Identification of Multi-DOF Loss Velocity Degeneracies for Redundant Manipulators

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1. Introduction

A velocity-degenerate (singular) configuration is a configuration in which a robot manipulator has lost at least one motion degree-of-freedom (DOF). In such a configuration, the manipulator is unable to execute an arbitrary instantaneous motion.

For a kinematically-redundant manipulator, singularities of the right Moore-Penrose pseudo-inverse of the Jacobian $\mathbf{J}$ can be examined to attempt to resolve degenerate joint displacements. This pseudo-inverse of the Jacobian, $\mathbf{J}^+$, is given by:

$$\mathbf{J}^+ = \mathbf{J}^{\mathrm{T}} \left(\mathbf{J}\mathbf{J}^{\mathrm{T}}\right)^{-1} \tag{1}$$

Velocity-degenerate configurations occur when the determinant of the $\mathbf{J}\mathbf{J}^{\mathrm{T}}$ portion of $\mathbf{J}^+$ is equal to zero. Although the matrix formed by $\mathbf{J}\mathbf{J}^{\mathrm{T}}$ is a square matrix, the form of expressions for its elements can be unwieldy. The resulting expression for $\left|\mathbf{J}\mathbf{J}^{\mathrm{T}}\right|$ can be difficult to simplify and analytical solutions to the velocity-degeneracy problem hard to find.

Nokleby and Podhorodeski [1] used the concept of reciprocity of screws to find all configurations causing one-DOF motion loss in redundant manipulators. This paper extends the reciprocity-based method to the problem of identifying multi-DOF motion loss velocity-degenerate configurations.

2. Methodology

2.1 Forward Velocity Solution

The forward velocity solution of a manipulator (given the joint rates $\dot{\mathbf{q}}$, what is the end-effector velocity $\mathbf{V} = \{\boldsymbol{\omega}^{\mathrm{T}};\ \mathbf{v}^{\mathrm{T}}\}^{\mathrm{T}}$) can be described using screws. A screw ($) is a line in space having an associated linear pitch. Screws can be represented as:

$$\$ = \{\ \mathbf{s}^{\mathrm{T}};\quad \mathbf{s}_o^{\mathrm{T}}\ \}^{\mathrm{T}} = \{\ \mathbf{l}^{\mathrm{T}};\quad \mathbf{l}_o^{\mathrm{T}} + p_{\scriptscriptstyle L}\mathbf{l}^{\mathrm{T}}\ \}^{\mathrm{T}} \tag{2}$$

where $\mathbf{s}$ and $\mathbf{s}_o$ are the unit screw coordinates, $\mathbf{l}$ and $\mathbf{l}_o$ are the Plücker coordinates of the line, and $p_{\scriptscriptstyle L}$ is the pitch of the screw [2]. A revolute joint can be represented by a zero pitch screw $\$ = \{\mathbf{l}^{\mathrm{T}};\ \mathbf{l}_o^{\mathrm{T}}\}^{\mathrm{T}}$ and a prismatic joint can be represented by an infinite pitch screw $\$ = \{\mathbf{0}_{3\mathrm{x}1}^{\mathrm{T}};\ \mathbf{l}^{\mathrm{T}}\}^{\mathrm{T}}$.

Letting $\$_i$ denote the screw coordinates of the i^{th} joint allows $\mathbf{V}$ to be determined by:

$$\mathbf{V} = \{\ \boldsymbol{\omega}^{\mathrm{T}};\quad \mathbf{v}^{\mathrm{T}}\ \}^{\mathrm{T}} = \sum_{i=1}^{k} \$_i \dot{q}_i \tag{3}$$

where k is the total number of joints and $\dot{q}_i$ is the joint rate of the i^{th} joint. In matrix form this can be expressed as:

$$\mathbf{V} = \{\ \boldsymbol{\omega}^{\mathrm{T}};\quad \mathbf{v}^{\mathrm{T}}\ \}^{\mathrm{T}} = \mathbf{J}\dot{\mathbf{q}} \tag{4}$$

where $\mathbf{J} = [\ \$_1\quad \$_2\quad \cdots\quad \$_k\ ]$ is the operator commonly referred to as the Jacobian matrix and $\dot{\mathbf{q}} = [\ \dot{q}_1\quad \dot{q}_2\quad \cdots\quad \dot{q}_k\ ]^{\mathrm{T}}$ is the vector of joint rates. Velocity degenerate configurations correspond to manipulator poses where the joint screws, $\$_i$, $i = 1, 2, \cdots, k$, do not span the 6-system of arbitrary spatial velocities.

