Optimization of Six-Bar Stephenson Dwell Mechanism Using Differential Evolution

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Abstract

This paper discusses optimum synthesis of mechanisms. In general mechanism synthesis, a linkage is designed for function generation, motion generation, or path generation. Function generation correlates the input and output link motions. A dwell mechanism output link remains stationary for some specified input motion range. This paper presents a method for designing dwell mechanisms using Differential Evolution. Differential Evolution is an evolutionary optimization scheme, enabling finding the global optimum of a design problem. The method uses two penalty functions: one for constraint violation, and one for relative accuracy. The developed methodology is applied to the synthesis of six-bar linkages for dwell and dual-dwell mechanisms. The six-bar mechanism is synthesized using two different approaches: four-bar extension to six-bar, and direct six-bar. The paper concludes with results demonstrating the successful application of the method and the two approaches.

Optimisation du Mécanisme Statique à Six-Barres Stephenson en Utilisant l'évolution Différentielle

Résumé

Cet article présente la synthèse optimum des mécanismes. Généralement dans la synthèse de mécanismes, un lien est conçu pour la génération de fonctions, la génération de mouvements, ou la génération de chemins. La génération de fonctions corrèle les liens d’entrée et de sortie des mouvements. Le lien de sortie d’un mécanisme statique reste stationnaire pour une gamme spécifique de mouvements d’entrée. Cet article présente une méthode pour concevoir des mécanismes statique en utilisant l' Évolution Différentielle. L'Évolution Différentielle est un arrangement évolutionnaire d'optimisation, permettant de trouver la solution globale optimum pour un problème de conception. La méthode emploie deux fonctions de pénalité - une pour la violation de contraintes et une pour l'exactitude relative. La méthodologie développée est appliquée à la synthèse des liens à six barres pour des mécanismes statique et dual-statique. Le mécanisme à six-barres est synthétisé en utilisant deux approches différentes: la prolongation de quatre-barres à la six-barres, et l’utilisation de six-barres directement. L’article conclut avec des résultats démontrant la réussite de l'application de la méthode et des deux approches.

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1 Introduction

Mechanism synthesis using computer optimization has been an active research area during the last 30 years. The most common mechanism studied is the four-bar linkage. In general, mechanism synthesis includes motion, function, and path generation. Both graphical and numerical techniques with and without prescribed timing are well studied [1]. The graphical techniques are limited to a small finite number of precision points and the solution accuracy is limited. Numerical techniques are commonly combined with various optimization schemes such as genetic algorithms [2, 3, 4], evolutionary techniques [5], interior-point method [6], and Gauss constrained method [7].

Limitations imposed on size, shape and force transmission ability at times require mechanisms that could meet complex design tasks [8]. The synthesis of six-bar linkages offers an alternative to cams to attain certain special requirements that are usually not satisfied by a four-bar mechanism. One such application is the use of a six-bar mechanism when there is a requirement to produce a dwell in the output link during predefined motion periods of the input link.

This paper will only study a six-bar linkage, comprised exclusively of binary links and revolute joints. A similar procedure developed for linkages with a slider joint exists, but will not be presented here. A dwell occurs in the output link when the output link remains stationary for non-zero input link motion. Cams are often used in generating dwells, but cams generally are expensive to manufacture, whereas the six-bar linkages can produce the dwell with relatively low cost and wear [9]. Thus, six-bar linkages are sometimes a viable alternative even considering the difficulties associated with its synthesis [9]. In the optimum synthesis of dwell mechanisms, the optimization problem is formulated as a minimization of the error in correlated input and output angles.

Numerous optimization methods applied to mechanism synthesis are found in the literature with somewhat dated surveys presented in [10, 11]. More modern approaches, including the use of evolutionary computation, linear and non-linear programming, and other techniques are found in [2- 5, 12-14, 16-24, 34, 35]. Four-bar mechanism synthesis is broadly reviewed in [4, 5]. A detailed review of mechanism optimization literature will not be presented herein. Because of its ease of use in scientific computing, MATLAB was used in all this research.

