Kinematic Analysis of a Spatial 3-DOF Parallel Manipulator

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1. Introduction

Compared with serial mechanisms, parallel mechanisms have potentially higher precision, greater structural rigidity, higher speed and acceleration, and larger capacity. Therefore, they have received increased interest from both researchers and industries. They are used for flight simulators, pointing devices, and more recently, for Parallel Kinematic Machines (PKMs).

In this paper, a spatial 3-dof parallel mechanism which can be used in several applications including machine tools is proposed, and the kinematic analysis with the consideration of link flexibility is conducted. A kinetostatic model of the 3-dof parallel mechanism is then established and analyzed using the lumped-parameter model. This model can be extended for optimal design and control of PKMs.

2. Geometric Modeling and Inverse Kinematics

As represented in Figure 1, the spatial three-degree-of-freedom mechanism consists of four kinematic chains, including three variable length legs with identical topology and one passive constraining leg, connecting the fixed base to a moving platform. In this 3-dof parallel mechanism, the kinematic chains associated with the three identical legs consist — from base to platform — of a fixed Hooke joint, a moving link, an actuated prismatic joint, a moving link and a spherical joint attached to the platform. The fourth chain connecting the base center to the platform center is a passive constraining leg and has a different architecture from the other three identical chains. It consists of a prismatic joint attached to the base, a moving link and a Hooke joint attached to the platform. This last leg is used to constrain the motion of the platform to only three degrees of freedom.

The lumped compliance model described in [1] will be used to establish a simple kinetostatic model for this mechanism.

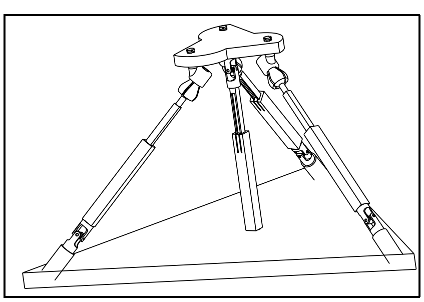

Figure 1: CAD model of the spatial 3-dof parallel mechanism with prismatic actuators.

3. Inverse Kinematics

In this 3-dof mechanism, only three of the six Cartesian coordinates of the platform are independent. In the present study, the independent coordinates have been chosen for convenience as $(z, \theta_{42}, \theta_{43})$, where θ_{42}, θ_{43} are the joint angles of the Hooke joint attached to the platform and z is the height of the platform. In order to solve the inverse kinematic problem, one must first consider the passive constraining leg as a serial 3-dof mechanism whose 3 Cartesian coordinates are known, which is a well known problem. Once the solution to the inverse kinematics of this 3-dof serial mechanism is found, the complete pose (position and orientation) of the platform can be determined using the direct kinematic equations for this serial mechanism. The inverse kinematic problem for the parallel component of the 3-dof platform can be written as

$$\rho_i^2 = (\mathbf{p}_i - \mathbf{b}_i)^T(\mathbf{p}_i - \mathbf{b}_i), \quad i = 1, 2, 3 \tag{1}$$

where $\mathbf{p}_i$ is the position vector of point P_i (vertices of the moving platform) expressed in the fixed coordinate frame whose coordinates are defined as (x_i, y_i, z_i), $\mathbf{b}_i$ is the vertices of the base in the Cartesian coordinate, ρ_i is the length of the ith leg, i.e. the value of the ith joint coordinate.

4. Jacobian Matrices for parallel components

After considering the parallel component of the mechanism, the relationship between Cartesian velocities and joint rates can be obtained by eq. (2).

$$\mathbf{A}\mathbf{t} = \mathbf{B}\dot{\boldsymbol{\rho}} \tag{2}$$

5. Jacobian Matrices for serial components

Rigid Model

For the passive leg with rigid links, one can obtain the Denavit-Hartenberg parameters, then, one has the velocity equation

$$\mathbf{J}_4\dot{\boldsymbol{\theta}}_4 = \mathbf{t} \tag{3}$$

Compliant Model

In this section, the equations for all the three identical legs are the same as in the rigid model, we only need to study the passive constraining leg with virtual joint.