2.2 Single-DOF Loss Velocity Degeneracies [1]

Let the screw quantity $\mathbf{A}$ ($\mathbf{A} = \{\mathbf{a}^{\mathrm{T}};\ \mathbf{a}_o^{\mathrm{T}}\}^{\mathrm{T}}$) represent the velocity of a body and the screw quantity $\mathbf{B}$ ($\mathbf{B} = \{\mathbf{b}^{\mathrm{T}};\ \mathbf{b}_o^{\mathrm{T}}\}^{\mathrm{T}}$) represent a force acting on the body. If a wrench acting on $\mathbf{B}$ contributes nothing to the rate of work being done to the body, $\mathbf{A}$ and $\mathbf{B}$ are said to be reciprocal to one another [2]. Mathematically, the two screws $\mathbf{A}$ and $\mathbf{B}$ are reciprocal if their reciprocal product is zero:

$$\mathbf{A} \circledast \mathbf{B} = \mathbf{a} \cdot \mathbf{b}_o + \mathbf{a}_o \cdot \mathbf{b} = 0 \tag{5}$$

Assuming a spatial task, i.e., 6-DOF, a manipulator is redundant if the number of joints (k) is greater than six ($k > 6$). Six joint screws ($\$_{sub_1}$, $\$_{sub_2}$, $\cdots$, $\$_{sub_6}$) are chosen to form a 6-joint sub-group Jacobian, $\mathbf{J}_{sub}$:

$$\mathbf{J}_{sub} = [\ \$_{sub_1}\quad \$_{sub_2}\quad \$_{sub_3}\quad \$_{sub_4}\quad \$_{sub_5}\quad \$_{sub_6}\ ] \tag{6}$$

Note that these six joints can not be inherently degenerate. This leaves $k - 6$ joint screws that can be considered as redundant joint screws $\$_{r_1}$, $\$_{r_2}$, $\cdots$, $\$_{r_{k-6}}$. Setting the determinant of $\mathbf{J}_{sub}$ to zero ($\mid \mathbf{J}_{sub} \mid = 0$) allows all conditions (say n in total) that cause the 6-joint sub-group to become velocity degenerate to be identified [3, 4].

In a singular configuration there exists a screw ($\mathbf{W}_{recip}$) that is reciprocal to all joint screws [5], i.e., in a singular configuration:

$$\mathbf{W}_{recip} \circledast \$_i = 0, \text{ for } i = 1 \text{ to } k \tag{7}$$

where $\$_i$ is the i^{th} joint screw and k is the total number of joints. Applying this to the six joints that comprise $\mathbf{J}_{sub}$, reciprocal screws, $\mathbf{W}_{recip_1}, \mathbf{W}_{recip_2}, \cdots, \mathbf{W}_{recip_n}$, can be found for each of the n velocity-degeneracy conditions using linear algebra techniques. The reciprocal screw, $\mathbf{W}_{recip_i}$, is reciprocal to the six joints that comprise $\mathbf{J}_{sub}$ when the i^{th} $\mathbf{J}_{sub}$ degeneracy condition is true, but will not necessarily be reciprocal to the redundant joints $\$_{r_1}, \$_{r_2}, \cdots, \$_{r_{k-6}}$. The redundant joints may still allow the manipulator to span the 6-system of general velocity. Generally, additional conditions will be required to cause $\mathbf{W}_{recip_i}$ to be reciprocal to all of the redundant joints $\$_{r_1}, \$_{r_2}, \cdots, \$_{r_{k-6}}$.

