bounds of $[f]$ are inflated due to two well known properties of interval analysis. First, the *wrapping effect* is a result of the axis-aligned representation of intervals. The solution $[f]$ will always be an axis-aligned *box* which contains $f([\mathbf{x}])$ and therefore introduces overestimation to the solution domain. Second, the *dependency problem* is a result of multiple occurrences of a variable appearing in the equation. This causes additional expansion of the solution domain. It is possible to minimize the effects of the dependency problem by properly writing the equation in a form best suited for interval analysis.

An interval analysis solving routine typically contains three phases evaluated in a loop, *simplification*, *existence*, and *bisection*. Let $[\mathbf{k}]$ denote the knowns and $[\mathbf{u}]$ denote the unknowns in a problem.

1. *Simplification*: In the simplification phase, various simplification routines (*e.g.*, 2B and 3B filtering [4], HC4 [5], ACID [6]) are applied to attempt to simplify the unknowns $[\mathbf{u}]$ to be more consistent with the knowns $[\mathbf{k}]$. The unknowns are considered to be simplified if their widths are reduced. If any unknown is simplified to an empty set, then no solution exists for these unknowns.

2. *Existence*: In the existence phase, existence methods (*e.g.*, Krawcyzk [7] or Newton-Kantorovitch [8]) are applied to determine if a unique solution $[\mathbf{u}^*]$ exists within the domains of the unknowns $[\mathbf{u}]$ which corresponds with the knowns $[\mathbf{k}]$. These methods are then able to tightly converge to the unique solution, where a unique solution corresponds to a region of non-separable solutions. The existence methods may also return that no solution exists.

3. *Bisection*: In the bisection phase, one unknown is bisected such that the original unknowns $[\mathbf{u}]$ are bisected into two subintervals $[\mathbf{u}_1]$ and $[\mathbf{u}_2]$. The union of the two subintervals results in the original interval, thus no combination of unknowns is skipped. The benefit of bisecting the unknowns is that the simplification and existence phases may have greater success when the unknowns have smaller widths. This is mainly attributed to the wrapping effect and dependency problem. It requires however to maintain a list of the bisected interval vectors. A common stopping criteria is to break when the width of all unknowns are less than a desired threshold ε. The algorithm completes when the list is empty.

When the knowns and unknowns of a system are properly selected, the interval analysis solving routine is able to guarantee the existence and uniqueness of every solution.

All mechanisms have uncertainties. These uncertainties are inherent in the manufacturing, assembly, and operation of the mechanism. Herein, the uncertainties on each of the design parameters are considered. The design parameters, represented as intervals, are denoted as $[\mathbf{d}]$.

$$[\mathbf{d}] = ([u], [v], [p], [q], [r], [s], [c], [e], [f])^T \tag{7}$$

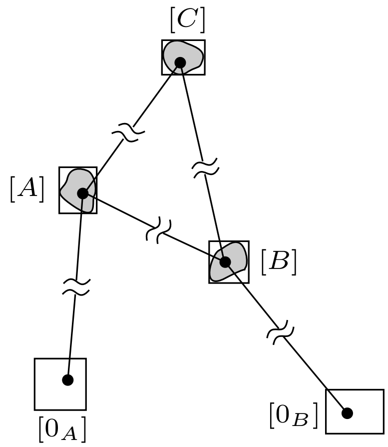


Fig. 2. The effect of geometrical uncertainties in the four-bar linkage.

It is no longer possible, nor relevant, to determine a discrete solution for the location of the coupler point for a given input angle. Instead, interval analysis will account for the uncertainty and provide guaranteed bounds for the location of the coupler point. For instance, Figure 2 depicts the effect of uncertainties on the linkage. The fixed locations O_A and O_B are known to lie in the corresponding boxes $[O_A]$ and $[O_B]$, respectively. For a given input and output angle, the locations A, B, and C will lie in some domain. With interval analysis, these domains are approximated by the intervals $[A]$, $[B]$, and $[C]$, respectively. The coupler point will always be inside the box $[C]$ for a given input and output angle. Based on this understanding, methods can be developed for the analysis and synthesis of linkages with geometrical uncertainties.