Based on four-bar, slider-crank, and inverted slider-crank, there are 21 six-bar configurations, which may be synthesized. This paper studies the six-bar based on the basic four-bar linkage, having R-R dyads exclusively (known as Stephenson's inversion III). This mechanism can generate dwells in the output linkage if there are circular arcs in the coupler curve of the primary four-bar linkage. The proposed method requires the user to prescribe the precision points on the coupler curve including the precision points located on circular arcs, which will be used to produce the dwells in the output link of the six-bar, the relationship between crank-angle to output-angle for the precision points on arcs, and the minimum allowable value of the transmission angle. The successive optimization methodology is applied to the synthesis of six-bar dwell mechanisms using two different approaches. First, a two-staged synthesis is used in the synthesis of a four-bar linkage and then the synthesis of remaining dyad of six-bar. Here, we consider the accuracy at each precision point on the coupler curve and the input-output angle correlation for circular arc regions. In the second method, the synthesis of the six-bar is performed directly, considering accuracy at precision points on the whole coupler curve and the input-output angle correlation for circular arc regions.

A recently developed evolutionary-based optimization technique was used in the optimization, referred to as Differential Evolution (DE). The method has been successfully applied to many diverse domains [25, 26]. The authors have previously used a DE-based optimization techniques for optimum robot design considering kinematic, dynamic, and structural constraints, and for optimum synthesis of four-bar mechanisms [5, 27]. Herein, we investigate the application of DE to the synthesis of six-bar mechanisms. The initial bounds for the design variables are defined based on the Geometric Centroid of the Precision Positions (GCPP) previously developed in [5]. In the presented technique iterations, the initial accuracy to be met at each precision position and the accuracy at the output angle are obtained. The results of the current stage are then used as initial guesses for subsequent levels of optimization in an attempt to improve the accuracy of the synthesized mechanism.

2 Optimization Problem Definition and Tools

A general optimization problem is defined as follows [28]

$$\begin{aligned} &\text{minimize } f(x) \\ &\text{subject to } g_i(x) \le 0 \end{aligned} \tag{1}$$

where, $f(x)$ is the cost function, $g_i(x)$ is the set of inequality constraints and $x \in |R^n$ is the real-valued design variable vector with n being the number of design variables. In DE terminology, the objective function is called the Cost-function. Figure 1 shows a schematic of the six-bar mechanism and the design variable used. Although it is possible to optimize mechanisms without these inequality constraints [34], and thus reduce computation time, the efficiency of the DE algorithm has been found by the authors' tests to be much faster than genetic algorithms with elitism, and thus, the computation times are relatively small. In fact, the authors' tests have shown the DE algorithm to often be 30 times faster than a conventional GA. Other tests have shown DE to be more accurate than GAs. The authors' experience has been that most computations are found in just a few minutes. However, the general technique describe herein could be improved using the techniques given in [34].

The cost function $f(x)$ for mechanism synthesis for path generation is expressed as an error quantity that defines the deviation of each evaluated coupler point $P_C(x, y)$ from the corresponding specified precision position. The cost function for the synthesis of dwell mechanism also includes the correlation between the input and output angles. The cost function employed here consists of the deviation of the precision points of the coupler curve and the deviation of the output angles from their respective desired values. These deviations are captured in the error function E. The error function directs the optimization search in regions of possible solutions. In addition, two different penalty functions are incorporated for the violation of imposed constrains. The cost function is found in two steps: first the error calculation, and second, the penalty for constraint violation. Note that only the precision points located on the circular arcs of the coupler curve are considered for input-output angle correlation because these points are used to generate the dwells.

