

# CLASSICAL WASHOUT USING QUATERNIONS

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## ABSTRACT

Washout algorithms for flight simulation, used to produce displacement cues within the relatively small workspace of a Gough-Stewart motion platform, are typically implemented for modest ranges of rotations. Due to the limited range of angular motion possible with conventional simulator motion platforms, Euler angles are commonly used for representing orientation. In this paper, for classical washout, Euler angles are replaced with quaternions to eliminate the singularities that would otherwise be present as the range of angular motion is increased. The resulting modified algorithm is then run through several test manoeuvres to compare its performance with the conventional algorithm, using a normalized Pearson correlation to determine the relative performance. There is some measurable degradation in performance for quaternion washout for some of the cases and future work is required to determine whether this may be acceptable due to the increase in angular range, and corresponding opportunities for simulator fidelity improvement, it permits.

**Keywords:** classical washout; quaternions; motion simulator.

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## WASHOUT CLASSIQUE À L'AIDE DE QUATERNIONS

### RÉSUMÉ

Les algorithmes de washout, utilisés pour produire des signaux de mouvement dans l'intervalle de mouvement d'un simulateur de mouvement, ont un mouvement angulaire limité. En raison d'une plage de mouvement angulaire limitée chez les simulateurs de plates-formes de mouvement classiques, les angles d'Euler sont couramment utilisés pour représenter l'orientation. Dans cet article, les angles d'Euler sont remplacés par des quaternions afin d'éliminer les singularités associées aux mouvements angulaires de grande envergure. L'algorithme modifié présenté est validé et comparé à l'algorithme conventionnel à l'aide de plusieurs tests de manœuvre. Une corrélation de Pearson normalisée est utilisée pour déterminer les caractéristiques de performance relative. Il existe une certaine dégradation mesurable de la performance de washout de quaternion dans certains cas, et des travaux futurs sont nécessaires pour déterminer s'il s'agit d'un résultat acceptable par rapport à l'augmentation de la plage angulaire et des possibilités correspondantes qu'il permet d'améliorer la fidélité du simulateur.

**Mots-clés :** washout classique ; quaternions ; simulateur de mouvement.

## 1. INTRODUCTION

Washout algorithms allow a Gough-Stewart motion simulator platform to produce a larger range of motion cues than would otherwise be possible in its limited motion envelope, by using gravity to simulate extended translational accelerations. Combined with visual flow, this is known to produce an immersive environment for the trainee. Washout algorithms are typically developed for a small range of angular motion, due to the hardware limits of a typical Gough-Stewart simulator. Reid and Nahon conducted an in-depth study of washout algorithms and how they were evaluated by pilots [1–3]. Classical washout is one of the three main algorithm types studied, and is still widely used. There has been minimal further development for classical washout since this study was completed in 1986 [4].

As large-range-of-motion simulators are being developed for applications such as light helicopters and edge-of-envelope flight conditions, it is desirable to extend the angular range of washout algorithms. The Atlas simulator developed by the Carleton University Simulator Project (CUSP) [5–7], which allows for unbounded rotation about any axis, is one example of the types of simulators being developed. To begin addressing the requirement for a larger angular range of motion, quaternions are added to classical washout to eliminate the singularities introduced by Euler angles.

## 2. CLASSICAL WASHOUT WITH EULER ANGLES

Washout algorithms allow for flight simulators to reproduce the sensation of aircraft flight in the limited motion envelope of a flight simulator [1]. This sensation is accomplished by restricting large translational or rotational motions. High-frequency translational accelerations and angular velocities are directly reproduced, although they may be scaled to remain within the simulator motion envelope. Following high-frequency translational accelerations, the platform is recentred to allow for future motion.

To reproduce sustained low-frequency translational acceleration for surge and sway motions, the simulator is slowly rotated to align the gravity vector with the direction of the inertia force associated with the sustained translational acceleration. By maintaining the rotation rate below 3 deg/s the pilot does not sense the rotation [1] and instead interprets the acceleration from the gravity vector as the sustained translational acceleration of the aircraft.