As an introduction to interval analysis solving routines, consider the problem of obtaining the coupler curve for a linkage with uncertainties in the design parameters. The coupler curve corresponding to a given set of design parameters $[\mathbf{d}]$ may be computed using an interval analysis solving routine. An outer loop iterates over every possible value of input angle (*e.g.*, $\forall[\theta] \in [-\pi, \pi]$). Inside this loop the knowns are considered as $[\mathbf{k}] = \{[\mathbf{d}], [\theta]\}$ and the unknowns may be selected as $[\mathbf{u}] = \{[B_x], [B_y], [C_x], [C_y]\}$. Since B is a function of the output angle ψ, the bisection phase will bisect $[\psi]$. When a unique solution is found, the corresponding coupler point $([C_x], [C_y])$ is saved. The coupler curve for the design parameters given in (8) is plotted in Figure 3 using a width of 0.005 rad for each $[\theta]$. An uncertainty of $\rho = [-0.0001, 0.0001]$ is added to each design parameter. A greater uncertainty reduces the successfulness of the existence methods, thus a small uncertainty is chosen here. The resulting linkage is classified as a 0π-double-rocker non-Grashof linkage. The linkage has a single circuit containing two different branch configurations, identified by different colours. Coupler points in the neighbourhood of the toggle position (the transition between branch configurations) do not yield unique solutions due to the existence method not being able to guarantee solutions. These regions are simply considered as unknown in terms of coupler point solutions. The set of intervals provides an outer approximation of the actual coupler curve, *i.e.*, the actual coupler curve is contained inside the union of the set of intervals.

$$
\begin{aligned}
[\mathbf{d}] &= ([0.0]+\rho, [0.0]+\rho, [0.4]+\rho, [0.0]+\rho, [0.24]+\rho, [0.24]+\rho, [0.2517]+\rho, \\
&\qquad [0.12585]+\rho, [0.15534]+\rho)^T
\end{aligned} \tag{8}
$$

TASK DESCRIPTION

The goal is to be able to synthesize the design parameters of four-bar linkages given the description of a desired coupler curve. The desired coupler curve will be described by a set of precision points and/or a set of trajectories. Each element, whether a precision point or a trajectory, is described with an allowable error. The allowable error is a requirement, since a discrete solution for the location of the coupler point is not possible.

The task element $\mathscr{P}$ corresponding to a precision point is represented by coupler point coordinates $[C_x]$ and $[C_y]$, and input and output angles $[\theta]$ and $[\psi]$ as

$$\mathscr{P} = \{[C_x],[C_y],[\theta],[\psi]\} \tag{9}$$

A desired trajectory, denoted $f([C_x],[C_y])$, can be described by parametric equations

$$f([C_x],[C_y]) = \{(C_x,C_y) \mid C_x = f_x(t),\ C_y = f_y(t),\ t \in [\underline{t},\overline{t}]\} \tag{10}$$

An allowable error on the trajectory can be described by $\alpha\hat{\mathbf{n}}$ where $\hat{\mathbf{n}}$ is the unit normal along $f([C_x],[C_y])$ and $\alpha = [\underline{\alpha},\overline{\alpha}]$ is the allowable error. The allowable trajectory can then be defined as

$$f([C_x],[C_y]) = \{(C_x,C_y) \mid C_x = f_x(t)+\alpha\hat{\mathbf{n}},\ C_y = f_y(t)+\alpha\hat{\mathbf{n}},\ t \in [\underline{t},\overline{t}],\ \alpha \in [\underline{\alpha},\overline{\alpha}]\} \tag{11}$$

It is necessary to specify start and finish end-points for the trajectory. These end-points will have a width δ and be defined as $[t_{start}] = [\underline{t}-\delta,\underline{t}]$ and $[t_{finish}] = [\overline{t},\overline{t}+\delta]$. The task element $\mathscr{T}$ corresponding to a trajectory is represented by the desired trajectory $f([C_x],[C_y])$, input and output angles $[\theta]$ and $[\psi]$, allowable error $[\alpha]$, the parametric equation parameter $[t]$, and end-point width δ.