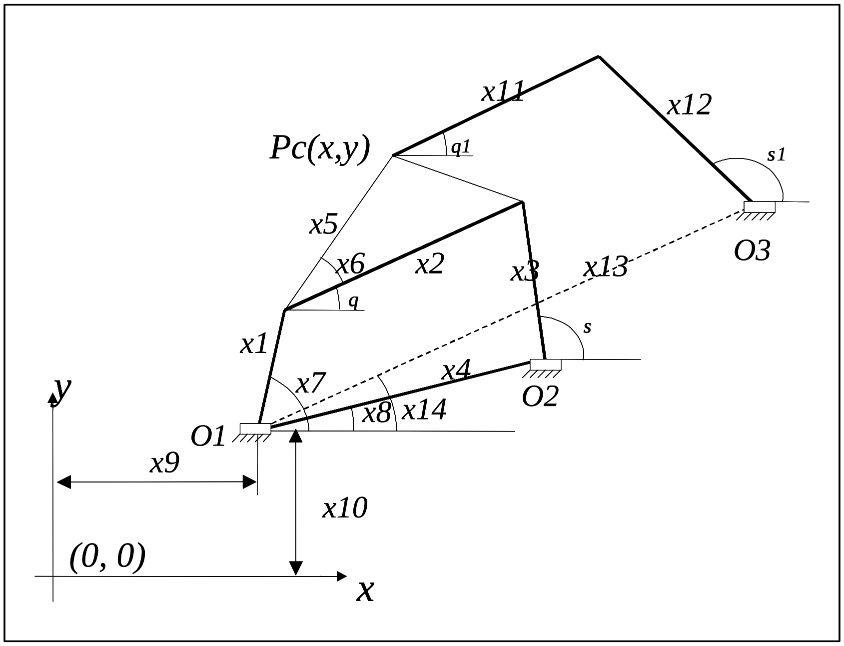

Figure1: Six-bar Mechanism Diagram and Design Variable Definition

To summarize, the objective is to synthesize a mechanism that will pass through the precision points, while meeting the coordinated requirement between input and output angles in the portion of dwell with the desired accuracy level. Therefore, the error function is the sum of the square of the error at each precision point and square of the error at output angle. The precision points error is

$$E_1 = \sum_{i=1}^{n_p} \left\{ (x_{di} - x_{gi})^2 + (y_{di} - y_{gi})^2 \right\} \tag{2}$$

where, n_p is the number of specified points, (x_{di}, y_{di}) and (x_{gi}, y_{gi}) are the coordinates of the desired and generated (actual) points. Error for the circular arc (dwell) is

$$E_2 = \sum_{i=1}^{n_{pd}} (s_d - s_g)^2 \tag{3}$$

where, n_{pd} is the number of specified points on a dwell period, s_d is the desired output angle, and s_g is the generated output angle.

In mechanism synthesis, an often imposed constraint is the satisfaction of the Grashoff criterion [1]. Depending upon the optimization method and application requirements, inequality constraints on the design variables are also defined, such as limits on the range of transmission angles, or limits on the mechanism link lengths. In this work, three constraint sets are imposed. These constraints could be easily replaced to address other requirements or Grashoff type kinematic inversions.

First, we consider positive magnitude constraints. All evolutionary based optimization techniques require the user to define bounds for the design variables. A property of DE allows the search to extend beyond these initial bounds. However, the user must carefully define constraints such that the physical characteristics of the problem and design variables are preserved. The positive magnitude constraints ensure that the link lengths of the linkages are positive.

$$\begin{gathered} g_1 = -x_1 < 0 \ ; \ g_2 = -x_2 < 0 \ ; g_3 = -x_3 < 0 ; \ g_4 = -x_4 < 0 ; \\ g_5 = -x_5 < 0 ; g_6 = -x_{11} < 0 \ ; \ g_7 = -x_{12} < 0 \ ; g_8 = -x_{13} < 0 \end{gathered} \tag{4}$$

Second, we consider the Grashoff criterion specifying a crank-rocker. This Grashoff criterion states that the designed mechanism must have at least one crank with complete rotation. This constraint enables the mechanism to be driven continuously, say by an electrical motor in a manufacturing situation where continuous motion is required in an inexpensive form. In four-bar mechanisms, the crank-rocker type linkage takes form when the input link has the smallest length and operates as a crank, while the output link just oscillates [1]. The Grashoff constraints for crank-rocker type four-bar mechanism are,

$$\begin{gathered} g_9 = (l + s) - (p + q) < 0 \\ g_{10} = x_1 - x_2 < 0 ; \ g_{11} = x_1 - x_3 < 0 \\ g_{12} = x_1 - x_4 < 0 \end{gathered} \tag{5}$$

where, l and s identify the longest and the shortest links. It is important to note that these constraints could be easily changed to address other requirements and this Grashoff criterion is only imposed to guarantee continuous motion of the crank.

Third, we consider the violation of transmission angle constraint. The condition of transmission angle is verified at each precision point. The goal is to keep the minimum transmission angle of the mechanism larger than the desired value when the mechanism passes through the precision points. The transmission angle is defined as an acute angle between the coupler and output links [1, 15]. Whenever the coupler and the output links of the mechanism are aligned, the mechanism losses its mobility [1]. A small transmission angle is deemed inadequate to transmit desired forces to the output link. The desired value of transmission angle depends upon the specific application [1]. In order to prevent the loss of mobility condition, the transmission angle constraint imposed is,

$$\begin{gathered} g_{13} = t_{fd} - t_{fo} < 0 \\ g_{14} = t_{sd} - t_{so} < 0 \end{gathered} \tag{6}$$

where t_{fd} and t_{sd} are the desired transmission angles of four-bar and six-bar mechanisms respectively, and t_{fo} and t_{so} are the obtained transmission angles of four-bar and six-bar mechanisms respectively.