Fig. 1 shows the block diagram of classical washout, where the three main channels, translational, tilt coordination, and rotational are indicated. The algorithm takes in inputs of the specific force and angular velocity experienced by the aircraft pilot, and produces set points of the translational and angular positions for the motion simulator.

### 2.1. Translational Channel

The translational channel's input is the specific force experienced by the pilot of the aircraft being simulated, in the aircraft frame. The specific force,  $\mathbf{f}_{AA}$ , includes  $-\mathbf{g}_S$  to account for gravity, while the acceleration measurement excludes gravity. The specific force is scaled, if desired, then converted to the inertial frame.  $\mathbf{L}_{IS}$  converts the specific force from the aircraft or simulator frame to the inertial frame using the Euler angle rotation matrix

$$\mathbf{L}_{IS} = \begin{bmatrix} \cos \theta \cos \psi & \sin \phi \sin \theta \cos \psi & \cos \phi \sin \theta \cos \psi \\ & -\cos \phi \cos \psi & +\sin \phi \sin \psi \\ \cos \theta \sin \psi & \sin \phi \sin \theta \sin \psi & \cos \phi \sin \theta \sin \psi \\ & +\cos \phi \cos \psi & -\sin \phi \cos \psi \\ -\sin \theta & \sin \phi \cos \theta & \cos \phi \cos \theta \end{bmatrix}, \quad (1)$$

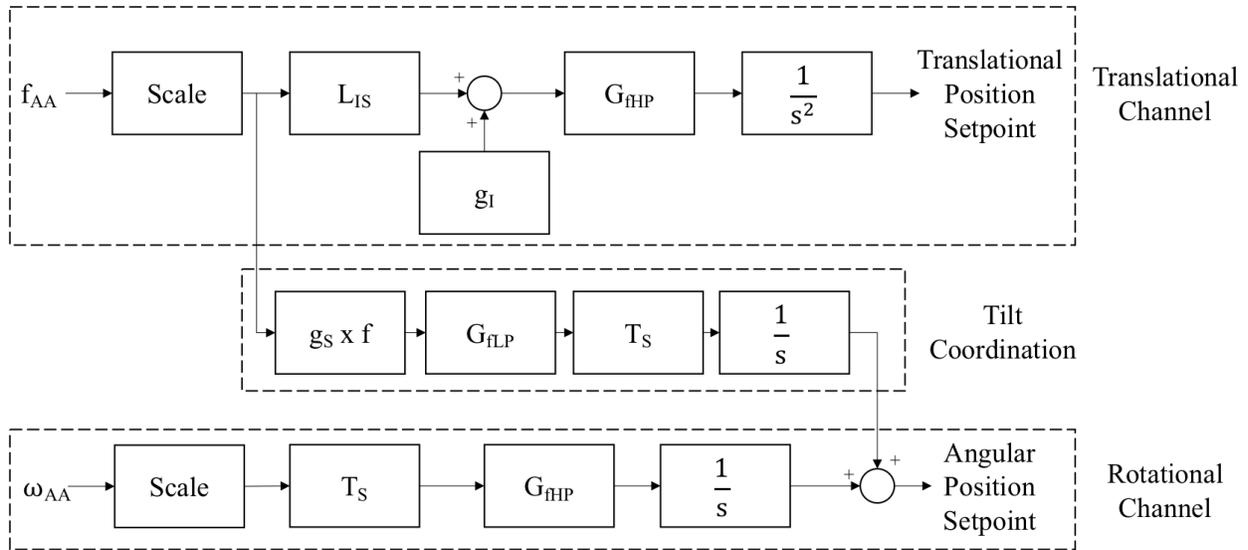


Fig. 1. Classical washout with Euler angles.

where  $\phi$ ,  $\theta$ , and  $\psi$  are the roll, pitch, and yaw angles based on the current orientation of the simulator [1]. Gravity is added in the inertial frame to convert the specific force to acceleration. The result is then passed through a high-pass filter  $\mathbf{G}_{fHP}$  of the form

$$\mathbf{G}_{fHP} = \frac{s^2}{(s + \omega_n)^2}, \quad (2)$$

where  $\omega_n$  is chosen based on the desired aircraft's performance characteristics. For this paper,  $\omega_n = 1.5$  for surge, and  $\omega_n = 3.5$  for sway and heave [2]. The output of the high-pass filter is then integrated to produce a position set point.