$$\mathscr{T} = \{f([C_x],[C_y]),[\theta],[\psi],[\alpha],[t],\delta\} \tag{12}$$

The description of a coupler curve may contain multiple precision point task elements $\mathscr{P}$ and multiple trajectory task elements $\mathscr{T}$. As an example, consider the desired coupler curve described in (13) with three precision points and two trajectories. The task elements are chosen, such that they correspond to the coupler curve already obtained from the design parameters given in (8). The desired task elements are plotted in Figure 3.

$$\begin{aligned}
&\mathscr{P}_1 = \{[0.24,0.26],[0.323706,0.343706],[-\pi,\pi],[-\pi,\pi]\}\\
&\mathscr{P}_2 = \{[0.19,0.21],[0.373706,0.393706],[-\pi,\pi],[-\pi,\pi]\}\\
&\mathscr{P}_3 = \{[0.14,0.16],[0.333706,0.353706],[-\pi,\pi],[-\pi,\pi]\}\\
&f([C_x],[C_y])_1 = \{(C_x,C_y) \mid C_x = t,\ C_y = \alpha - 0.065,\ t \in [\underline{t},\overline{t}]\}\\
&\mathscr{T}_1 = \{f([C_x],[C_y])_1,[-\pi,\pi],[-\pi,\pi],[-0.01,0.01],[0.13,0.17],0.005\}\\
&\mathscr{T}_2 = \{f([C_x],[C_y])_1,[-\pi,\pi],[-\pi,\pi],[-0.01,0.01],[0.19,0.23],0.005\}
\end{aligned} \tag{13}$$

APPROPRIATE DESIGN METHODOLOGY

The main idea behind the appropriate design methodology is that uncertainties may be accounted for in order to yield reliable results. The flowchart in Figure 4 summarizes the appropriate design methods utilized for dimensional synthesis. A design and a task are provided by a user, and the appropriate design methodology is applied to synthesize the design solutions found within the design space which accomplish the task. The development of a routine for synthesizing appropriate designs may be simplified into several subroutines, namely: *verify precision points*, *verify trajectories*, and *verify appropriate design*. Initially, a task is given as a set of precision point elements and/or a set of trajectory elements, and the initial domains of the design parameters are given. A list $\mathscr{L}_{designs}$ is created and the initial domains of the design parameters

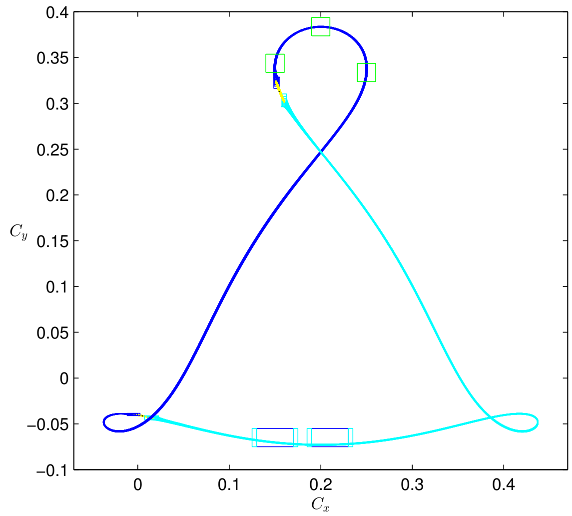


Fig. 3. The coupler curve corresponding to the design parameters in (8) and the desired task elements in (13).

are added. This list is further populated from bisections of the design parameters. Solutions, non-solutions, and boundaries are removed this list, such that the routine stops when the list is empty.

The purpose of the *verify precision points* subroutine is to verify if a set of design parameters $[\mathbf{d}_i]$ satisfy the requirements of the precision point elements of the task. The subroutines returns YES, NO, or MAYBE, where YES indicates that the design parameters $[\mathbf{d}_i]$ yield solutions for the coupler point which fall inside of each precision point.

