MESHING CHARACTERISTICS OF HYPOCYCLOIDAL GEARS

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1. Abstract

The paper deals with kinematic characteristics of hypocycloidal gearsets generated by conjugate action of a known pinion with cylindrical teeth. The number of teeth of pinion is for one les than the number of teeth of gear. The line of contact of teeth is analytically formulated and its trends discussed in detail. The effects of the gearset parameters upon the locus of contact points are analytically determined and graphically plotted for the three typical gearsets.

2. Introduction

Cyloidal gearsets are characterized by a large gear ratio when they are designed in a planetary arrangement. The large reductions of input velocity (10 to 50 times) result from the difference of one tooth in numbers of teeth of the pinion and the internal gear. The sizes of the cycloidal gearsets are relatively smaller than those with other shape of tooth profile, including the involute profile, for the same gear ratios and the load ratings. Besides, the loss of power due to friction in these gearsets is reduced to a minimum. These characteristics of cycloidal gears have lead to their increased application in engineering practice [1].

A gearset containing a compound pinion with cylindrical teeth and an internal gear meshing together is shown in Fig.1 Tooth numbers of gear and pinion differ by one.

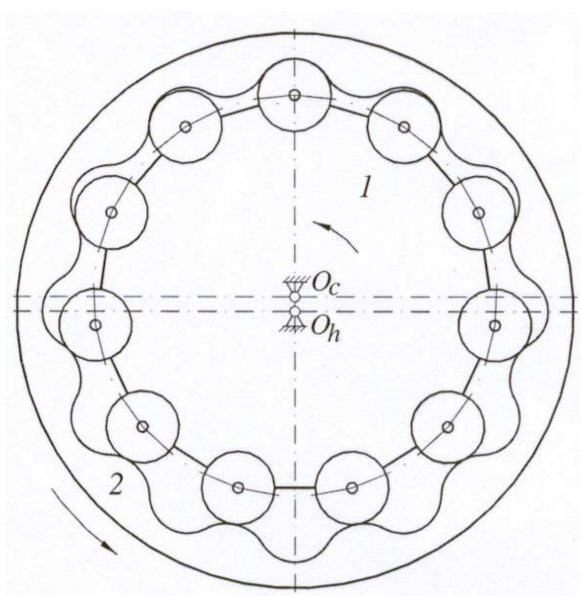
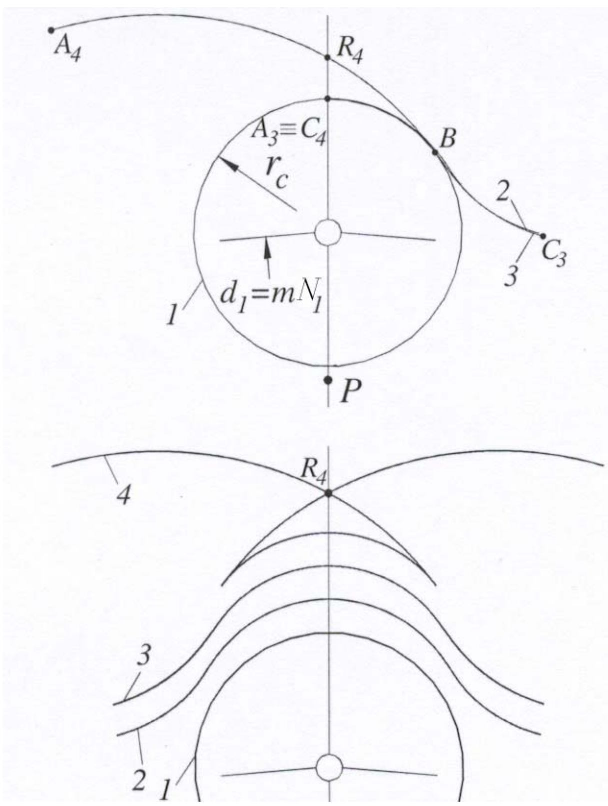