## 2.2. Tilt Coordination

Tilt coordination is the mechanism that aligns the gravity vector with the direction of the specific force. Reid and Nahon [1] discuss two methods of tilt coordination, and here the second is used, as it does not rely on small angle approximations. The cross product of  $\mathbf{g}_S$ , the gravity vector in the simulator frame, and the scaled specific force will align the gravity vector with the specific force set point without overshoot. As the simulator's rotation rate must be below the threshold that the pilot will detect, it is passed through a low-pass filter of the form

$$\mathbf{G}_{fHP} = \frac{(2\omega_n)^2}{s + 4\omega_n s^2 + (2\omega_n)^2}, \quad (3)$$

with the same values for  $\omega_n$  as those in the high-pass filter in the rotational channel [2]. A rate limiter can also be added if required to ensure the rotation produced by tilt coordination is below the pilot's perception threshold. The result is converted to Euler angular rates using

$$\mathbf{T}_S = \begin{bmatrix} 1 & \sin \phi \tan \theta & \cos \phi \tan \theta \\ 0 & \cos \phi & -\sin \phi \\ 0 & \sin \phi \sec \theta & \cos \phi \sec \theta \end{bmatrix}, \quad (4)$$

based on the current orientation of the simulator, and then integrated to update the Euler angles [1].

### 2.3. Rotational Channel

The rotational channel takes the input of angular velocity in the aircraft frame. It is scaled, if desired, and then converted to Euler angular rates using Equation (4). The Euler angular rates are then passed through a high-pass filter of the form

$$\mathbf{G}_{\text{fHP}} = \frac{s}{s + \omega_n}, \quad (5)$$

with  $\omega_n = 0.5$  for the pitch, roll, and yaw directions [2]. The output of  $\mathbf{G}_{\text{fHP}}$  is integrated to produce angular position, and then added to the Euler angles produced from the tilt coordination [1]. The sum of the Euler angles is the angular position set point.

### 2.4. Limitations

Classical washout was designed for small angular motions; therefore, problems arise as the range of angular displacement increases. Euler angles, regardless of the sequence used, will always have a singularity when the middle angle of the sequence reaches  $\frac{\pi}{2} \pm i\pi$ , where  $i$  is any positive integer. Furthermore, the addition of the Euler angles from the tilt coordination and rotational channels to produce the angular position set point, which was an acceptable approximation at small angles [1], will cause significant error with larger angular displacements.

## 3. QUATERNION MATHEMATICAL OPERATIONS

Quaternions are a useful replacement for Euler angles in classical washout. Quaternions can be used to represent rotations in three-dimensional space, and do not have the singularities present with Euler angles. They can be represented as

$$q = q_0 + q_1i + q_2j + q_3k, \quad (6)$$

where  $q_0, q_1, q_2, q_3$  are real and  $i, j, k$  are three imaginary numbers defined as

$$i^2 = j^2 = k^2 = -1. \quad (7)$$

The quaternion can also be represented as

$$\mathbf{q} = \begin{bmatrix} q_0 \\ \mathbf{q}_v \end{bmatrix} = \begin{bmatrix} q_0 \\ q_1 \\ q_2 \\ q_3 \end{bmatrix}, \quad (8)$$

where  $q_0$  is the scalar or real part and  $\mathbf{q}_v$  is the vector or imaginary part [8].

### 3.1. Product

The quaternion product can be written in vector form

$$\mathbf{p} \otimes \mathbf{q} = \begin{bmatrix} p_0q_0 - p_1q_1 - p_2q_2 - p_3q_3 \\ p_0q_1 + p_1q_0 + p_2q_3 - p_3q_2 \\ p_0q_2 - p_1q_3 + p_2q_0 + p_3q_1 \\ p_0q_3 + p_1q_2 - p_2q_1 + p_3q_0 \end{bmatrix}. \quad (9)$$

It is important to note that the quaternion product is not commutative [8].