The *verify trajectories* subroutine verifies if the set of design parameters $[\mathbf{d}_i]$ satisfy the requirements of the trajectory elements of the task. The subroutine returns YES, NO, or MAYBE, where YES indicates that the design parameters $[\mathbf{d}_i]$ yield solutions for the coupler point which fall inside both end-points of each trajectory, while also ensuring that the coupler curve is continuous between the end-points and remains inside the allowable error of the trajectory. The end-points are necessary to initialize the trajectory verification. The solution for each end-point will have an associated input angle $[\theta]$ and output angle $[\psi]$. Knowing the start and finish values of the input angle as $[\theta_s]$ and $[\theta_f]$, respectively, the problem is to ensure that each $\theta \in \square([\theta_s] \cup [\theta_f])$ yields a solution for the coupler point which falls inside the limits of the trajectory. If this is true, then it is guaranteed that the design parameters $[\mathbf{d}_i]$ satisfy the trajectory requirement.

The *verify appropriate design* subroutine is simply a wrapper for the precision point and trajectory verification subroutines, which again returns YES, NO, or MAYBE. If either of the verification subroutines fail (return NO), then this subroutine must also return NO. A YES is only returned if both verification subroutines return YES, as this indicates that all of the task requirements are satisfied. The return value from either subroutine is ignored if there are no associated task elements.

The *synthesize appropriate designs* routine iterates over the domains of the design parameters and calls *verify appropriate design* to determine if the design parameters satisfy the task requirements. If the verification subroutine succeeds (return YES), the current design parameters $[\mathbf{d}_i]$ are considered as a solution for the desired task. If the verification subroutine fails (return NO) then the current design parameters are removed from the search. Otherwise, if the verification subroutine returns MAYBE, then bisection is applied to reduce the width of the design parameters (the desired resolution is δ, which is usually given as the desired manufacturing accuracy). The synthesis routine loops until the list $\mathscr{L}_{designs}$ is empty.

To demonstrate the *synthesize appropriate designs* routine, the domains of the design parameters $[p]$