Fig. 1 Gearset of Pinion with Cylindrical Teeth and Epicycloidal Gear with m= 5 mm, $N_1 = 11$, $N_2 = 12$, x = 0.2 and $r_c^* = 1$

In the previous work [3] it has been shown that, besides this gearset, the other two gearsets can be generated, where compound pinion with cylindrical teeth has been replaced by an integral pinion with hypocycloidal external teeth. In this set the tooth profile of the pinion is generated using the conjugate action and external and internal envelope curve of the successive positions of the tooth profile of the hypocycloidal gear in the plain perpendicular to exes of the cenrodes that roll on each other with no slippage.
The objective can be achieved either by use of the theory of envelopes or by utilizing the law of gearing that assumes the instantaneous center of the pinion and gear to be fixed on the line of centers in order to have a constant angular velocities ratio.

In this article the authors aim to investigate the characteristics of meshing of the two hypocycloidal gearsets generated in the above mentioned manner. The locus of contact points is determined for each gearset and the effect of gear parameters upon the line of contact are analytically formulated and discussed. Also the loci of contact points are graphed for the three typical gearsets in Fig. 5.



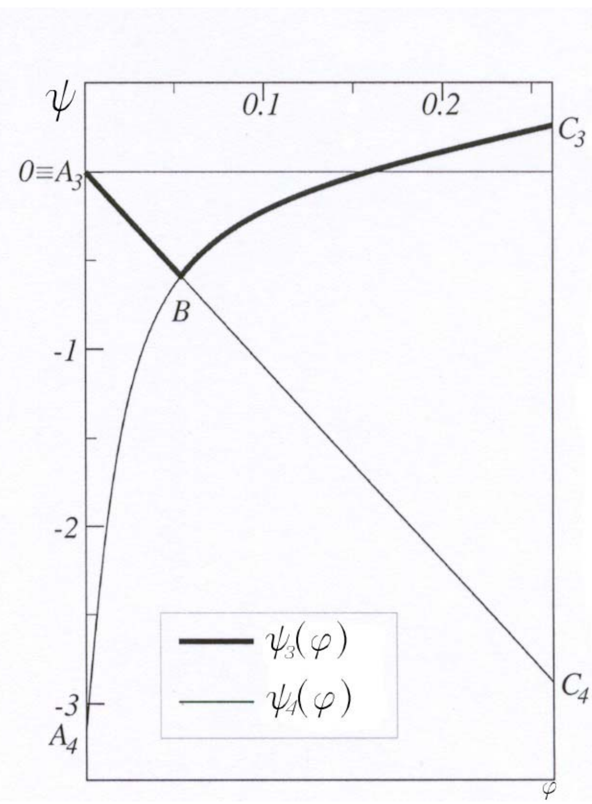


Fig. 2 Conjugate Tooth Profiles of Gearsets 1-2, 3-2 and 4-2 with N_1=N_3=N_4=11, N_2=12, x = 0.2 and $r_c^* = 1$

Fig 3 Relation Between Parameters ψ and φ for N_1=N_3=N_4=11, N_2=12 and x = 0.2

3. Determination of Line of Contact

Equations of the tooth profile of hypocycloidal wheel 2 for a typical hypocycloidal gearset are given by:

$$\xi_2 = \frac{m}{2}[(N_2-1)\sin\varphi-(1-x)\sin(N_2-1)\varphi+2r_c^*\frac{(1-x)\sin(N_2-1)\varphi+\sin\varphi}{\sqrt{1-2(1-x)\cos N_2\varphi+(1-x)^2}}] \quad (1a)$$

$$\eta_2 = \frac{m}{2}[(N_2-1)\cos\varphi-(1-x)\cos(N_2-1)\varphi+2r_c^*\frac{(1-x)\cos(N_2-1)\varphi+\cos\varphi}{\sqrt{1-2(1-x)\cos N_2\varphi+(1-x)^2}}] \quad (1b)$$

where *m, N, x* and r_c*=*1* are module, number of teeth, coefficient of modification (withdrawal) and coefficient of the generating circle radius respectively. The parameter φ varies within interval $[0,\pi/N_2]$. Pinion wheel *1* is of a compound design. It consists of a cylindrical hub and $N_1 = N_2 - 1$ cylinrical teeth of radius $r_c = m\, r_c^*$ with centres located on a circle with a radius $r = mN_1$. The centre distance is equal to $a_w = 0.5m(1-x)$.