### 3.2. Rotation

A quaternion can be used to rotate a vector with the equation

$$r(\mathbf{v}) = \mathbf{q} \otimes \mathbf{v} \otimes \mathbf{q}^*, \quad (10)$$

where  $\mathbf{q}^*$  is the conjugate of the quaternion defined as

$$\mathbf{q}^* = \begin{bmatrix} q_0 \\ -\mathbf{q}_v \end{bmatrix}. \quad (11)$$

Using the inverse quaternion, defined as

$$\mathbf{q}^{-1} = \frac{\mathbf{q}^*}{\|\mathbf{q}\|^2}, \quad (12)$$

in Equation (10) will rotate the vector in the opposite direction as the rotation with  $\mathbf{q}$ .

### 3.3. Integration

Quaternion integration can be accomplished given the angular rate,  $\omega$ , and the previous angular position,  $\mathbf{q}_{n-1}$ , using the equation

$$\mathbf{q}_n = \mathbf{q}_{n-1} \otimes \begin{bmatrix} \cos(\|\omega\| \Delta t / 2) \\ \frac{\omega}{\|\omega\|} \sin(\|\omega\| \Delta t / 2) \end{bmatrix}, \quad (13)$$

where  $\Delta t$  is the timestep [8].

## 4. CLASSICAL WASHOUT WITH QUATERNIONS

Beginning with the form of classical washout illustrated in Fig. 1, one of the primary issues to address is the singularities that are introduced by the use of Euler angles. Quaternions provide the means to address this issue, as they represent orientation without the potential singularities that are imposed by the Euler angles. Fig. 2 shows the block diagram for the classical washout which has been modified to use quaternions. The new quaternion washout is similar to classical washout with Euler angles. It has the translational channel, tilt coordination, and the rotational channel, and uses the same inputs and outputs. However, all rotations are conducted with quaternions instead of Euler angles, and the order of some of the blocks has been rearranged to apply the high-pass filter to angular velocity instead of Euler angular rates.

### 4.1. Translational Channel

The quaternion washout translational channel is almost identical to the original Euler angle classical washout. The difference is that instead of  $\mathbf{L}_{IS}$  as the rotation matrix in Equation (1), instead the scaled specific force is converted from the simulator frame to the inertial frame using a quaternion rotation as described in Equation (10)

$$r(\mathbf{f}) = \mathbf{q} \otimes \mathbf{f} \otimes \mathbf{q}^*, \quad (14)$$

where  $\mathbf{q}$  is the inverse of the current orientation of the simulator, calculated using Equation (12), and  $\mathbf{f}$  is the scaled specific force.

### 4.2. Tilt Coordination

The substantial change to tilt coordination is that instead of outputting Euler angles as was done in classical washout with Euler angles, the output is now an angular velocity. This change eliminates the  $\mathbf{T}_S$  and integration blocks that were present in the Euler angle version. The changes allow the addition of the set points from the tilt coordination and the rotational channel as angular velocities.