Forth, we consider the accuracy constraint. The accuracy constraint is applied at each precision point. It compares the distance between the desired and the generated precision points with a desired accuracy.

$$g_k = d_i - a < 0$$
$$d_i = \left\{(x_{di} - x_{gi})^2 + (y_{di} - y_{gi})^2\right\}^{1/2} \tag{7}$$

The accuracy level is also applied to the output angle

$$g_k = \bar{s}_i - a < 0 \tag{8}$$

where $\bar{s}_i$ is the difference between the desired and generated output angles at the precision point located on the dwell and $i = 1,2,.....n_{pd}$, $k = 15 + np,.....15 + n_p + npd$.

Two differently weighted penalty functions penalizing the violation of constraints and the relative accuracy of the generated coupler point are implemented. The first three constraint sets are required for the valid assembly of a continuous motion crank-rocker four-bar mechanism with satisfactory transmission angles for each precision point. Therefore, any violation is heavily penalized, regardless of the magnitude of the violation. This penalty is referred to as a dominating penalty. The cumulative dominating penalty for the violations in these constraint sets is calculated according to

$$\begin{aligned}
&P_d = 0\\
&for\ j = 1:14\\
&\quad if\ g_j > 0\\
&\qquad P_d = P_d + z\\
&\quad end\\
&end
\end{aligned} \tag{9}$$

where, z is a large positive number.

The violation of an accuracy constraint is penalized considering the magnitude (measure or severity) of violation. The implemented strategy calculates the percentage of the violation with respect to the desired accuracy. Thus, the cumulative accuracy penalty is evaluated according to

$$\begin{aligned}
&P_a = 0; i = 1;\\
&for\ k = 15:15 + np + npd\\
&\quad if\ g_k > 0\\
&\qquad P_a = P_a + G_i^2\\
&\qquad i = i + 1\\
&\quad end\\
&end
\end{aligned} \tag{10}$$

In summary, the mathematical formulation of the optimization problem as applied to this work is as follows:

Minimize,

$$F = s \times E + P_t$$

$$P_t = \begin{cases} 0, & \text{if all constraints are satisfied} \\ \text{calculated penalty}, & \text{otherwise} \end{cases} \tag{11}$$

$$E = E_1 + E_2$$

where, $P_t = P_a + P_d$, and s denotes a scaling factor selected in such a way that the search is always directed towards decreasing error. This is achieved by weighing E more than P_a such that $s >> P_a$.

The problem definition begins by defining the desired set of precision points, desired accuracy, precision points located on the dwell portion of coupler curve, and the minimum allowable value of transmission angle. We begin the problem by specifying the set of precision points, indicate which precision points define sections of the coupler curve used in dwell, the required accuracy, and the minimum transmission angle. The defined values are used in the analysis routines to define the initial bounds and values for the design variables. The design variables are checked for constraint violations, and then the cost function is calculated. The optimization routine uses this information to generate a new set of values for the design variables. One function evaluation is completed when one set of design variables is analyzed. The calculated error provides a cost evaluation for optimization. The cost is evaluated as given in equation (11). This process is repeated until certain criteria are met, such as meeting the desired accuracy or a predefined number of generations. Figure 2 shows a flowchart of the optimization process.

The approach developed in this study requires the user to specify all the desired precision points on the coupler curve, the precision points located on the circular arc regions, and the corresponding crank angles to guarantee that the precision points are visited in the correct order. The crank angles for the precision points are calculated according to,

$$x_{ci} = x_7 + d_i \qquad i = 1,2,3 \ldots n_p \tag{12}$$

The optimum topology of a four-bar linkage is synthesized with ten real valued design variables. The design variables x_1, x_2, x_3, x_4, and x_5 represent the magnitudes of their relative vectors, x_7 and x_8 are the crank and ground link angles, and x_6 is the angle between the vectors $\mathbf{x}_2$ and $\mathbf{x}_5$. The coordinates of the first ground point are relative to the origin where those for the second ground point are calculated relative to O_1

$$O_2(x, y) = \{(x_4 \cos(x_8) + O_{1x}, x_4 \sin(x_8) + O_{1y}\} \tag{13}$$

The coordinates of the third ground pivot point are calculated relative to O_1

$$O_3(x, y) = \{(x_{13} \cos(x_{14}) + O_{1x}, x_{13} \sin(x_{14}) + O_{1y}\} \tag{14}$$

The coordinates of a generated precision point for a four-bar mechanism are calculated using

$$x_g = x_9 + x_1 \cos(x_7) + x_5 \cos(x_6 + q)$$
$$y_g = y_9 + x_1 \sin(x_7) + x_5 \sin(x_6 + q) \quad (15)$$

The mathematical relationships for mechanism synthesis used in this work are the standard mechanism analysis and synthesis algorithms [1, 29, 30].