Two new hypocycloidal gearsets are generated by the pinion with cylindrical teeth, and they consist of hypocycloidal gear *2* with internal teeth , and pinion *3* with external teeth. The hypocycloidal wheel *2* with external teeth and gear *4* with internal teeth as per Fig.2. The tooth profile of the wheels replacing the wheel with cylindrical teeth of the same number of teeth is described in terms of parametric equations:

$$x_{3,4} = \frac{m}{2}[N_1\sin(\varphi+\frac{\psi_{3,4}}{N_1})-\lambda\sin(N_1\varphi-\frac{\psi_{3,4}}{N_1})-\lambda\sin(\frac{N_2}{N_1}\psi_{3,4})+2r_c^*\frac{\lambda\sin(N_1\varphi-\frac{\psi_{3,4}}{N_1})-\sin(\varphi+\frac{\psi_{3,4}}{N_1})}{\sqrt{1-2\lambda\cos N_2\varphi+\lambda^2}}] \quad (2a)$$

$$y_{3,4} = \frac{m}{2}[N_1\cos(\varphi+\frac{\psi_{3,4}}{N_1})-\lambda\cos(N_1\varphi-\frac{\psi_{3,4}}{N_1})-\lambda\cos(\frac{N_2}{N_1}\psi_{3,4})-2r_c^*\frac{\lambda\cos(N_1\varphi-\frac{\psi_{3,4}}{N_1})-\cos(\varphi-\frac{\psi_{3,4}}{N_1})}{\sqrt{1-2\lambda\cos N_2\varphi+\lambda^2}}] \quad (2b)$$

where $\lambda = 1-x$ is a factor les than 1, and ψ is the angle of rotation of the hypocycloidal wheel about instantaneous center of velocitiy P. The angle ψ is defined by equation:

$$\psi_{3,4}(\varphi) = \varphi+\cos^{-1}\frac{-\lambda\sin^2 N_2\varphi\mp\sqrt{\lambda^2\sin^4 N_2\varphi-[\lambda^2-2\lambda\cos N_2\varphi+1][2\lambda-(1+\lambda^2)\cos N_2\varphi]\cos N_2\varphi}}{\lambda^2-2\lambda\cos N_2\varphi+1}-\pi \quad (3)$$

The locus of contact points is described by parametric equations:

$$x_k = x_{3,4}\cos(\frac{N_2\psi_{3,4}}{N_1})-y_{3,4}\sin(\frac{N_2\psi_{3,4}}{N_1}) \quad (4a)$$

$$y_k = x_{3,4}\sin(\frac{N_2\psi_{3,4}}{N_1})+y_{3,4}\cos(\frac{N_2\psi_{3,4}}{N_1}) \quad (4b)$$

It is obvious that the tooth-profiles of the wheels 3 and 4, relative to points of contact depend on parameters N_1, *x* and r_c^*. In order to define the effect of these parameters, it is necessary to examine the relation (3) between the parameters ψ and φ. It can be concluded:

$$\psi_3(\varphi) = -N_1\varphi, \qquad \varphi \in \left[0, \frac{\cos^{-1}(\lambda)}{N_2}\right] \tag{5a}$$

$$\psi_3(\varphi) = \varphi + \cos^{-1}\frac{-\lambda\sin^2 N_2\varphi + (\lambda - \cos N_2\varphi)(\lambda\cos N_2\varphi - 1)}{\lambda^2 - 2\lambda\cos N_2\varphi + 1} - \pi; \quad \varphi \in \left[\frac{\cos^{-1}(\lambda)}{N_2}, \frac{\pi}{N_2}\right] \tag{5b}$$