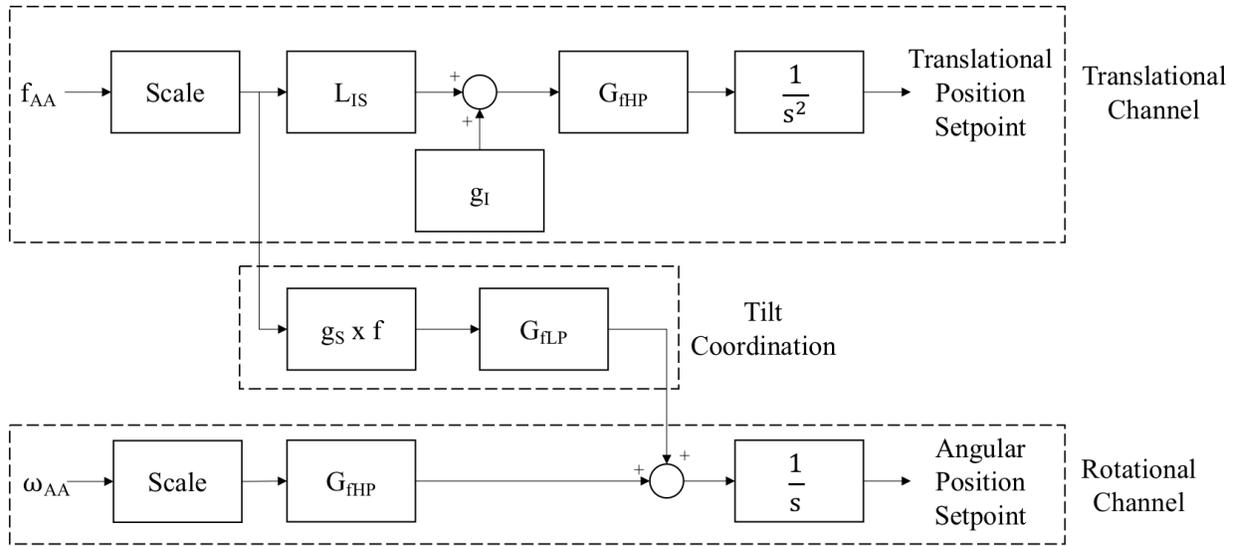


Fig. 2. Classical washout with quaternions.

### 4.3. Rotational Channel

The implementation of the rotational channel of quaternion washout differs from the classical washout with Euler angles illustrated in Fig. 1 in several ways, due to the conversion to quaternions and to accommodate large angular motions. The high-pass filter, Equation (5), is applied to the angular velocity in the simulator frame, instead of the rate of change of Euler angles in the inertial frame. Reid and Nahon concluded that the low-pass filter performs best when applied in the inertial frame [1]; however, the change was made to allow for the filtering to be completed with the angular velocity set point, as opposed to using Euler angles, which are to be avoided.

The angular velocity output from tilt coordination, as shown in Fig. 2, is then added to the angular velocity output of the high-pass filter, to produce an overall angular velocity set point for the simulator. A new angular position set point is calculated from the angular velocity and the current orientation of the simulator, using quaternion integration as described in Equation (13), instead of converting to Euler angular rates and then integrating. The addition of angular velocities eliminates the error present in classical washout with Euler angles due to the addition of Euler angles, which will cease to work in the future as washout is used with larger angles.

## 5. RESULTS

To evaluate the relative performance of the two algorithms, both were run for a set of seven different manoeuvres. In each case, the simulation was run for 40 seconds. The first six are step inputs in one degree of freedom, with the remaining degrees of freedom held at zero. For surge, sway, and heave, a step of  $2 \text{ m/s}^2$  of specific force sustained for 20 seconds is used. For roll, pitch, and yaw, the step is an angular velocity set point of  $10 \text{ deg/s}$  for 20 seconds. As well as these, the Turn Entries manoeuvre (M1) from [2] is also run to test the algorithms for more realistic inputs. Finally, to evaluate a large angular motion, a step of  $5 \text{ deg/s}$  of roll and  $120 \text{ deg/s}$  of pitch, sustained for 3 seconds, is used. The resulting simulator output traces must be compared with the desired specific force and angular velocity traces.

Typically, motion simulator performance is evaluated subjectively by pilots using the qualitative factor of

Parameter	rads/s
Surge $\omega_n$	1.5
Sway $\omega_n$	3.5
Yaw $\omega_n$	3.5
Roll $\omega_n$	0.5
Pitch $\omega_n$	0.5
Yaw $\omega_n$	0.5

Table 1. Parameters for classical washout test cases.