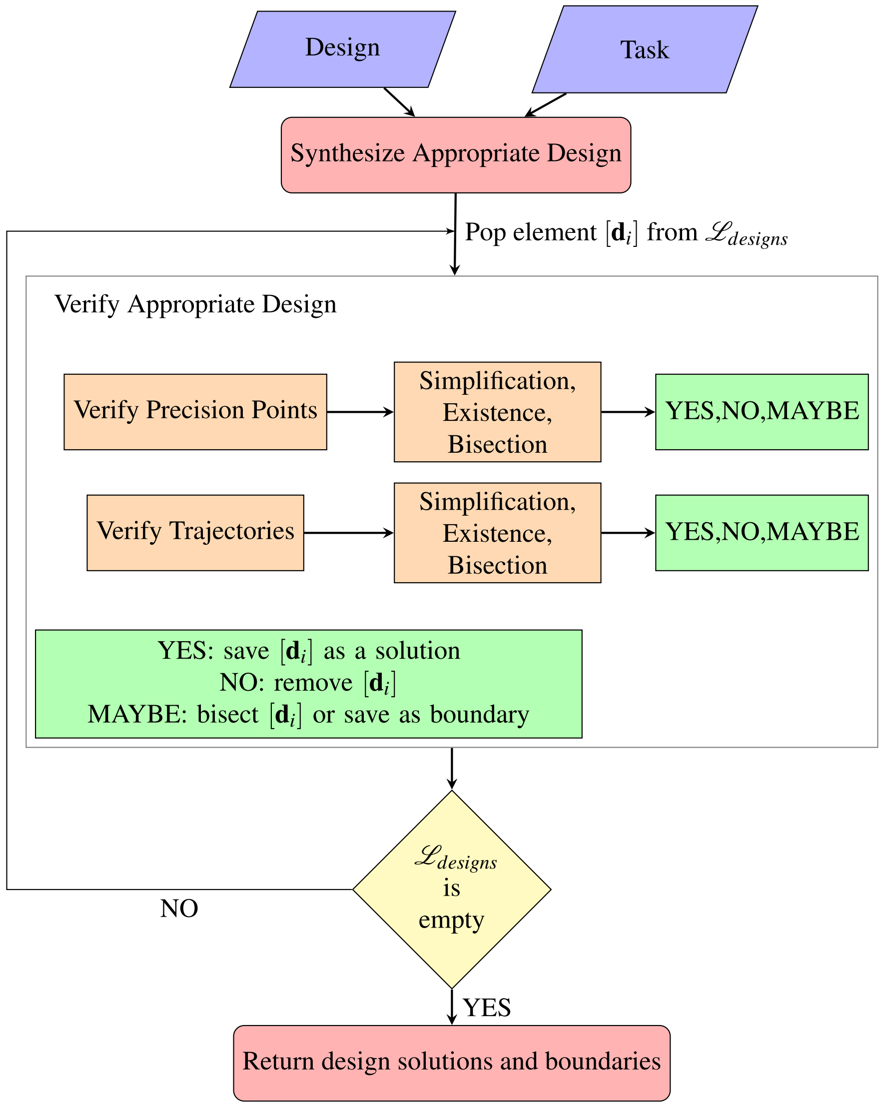
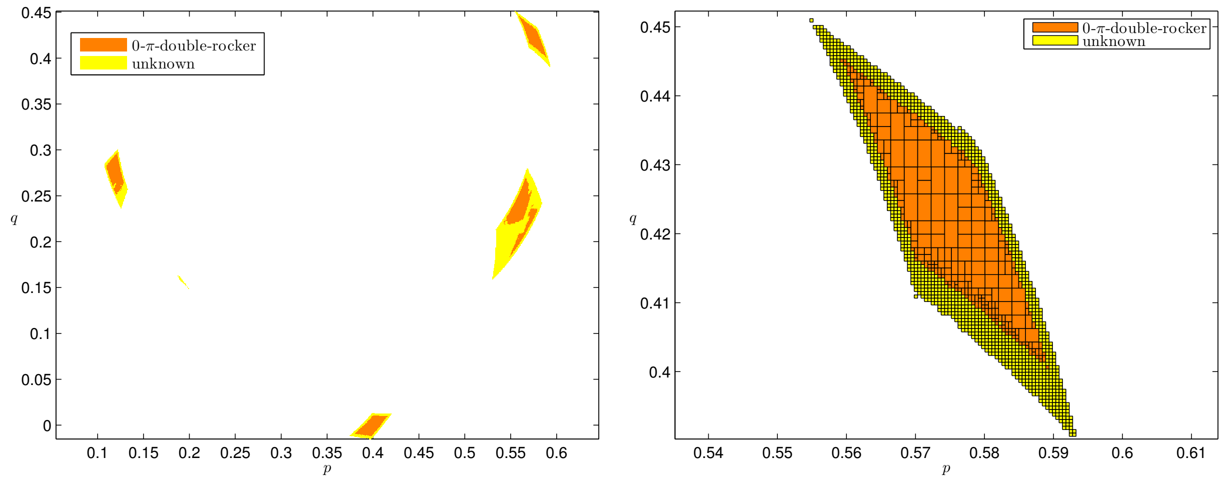


Fig. 4. Flowchart of appropriate synthesis routine.

and $[q]$ are initialized as $[-1.0, 1.0]$. The remaining design parameters are the same as those in (8). A threshold value of $\delta = 0.001$ is used to synthesize the appropriate solutions for $[p]$ and $[q]$. The complete set of appropriate design solutions considering the precision point task elements from (13) are shown in Figure 5a. These solutions do not apply restrictions to the branch configuration. Any design which achieves the desired precision points, regardless of branch configuration is accepted as a solution (a single branch configuration may optionally be enforced). This generates several disconnected allowed regions. Figure 5b zooms in on one of the disconnected allowed regions. Selecting the values of $[p] = [0.5699, 0.5701]$ and $[q] = [0.4299, 0.4301]$ from the allowed region, the corresponding coupler curve and desired precision points are plotted in Figure 6. The resulting linkage is classified as a 0π-double-rocker non-Grashof linkage, which has a single circuit containing two different branch configurations.