From Figure 2, one can obtain the Denavit-Hartenberg parameters. And, we have the velocity equation using the same method as for the rigid links.

$$\mathbf{J'}_4\dot{\boldsymbol{\theta}}'_4 = \mathbf{t} \tag{4}$$

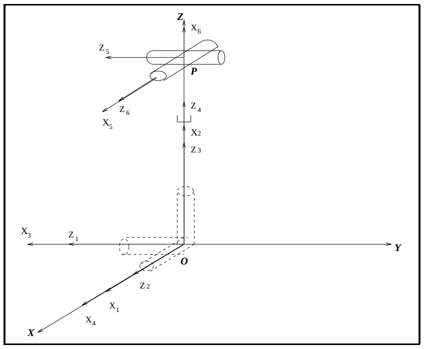

Figure 2: The passive constraining leg with flexible links.

6. Kinetostatic Models

According the principle of virtual work, one has

$$\boldsymbol{\tau}^T \dot{\boldsymbol{\rho}} = \mathbf{w}^T \mathbf{t} \tag{5}$$

where $\boldsymbol{\tau}$ is the vector of actuator forces applied at each actuated joint and $\mathbf{w}$ is the wrench (torque and force) applied to the platform and where it is assumed that no gravitational forces act on any of the intermediate links.
With eqs. (3) and (2), one has

$$\boldsymbol{\tau}^T \mathbf{B}^{-1} \mathbf{A} \mathbf{J}_4 \dot{\boldsymbol{\theta}}_4 = \mathbf{w}^T \mathbf{J}_4 \dot{\boldsymbol{\theta}}_4 \tag{6}$$

The latter equation must be satisfied for arbitrary values of $\dot{\boldsymbol{\theta}}_4$ and hence one can write

$$(\mathbf{A}\mathbf{J}_4)^T \mathbf{B}^{-T} \boldsymbol{\tau} = \mathbf{J}_4{}^T \mathbf{w} \tag{7}$$

The latter equation relates the actuator forces to the Cartesian wrench, $\mathbf{w}$, applied at the end-effector in static mode. Since all links are assumed rigid, the compliance of the mechanism will be induced solely by the compliance of the actuators. An actuator compliance matrix $\mathbf{C}$ is therefore defined as

$$\mathbf{C}\boldsymbol{\tau} = \Delta\boldsymbol{\rho} \tag{8}$$

where $\Delta\boldsymbol{\rho}$ is the induced joint displacement. Matrix $\mathbf{C}$ is a (3×3) diagonal matrix whose ith diagonal entry is the compliance of the ith actuator.
Now, eq. (7) can be rewritten as

$$\boldsymbol{\tau} = \mathbf{B}^T (\mathbf{A}\mathbf{J}_4)^{-T} \mathbf{J}_4{}^T \mathbf{w} \tag{9}$$

The substitution of eq. (9) into eq. (8) then leads to

$$\Delta\boldsymbol{\rho} = \mathbf{C}\mathbf{B}^T (\mathbf{A}\mathbf{J}_4)^{-T} \mathbf{J}_4{}^T \mathbf{w} \tag{10}$$

Moreover, for a small displacement vector $\Delta\boldsymbol{\rho}$, eq. (2) can be written as

$$\Delta\boldsymbol{\rho} \simeq \mathbf{B}^{-1} \mathbf{A} \Delta\mathbf{c} \tag{11}$$

Similarly, for small displacements, one has

$$\mathbf{J}_4 \Delta\boldsymbol{\theta}_4 \simeq \Delta\mathbf{c} \tag{12}$$

where $\Delta\boldsymbol{\theta}_4$ is a vector of small variations of the joint coordinates of the constraining leg.