*feel*. To quantitatively assess the performance of the algorithms in the absence of pilot testing two methods are used, the normalized Pearson correlation, and the area between the aircraft and simulator response curves. The normalized Pearson correlation (NPC) can be used to quantify how well a washout algorithm performs by comparing the closeness of the two time histories [4]. The Pearson correlation is given by

$$\text{PC}(x, y) = \frac{\sum_{i=1}^n (x_i - \bar{x})(y_i - \bar{y})}{\sqrt{\sum_{i=1}^n (x_i - \bar{x})^2} \sqrt{\sum_{i=1}^n (y_i - \bar{y})^2}}, \quad (15)$$

where  $x$  and  $y$  represent the aircraft and simulator responses respectively,  $\bar{x}$  and  $\bar{y}$  are the mean of  $x$  and  $y$ , and  $n$  is the number of samples. It is then normalized with

$$\text{NPC}(x, y) = \frac{K}{1 + \text{PC}(x, y)} + 1 - \frac{K}{2}, \quad (16)$$

where  $K = 1$ . The closer the result of the NPC is to 1, the better the performance of the washout algorithm in reproducing the motion cues of the aircraft. To test the relative performance of classical washout with quaternions compared to the original, both use the parameters associated with the classical washout version CW1, which are listed in Table 1.

Fig. 3 shows the inputs for each degree of freedom for the M1 manoeuvre, with the specific force and angular velocity inputs to the washout algorithm. Table 2 lists the NPC for each degree of freedom as well as the average NPC, for classical washout with Euler angles and with quaternions. The first value in each cell represents the NPC value for that degree of freedom for the manoeuvre using classical washout with Euler angles. The second value in the cell represents the NPC value for classical washout using quaternions.

While the NPC results for most of the cases are similar for both the original classical washout and the classical washout using quaternions, in the M1 case the quaternion version of the algorithm performs worse than the original, with all degrees of freedom showing better performance for the original algorithm. For the large angle, the quaternion version performs better in the roll direction, with similar results in the other degrees of freedom.

Due to limitations of the normalized Pearson correlation when the set points are zero or close to zero, the integral of the error between the scaled set points and the simulator response is also considered. Table 3 lists the Euler angle and quaternion values for each degree of freedom, and Table 4 lists the average error for the translational and rotational degrees of freedom.

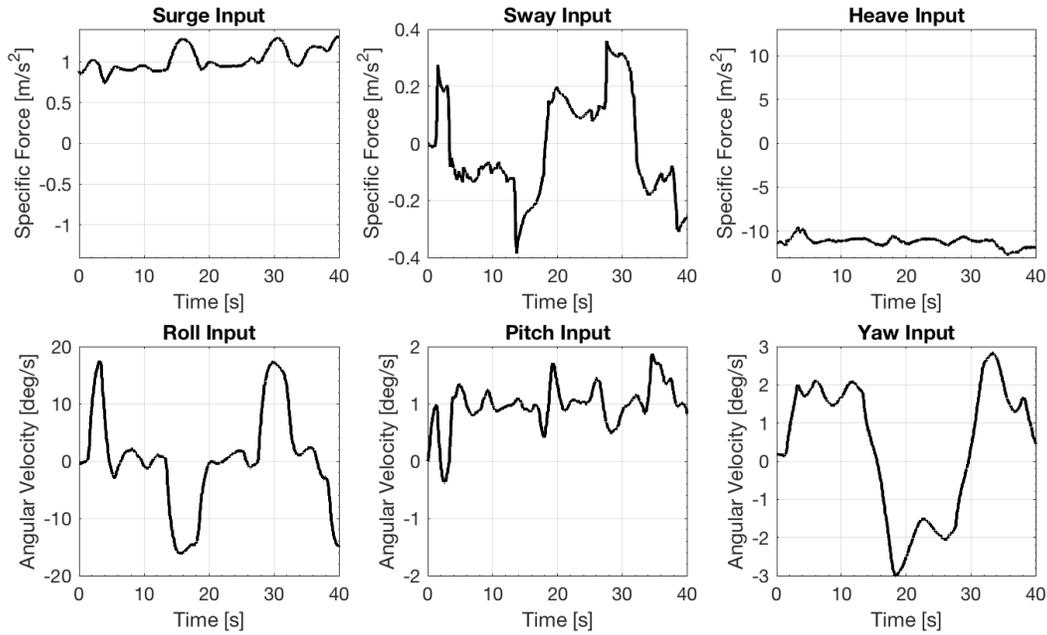


Fig. 3. Turn Entries (M1) case inputs.