Considering only the trajectory elements from (13), Figure 7a shows the complete set of appropriate design solutions. Several disconnected allowed regions are found. Selecting the values of $[p] = [2999, 0.3001]$ and $[q] = [0.0199, 0.0201]$ from the allowed region, the corresponding coupler curve with desired tra-



(a) (b)

Fig. 5. a) Appropriate design solutions for p and q for precision points task elements in (13) (the edges have been removed for clarity); b) Zoomed-in view of one of the disconnected allowed regions from Figure 5a

jectories and the linkage with associated uncertainties are plotted in Figure 8a. Selecting the values of $[p] = [0.2499, 0.2501]$ and $[q] = [-0.4401, -0.4399]$ from the allowed region, the corresponding coupler curve and desired trajectories are plotted in Figure 8b. These two coupler curves are quite different, yet are able to achieve the same objective. Lastly, using all task elements from (13), Figure 7b shows the complete set of appropriate design solutions to the problem. This is equal to the intersection of the allowed regions from the precision point solutions and trajectory solutions. The original design parameters from (8) are contained inside the allowed region. These examples help to demonstrate the usefulness of the appropriate design methodology.

CONCLUSION

In this work, the appropriate design methodology was utilized to develop routines which are capable of yielding reliable solutions for the analysis and synthesis of the four-bar linkage. These solutions are reliable since they are able to account for uncertainties. The appropriate design solutions for the synthesis problem were presented in the form of allowed regions. Any design which is contained in the allowed region is guaranteed to achieve the desired coupler curve characteristics, modelled as a set of precision points and/or trajectories with an allowable amount of error. This work shows great potential for the appropriate design methodology. Future work includes refinement to the methods presented here and determining techniques to improve the performance on higher dimensional synthesis problems.

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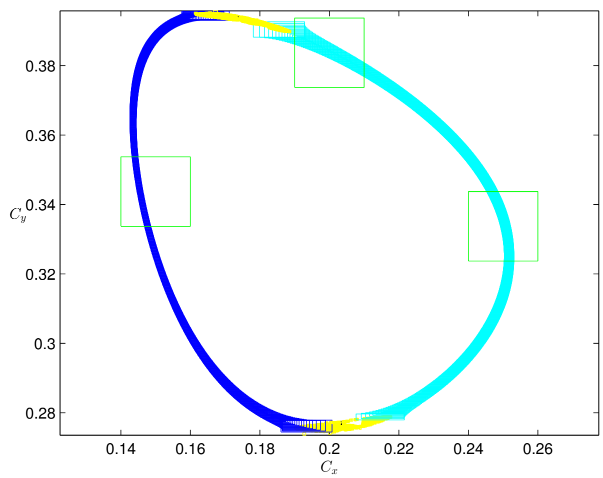
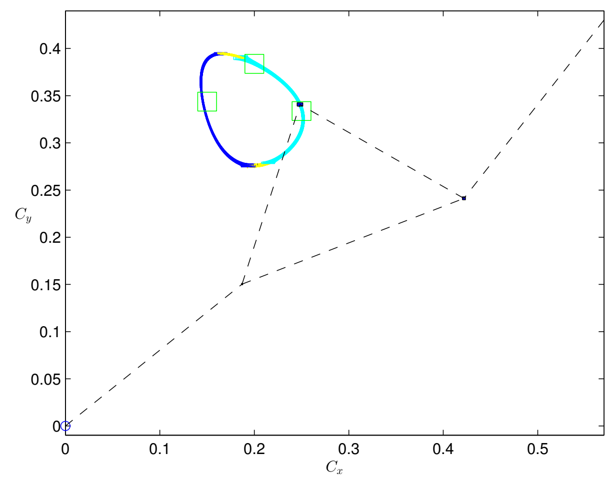


(a)



(b)

Fig. 6. a) The coupler curve and b) the linkage with associated uncertainties, corresponding to a solution from the allowed region.