$$\psi_4(\varphi) = \varphi + \cos^{-1}\frac{-\lambda\sin^2 N_2\varphi + (\lambda - \cos N_2\varphi)(\lambda\cos N_2\varphi - 1)}{\lambda^2 - 2\lambda\cos N_2\varphi + 1} - \pi; \quad \varphi \in \left[0, \frac{\cos^{-1}(\lambda)}{N_2}\right] \tag{5c}$$

$$\psi_4(\varphi) = -N_1\varphi, \qquad \varphi \in \left[\frac{\cos^{-1}(\lambda)}{N_2}, \frac{\pi}{N_2}\right] \tag{5d}$$

Obviously within the range$\varphi \in [0, \cos^{-1}(\lambda)/N_2]$ function $\psi_3(\varphi)$ decreases linearly. The same is true for function $\psi_4(\varphi)$ within the range $\varphi \in [\cos^{-1}(\lambda)/N_2, \pi/N_2]$. In order to examine the trend of change of function $\psi_3(\varphi)$ within the range $\varphi \in [\cos^{-1}(\lambda)/N_2, \pi/N_2]$, and of function $\psi_4(\varphi)$ within the range $\varphi \in [0, \cos^{-1}(\lambda)/N_2]$, it is necessary to check the derivative:

$$\frac{d\psi_3(\varphi)}{d\varphi} = \frac{d\psi_4\varphi)}{d\varphi} = \frac{d[\varphi + \cos^{-1}\dfrac{-\lambda\sin^2 N_2\varphi + (\lambda - \cos N_2\varphi)(\lambda\cos N_2\varphi - 1)}{\lambda^2 - 2\lambda\cos N_2\varphi + 1} - \pi]}{d\varphi} \tag{6}$$

After few rearrangements one can obtain the following:

$$\frac{d\psi(\varphi)}{d\varphi} = 1 - \frac{N_2(\lambda^2 - 1)}{\lambda^2 - 2\lambda\cos N_2\varphi + 1} \quad ; \quad \varphi \in [0, \frac{\pi}{N_2}]$$

(7)

Since $N_2(\lambda^2 - 1) < 0$, and $\lambda^2 - 2\lambda\cos(N_2\varphi) + 1 > 0$ for ($\lambda \in (0,1)$), one can conclude that $d\psi(\varphi) > 1 > 0$, which means that the function $\psi_3(\varphi)$ within the range $\varphi \in [\cos^{-1}(\lambda)/N_2, \pi/N_2]$ as well as the function $\psi_4(\varphi)$ within the range $\varphi \in [0, \cos^{-1}(\lambda)/N_2]$ are monotonically nondecreasing functions.

The function $\psi_3(\varphi)$ has three extremes as shown in Fig.3. In the range $\varphi \in [0, \cos^{-1}(\lambda)/N_2]$ the function is monotonically decreasing. At $\varphi = 0$ the function has a local maximum equal to zero and at the end of the range at $\varphi_B = \cos(\lambda)/N_2$, it reaches a global minimum equal to $-\cos(\lambda)N_1/N_2$. Within the range $\varphi \in [\cos^{-1}(\lambda)/N_2, \pi/N_2]$ the function is increasing and at the end of the range at value $\varphi = \pi/N_2$ the function reaches a global maximum equal to π / N_2.

The function $\psi_4(\varphi)$ also has three extremes as shown in Fig 3.. In the range $\varphi \in [0, \cos^{-1}(\lambda)/N_2]$ the function is a monotonically nondecreasing function. At $\varphi = 0$ the function has a global minimum equal to $-\pi$. At $\varphi_B = \cos^{-1}(\lambda)/N_1)$ the function reaches a global maximum equal to $-\cos^{-1}(\lambda)N_1/N_2$.Within the range$\varphi \in [\cos^{-1}(\lambda)/N_2), \pi/N_2]$ the function is a monotonically nonincreasing and at $\varphi = \pi/N_2$ it reaches a local minimum equal to $\pi N_1/N_2$.