Substituting eq. (11) into eq. (10), one gets

$$\mathbf{B}^{-1} \mathbf{A} \Delta\mathbf{c} = \mathbf{C}\mathbf{B}^T (\mathbf{A}\mathbf{J}_4)^{-T} \mathbf{J}_4^T \mathbf{w} \tag{13}$$

Premultiplying both sides of eq. (13) by $\mathbf{B}$, and substituting eq. (12) into eq. (13), one obtains,

$$\mathbf{A}\mathbf{J}_4 \Delta\boldsymbol{\theta}_4 = \mathbf{B}\mathbf{C}\mathbf{B}^T (\mathbf{A}\mathbf{J}_4)^{-T} \mathbf{J}_4^T \mathbf{w} \tag{14}$$

Then, premultiplying both sides of eq. (14) by $(\mathbf{A}\mathbf{J}_4)^{-1}$, one obtains,

$$\Delta\boldsymbol{\theta}_4 = (\mathbf{A}\mathbf{J}_4)^{-1} \mathbf{B}\mathbf{C}\mathbf{B}^T (\mathbf{A}\mathbf{J}_4)^{-T} \mathbf{J}_4^T \mathbf{w} \tag{15}$$

and finally premultiplying both sides of eq. (15) by $\mathbf{J}_4$, one obtains,

$$\Delta\mathbf{c} = \mathbf{J}_4 (\mathbf{A}\mathbf{J}_4)^{-1} \mathbf{B}\mathbf{C}\mathbf{B}^T (\mathbf{A}\mathbf{J}_4)^{-T} \mathbf{J}_4^T \mathbf{w} \tag{16}$$

Hence, the compliance matrix for the rigid model can be written as

$$\mathbf{C}_c = \mathbf{J}_4 (\mathbf{A}\mathbf{J}_4)^{-1} \mathbf{B}\mathbf{C}\mathbf{B}^T (\mathbf{A}\mathbf{J}_4)^{-T} \mathbf{J}_4^T \tag{17}$$

where $\mathbf{C} = diag[c_1, c_2, c_3]$, with c_1, c_2 and c_3 the compliance of the actuators and $\mathbf{J}_4$ is the Jacobian matrix of the constraining leg in this 3-dof case.
Similarly, with the principle of virtual work, one can write

$$\mathbf{w}^T \mathbf{t} = \boldsymbol{\tau}_4^T \dot{\boldsymbol{\theta}}'_4 + \boldsymbol{\tau}^T \dot{\boldsymbol{\rho}} \tag{18}$$

where $\boldsymbol{\tau}_4$ is the vector of joint torques in the constraining leg. The stiffness matrix for the mechanism with flexible links can be finally written as

$$\mathbf{K} = [(\mathbf{J}'_4)^{-T} \mathbf{K}_4 (\mathbf{J}'_4)^{-1} + \mathbf{A}^T \mathbf{B}^{-T} \mathbf{K}_J \mathbf{B}^{-1} \mathbf{A}] \tag{19}$$

with

$$\mathbf{K}_4 = diag[k_{41}, k_{42}, k_{43}, 0, 0, 0] \tag{20}$$

where k_{41}, k_{42} and k_{43} are the stiffnesses of the virtual joints introduced to account for the flexibility of the links in the constraining leg. The architecture of the constraining leg including the virtual joints is represented in Figure 2, and $\mathbf{J}'_4$ is the Jacobian matrix of the constraining leg in this 3-dof case.

7. Conclusions

A spatial 3-DOF parallel mechanism with one passive constraining leg is presented in this paper. The kinematic analysis of this spatial parallel 3-degree-of-freedom mechanism has been presented. The Jacobian matrices obtained have been used to establish the kinetostatic model of the mechanism. Finally, the kinetostatic analysis of the mechanism is conducted.

References

[1] D. Zhang, Kinetostatic Analysis and Optimization of Parallel and Hybrid Architectures for Machine Tools. Ph.D. thesis, Laval University, Québec, Canada, 2000.