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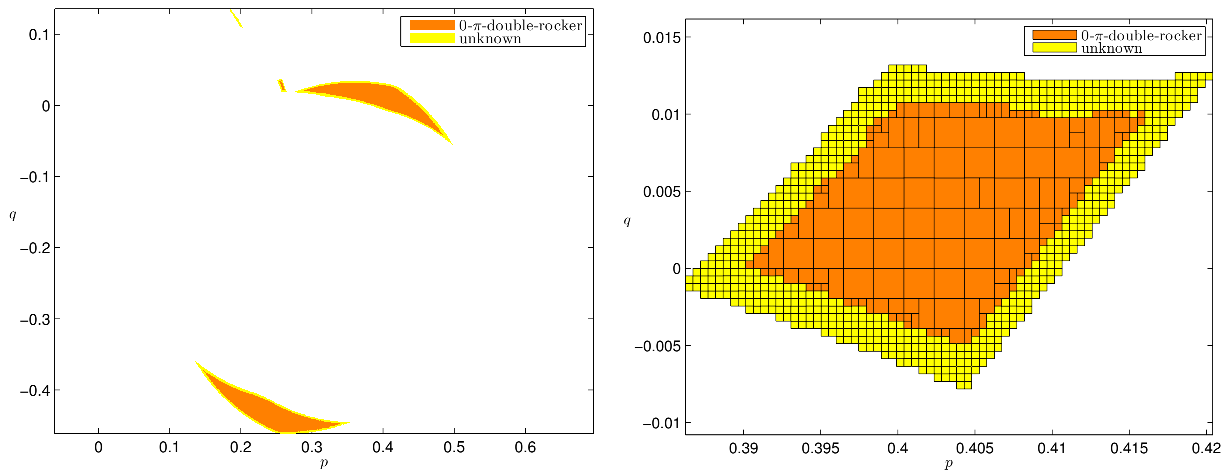
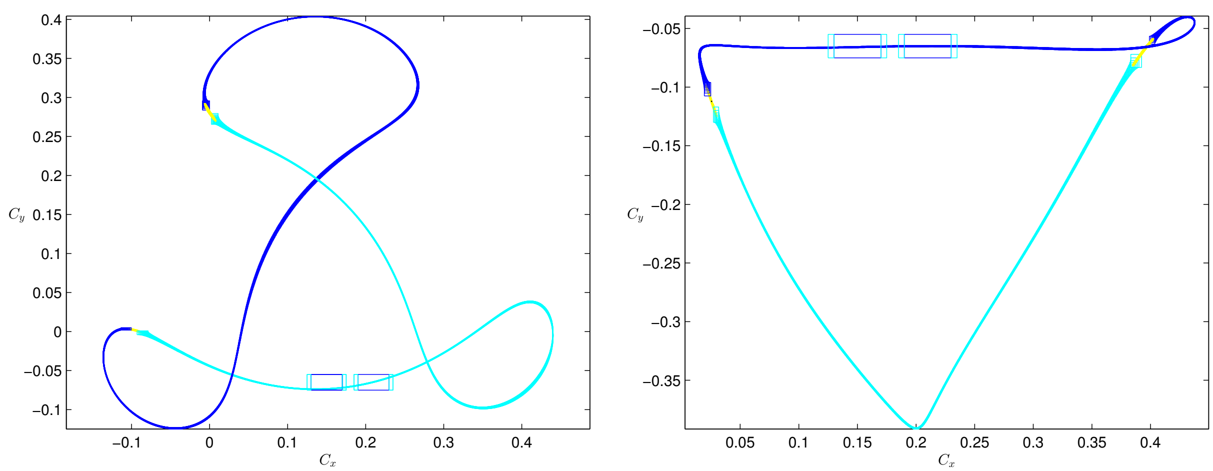


(a) (b)

Fig. 7. a) Design solutions for p and q for the trajectory task elements in (13) (the edges have been removed for clarity); b) Design solutions for p and q for all task elements in (13).



(a) (b)

Fig. 8. a-b) Coupler curves corresponding to a solution from the allowed region.