Fig.4 shows the lines of contact of two hypocycloidal gears with m = 5 mm, $N_1 = 11$, $r_c^* = 1$ and $x = 0.2$. Portion of contact line A_3BC_4 corresponding to a linear relation between the parameters ψ and φ, in both epicycloidal gears is equal to the line of contact of the pinion with cylindrical teeth and the hypocycloidal internal gear. Portion of contact line BC_3 of gerset *3-2* corresponds to line BC_3 of function $\psi_3(\varphi)$ and part BA_4 of gearset *2-4* corresponds to line BA_4 of function $\psi_4(\varphi)$. Gearset *3-2* exhibits two contact points within the range as shown in Fig.4 (a). When $\psi_3(\varphi)$ is changed within the range $\psi_{C3} = \pi/N_4$ to zero one contact point exists, while within the range from zero to $\psi_B = -\cos^{-1}(\lambda)N_1/N_2$ two contact points occur. Within the range of $\psi_B = -\cos^{-1}(\lambda)N_1/N_2$ to $-\pi N_1/N_2$ only one contact point exists.

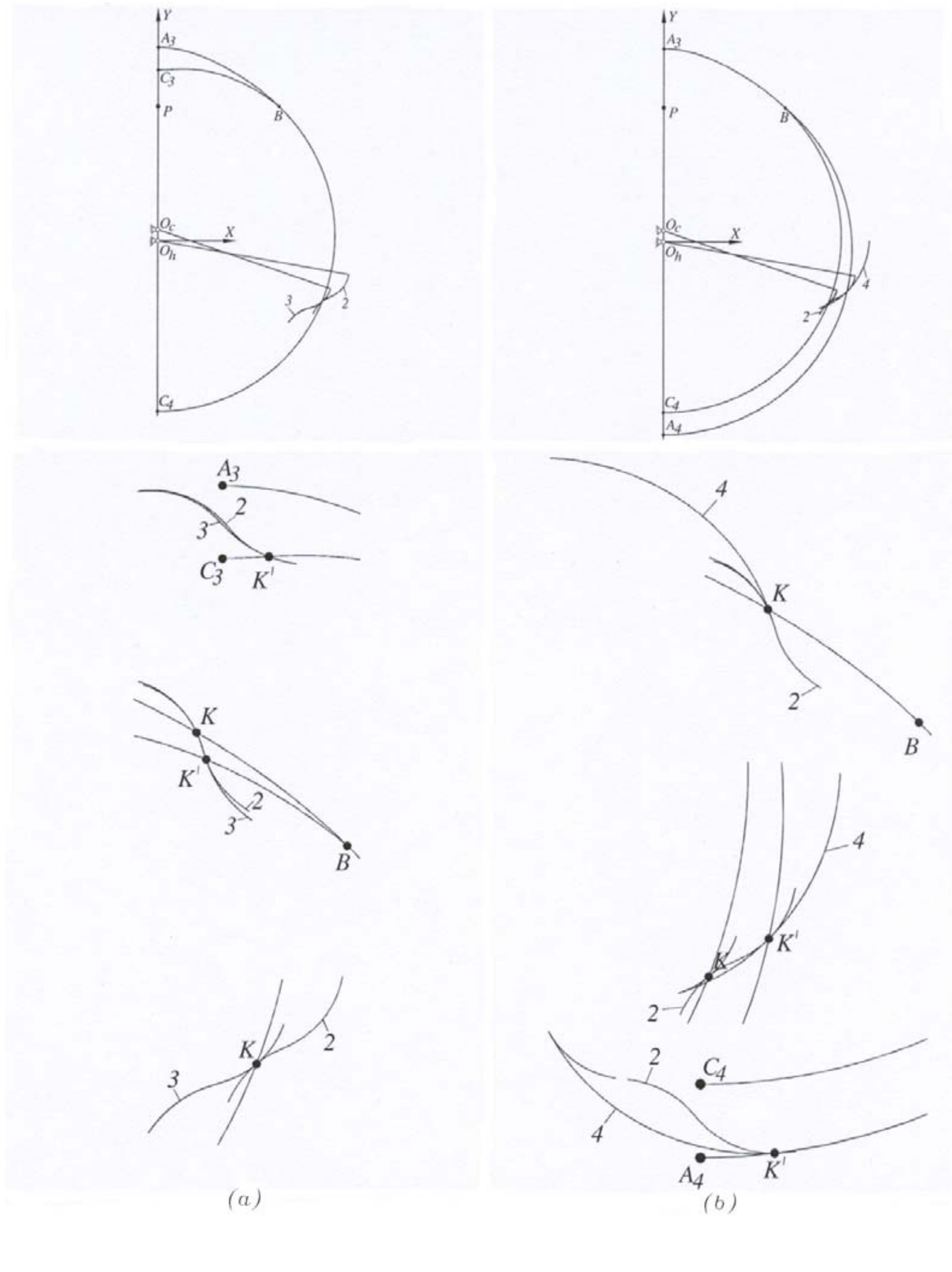


Fig. 4 Lines of Contact of Two Hypocycloidal Gearsetss with m = 5 mm, $N_1 = 11$, x = 0.2 and $r_c^* = 1$

In gearset *2-4* $\psi_4(\varphi)$ also passes through contact ranges with one and two contact points as shown in Fig.4.(b). When $\psi_4(\varphi)$ changes from zero to $\psi_B = -\cos^{-1}(\lambda)N_1/N_2)$ one contact point exists. Within the range from $\psi_B = -\cos^{-1}(\lambda)N_1/N_2$ to $-\pi N_1/N_2$ two contact points occur. Within the range from $-\pi N_1/N_2$ to π only one contact point exists. Due to the fact that the profile of gear wheel 4 makes a loop which is undercut so that only part RA_4 remains, there is only one point of contact.

Fig. 5 shows the loci of contact points drown by a continuous lines for a gearset consisting of a compound pinion with cylindrical teeth and a hypocycloidal pinion and the two hypocycloidal gear generated with m = 5 mm, N_1 = 11, x = 0.2 and r_c*=1.