Taking reciprocal products of $\mathbf{W}_{recip_1}$ and each redundant joint $\$_{r_1}, \$_{r_2}, \cdots, \$_{r_{k-6}}$ and setting the results to zero:

$$\mathbf{W}_{recip_1} \circledast \$_{r_i} = 0, \text{ for } i = 1 \text{ to } k-6 \qquad (8)$$

yields all additional conditions necessary for $\mathbf{W}_{recip_1}$ to be reciprocal to all of the redundant joints $\$_{r_1}, \$_{r_2}, \cdots, \$_{r_{k-6}}$, simultaneously. The first condition causing $\mathbf{J}_{sub}$ to be degenerate combined with the additional conditions identified through the reciprocal products of equations (8) defines sets of conditions causing the redundant manipulator to be velocity degenerate. This procedure is repeated for each of the reciprocal screws, $\mathbf{W}_{recip_2}, \mathbf{W}_{recip_3}, \cdots, \mathbf{W}_{recip_n}$ of the remaining degenerate configurations. The procedure allows all sets of conditions (say m in total) that result in the redundant manipulator losing a single motion DOF to be identified.

2.3 Multi-DOF Loss Velocity Degeneracies

In order for a manipulator to enter a multi-DOF loss velocity degeneracy, it must first enter a single-DOF loss velocity degeneracy. Therefore, all multi-DOF loss velocity degeneracies are based on at least one single-DOF loss velocity degeneracy. This fact is used to extend the methodology of Nokleby and Podhorodeski [1] to the problem of identifying multi-DOF loss velocity degeneracies.

Applying the method of Section 2.2 to a redundant manipulator yields m sets of conditions that result in a single-DOF loss velocity degeneracy. In addition, a reciprocal screw for each of those m velocity degeneracies is generated (i.e., $\mathbf{W}_1$ to $\mathbf{W}_m$).

Define $\mathbf{W}^*$ as the screw which in the chosen reference frame can be obtained from $\mathbf{W}$ as $\mathbf{W}^* = \{\mathbf{w}_o^{\text{T}}; \mathbf{w}^{\text{T}}\}^{\text{T}}$ where $\mathbf{W} = \{\mathbf{w}^{\text{T}}; \mathbf{w}_o^{\text{T}}\}^{\text{T}}$. Since $\mathbf{W}$, by definition, can not be a zero screw, the reciprocal product between $\mathbf{W}$ and $\mathbf{W}^*$ can never be zero ($\mathbf{W} \circledast \mathbf{W}^* \neq 0$).

Set the j^{th} set of degeneracy conditions to be true. Choosing five joint screws along with $\mathbf{W}_j^*$, allows a new sub-Jacobian to be defined as:

$$\mathbf{J}_{sub_j}^* = \left[\begin{array}{cccccc} \$_{sub_{j_1}} & \$_{sub_{j_2}} & \$_{sub_{j_3}} & \$_{sub_{j_4}} & \$_{sub_{j_5}} & \mathbf{W}_j^* \end{array} \right] \qquad (9)$$

The only condition on the choice of the screws for $\mathbf{J}_{sub_j}^*$ is that $\$_{sub_{j_1}}, \$_{sub_{j_2}}, \$_{sub_{j_3}}, \$_{sub_{j_4}}, \$_{sub_{j_5}}$ and $\mathbf{W}_j^*$ can not be inherently degenerate. This leaves $(k-6)+1 = k-5$ screws that can be thought of as redundant screws $\$_{r_{j_1}}, \$_{r_{j_2}}, \cdots, \$_{r_{j_{k-5}}}$. The same method outlined in Section 2.2 can now be used to find all unique sets of conditions that lead to a two-DOF loss velocity degeneracy, provided the j^{th} set of degeneracy conditions are also true. As before, the reciprocal screws for these new velocity-degenerate configurations are generated. Using $\mathbf{W}_j^*$ in the sub-Jacobian ensures that the new reciprocal screw will be a screw that is not a linear multiple of $\mathbf{W}_j$. Repeating the procedure for the $j = 1$ to m sets of conditions allows all sets of conditions resulting in a double-DOF loss velocity degeneracy to be identified.

Based on the fact that to enter a n-DOF loss velocity-degenerate configuration, a manipulator must first enter a $(n-1)$-DOF loss velocity-degenerate configuration, the same procedure can be extended to yield all sets of conditions resulting in velocity degeneracies of higher DOF loss.

3. Conclusions

The methodology of Nokleby and Podhorodeski [1] for determining single-DOF loss velocity-degenerate configurations for redundant manipulators based on reciprocity of screws was extended to solve the problem of determining multi-DOF loss velocity-degenerate configurations for redundant manipulators.

References

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