Manoeuvre	Surge	Sway	Heave	Roll	Pitch	Yaw	Average
Surge	1.01 / 1.02	1.00 / 1.00	1.50 / 1.50	1.00 / 1.00	1.50 / 1.50	1.00 / 1.00	1.17 / 1.17
Sway	1.00 / 1.00	1.02 / 1.02	1.50 / 1.50	1.50 / 1.50	1.00 / 1.00	1.00 / 1.00	1.17 / 1.17
Heave	1.00 / 1.00	1.00 / 1.00	1.51 / 1.51	1.00 / 1.00	1.00 / 1.00	1.00 / 1.00	1.09 / 1.09
Roll	1.00 / 1.00	1.48 / 1.48	1.13 / 1.13	1.19 / 1.19	1.00 / 1.00	1.00 / 1.00	1.13 / 1.13
Pitch	1.52 / 1.51	1.00 / 1.00	1.13 / 1.13	1.00 / 1.00	1.19 / 1.19	1.00 / 1.00	1.14 / 1.14
Yaw	1.50 / 1.50	1.50 / 1.50	1.12 / 1.12	1.00 / 1.00	1.00 / 1.00	1.19 / 1.19	1.22 / 1.22
M1	1.19 / 1.25	1.57 / 1.76	1.42 / 1.59	1.09 / 1.29	1.50 / 1.66	1.18 / 1.31	1.33 / 1.48
Large Angle	1.51 / 1.51	1.49 / 1.49	1.45 / 1.44	1.35 / 1.07	1.06 / 1.07	1.50 / 1.50	1.39 / 1.35

Table 2. Normalized Pearson correlation results for classical washout with Euler angles and with quaternions (Euler / quaternion).

Manoeuvre	Surge	Sway	Heave	Roll	Pitch	Yaw
Surge	8.64 / 2.93	0.00 / 0.00	0.47 / 1.08	0.00 / 0.00	0.12 / 0.23	0.00 / 0.00
Sway	0.00 / 0.00	5.62 / 2.55	0.86 / 1.14	0.16 / 0.21	0.00 / 0.00	0.00 / 0.00
Heave	0.00 / 0.00	0.00 / 0.00	20.31 / 20.31	0.00 / 0.00	0.00 / 0.00	0.00 / 0.00
Roll	0.00 / 0.00	215.44 / 216.02	202.91 / 203.69	17.44 / 17.44	0.00 / 0.00	0.00 / 0.00
Pitch	34.28 / 34.42	0.00 / 0.00	193.50 / 193.48	0.00 / 0.00	1.74 / 1.74	0.00 / 0.00
Yaw	0.14 / 0.15	0.01 / 0.01	196.22 / 196.22	0.00 / 0.00	0.00 / 0.00	1.74 / 1.74
M1	5.73 / 4.52	32.31 / 7.09	31.15 / 28.29	1.59 / 1.93	0.33 / 0.42	0.50 / 0.88
Large Angle	37.42 / 52.79	34.57 / 2.20	187.13 / 183.12	1.40 / 0.13	2.78 / 3.14	1.58 / 0.00

Table 3. Integrated error results for classical washout with Euler angles and with quaternions (Euler / quaternion).

Manoeuvre	Specific Force	Angular Velocity
Surge	3.04 / 1.34	0.04 / 0.08
Sway	2.16 / 1.23	0.05 / 0.07
Heave	6.77 / 6.77	0.00 / 0.00
Roll	139.91 / 139.91	5.81 / 5.81
Pitch	75.93 / 75.97	0.58 / 0.58
Yaw	65.46 / 65.46	0.58 / 0.58
M1	23.06 / 13.30	0.81 / 1.08
Large Angle	86.37 / 79.37	1.92 / 1.09

Table 4. Average values of the specific force and angular velocity integrated error rates for classical washout with Euler angles and with quaternions (Euler / quaternion).