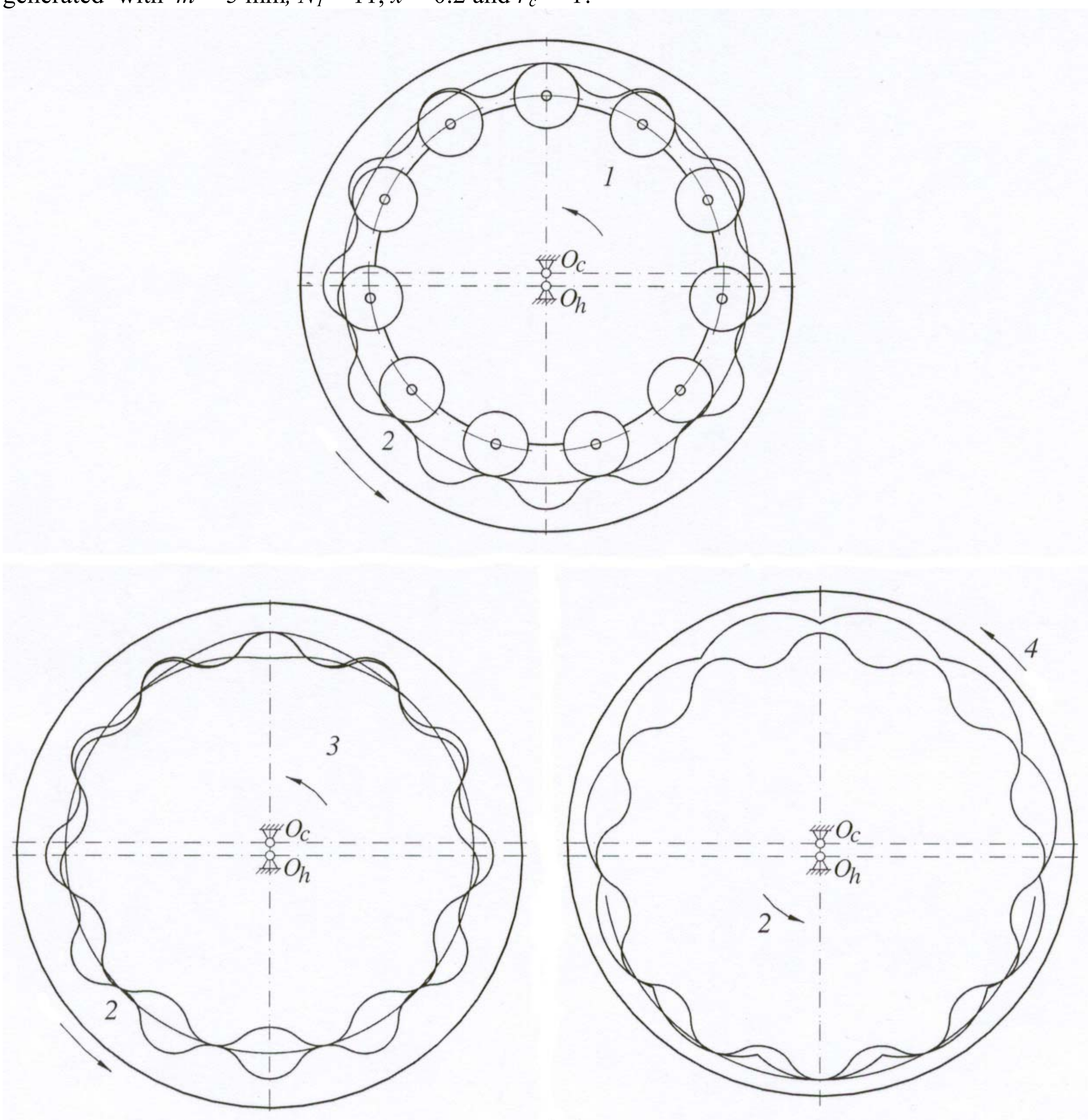


Fig. 5 Lines of Contact for Three Hypocycloidal Gearsets with
m = 5 mm, N_1= 11, N_2= 12, x = 0.2 and r_c^*=1

4. Conclusions

The objective of the study that had been set to generate and analytically formulate parameters of the gearsets based on a pinion with cylindrical tooth profile using the conjugate action and replace the pinion with an integral wheel with hypocycloidal tooth profile has been accomplished.. The line of contact has been formulated analytically. The characteristic portions of the line of contact have been critically analyzed and the effect of geaset basic parameters on the line of contact have been established.

As the most important conclusion from the results of this analysis is that generated gearset *3-2* would have an increased load rating because the contact of a convex and a concave surfaces occurs that should yield a lower Hertzian contact stress. In addition a multiple contact points occur along line of contact and an increased load rating could be claimed too. This aspect remains to be investigated by a contact stresses determination. From the property of cycloidal curves the curvature of the surfaces in contact are readily available.

Since the common normal from each contact point must pass through the instantaneous center it is obvious that the pressure angle vary and the transmitted torque is expected to vary. Since there are a half of teeth in contact at any instant it is hoped that the torque will average to a value close to constant.

Gearset *2-4* resulting from the conjugate generating process represents an unusual phenomenon in the theory of the toothed wheels since the number of teeth of wheel 2 is for one larger than the number of teeth of wheel 4, hence designation of the pinion and the gear does not hold anymore. Due to unfavorable pressure angles, as result of undercutting of tooth profile of wheel 4 this gearset is unsuitable for power transmission. Yet, by itself, it is an interesting kinematic phenomenon

References

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