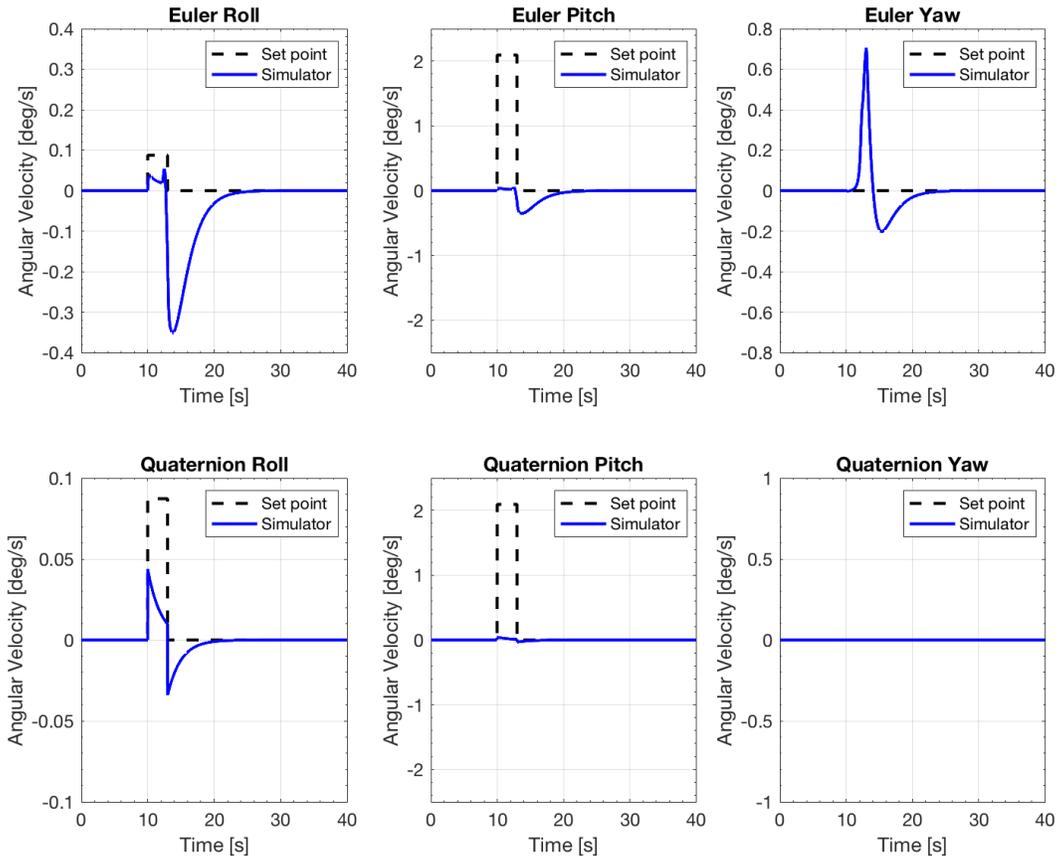


Fig. 4. Angular velocity response for Euler and quaternion washout for the large angle test case.

The integrated error shows similar performance on average for the two algorithms. For the M1 case, the quaternion version of the algorithm provides measurably better performance in the specific force errors, and worse performance for the angular velocities. For the roll, pitch, yaw, and large angle tests there are large specific force errors for surge, sway, and heave for both algorithms. This is due to the construction of the test cases, as all the test cases except M1 do not reflect the way the set points of the other degrees of freedom would respond to the changing motion of the aircraft.

For further qualitative analysis of the large-angle case, Fig. 4 shows the angular velocity response for both versions of the algorithm. It shows that for this particular large-angle case, the Euler angle washout has larger negative cues for roll and pitch, which are detrimental to the pilot's experience in the simulator [4], and there is undesirable motion in yaw as well all producing negative pilot training. The quaternion version reduces these undesirable cues, and eliminates the risk of singularities that would be present with the Euler-angle version.

## 6. CONCLUSIONS

In conclusion, while classical washout with quaternions eliminates singularities that Euler-angles introduce over extended angular displacements, it can result in reduced performance during small-angle simulations, and improved angular performance in large-angle conditions, while also removing the risk of singularities. The selection of small- or large-angle implementation of the algorithm must consider the relative merits of fidelity for small angular inputs and the ability to accommodate an expanded simulator angular motion envelope.

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