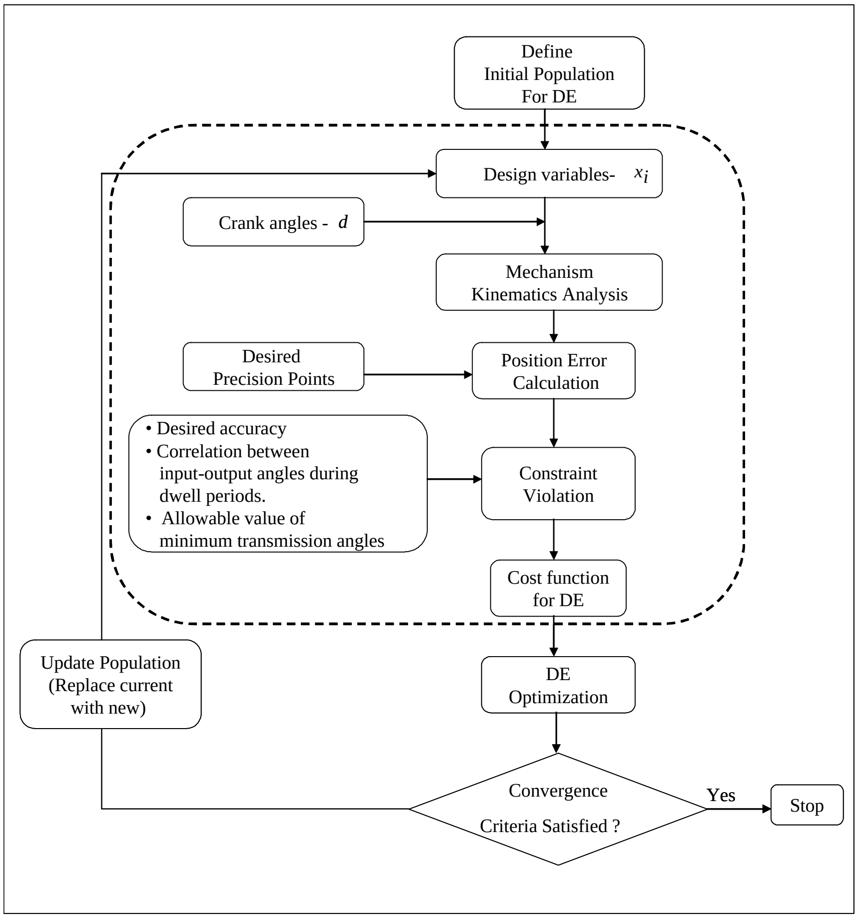

Figure 2: Analysis flowchart

3 DE and Geometric Centroid of Precision Positions

This section presents a short introduction to the general operation of DE. The interested reader is referred to the literature for a detailed description of DE [26, 31, 32, 33]. DE uses the floating-point representations of design variables, which accelerates the manipulation of data. This property also makes DE independent of defined values of precision, and improves the resolution [31]. The DE approach contains the same processes of population initialization, mutation, crossover, and selection like Genetic Algorithms (GA), and emphasizes direct use of the objective function. In DE, unlike traditional GA, the genetic operation of mutation uses the vector differentiation method (adding the weighted difference between two population vectors to a third vector) to generate a new vector. In our work, we used the control parameters suggested by Storn [33], a crossover probability of 0.9 and weighting factor of 0.4. The population size is generally set at ten times the number of design variables, but increasing the population size could be beneficial in some cases [33]. The number of generations for each case is selected based on desired convergence characteristics.

Evolutionary-based approaches require the initial definition of bounds for each design variable to commence the optimization, in the same manner that classical optimization techniques require initial guesses for each design variable. However, there is no guarantee that a solution would exist in the initial range of design variables or that a solution will be reached starting from the initial guess. Thus, the quality of the final result greatly depends upon the definition of the initial guess or design variable bounds [31]. One of the most promising features of DE is its capability to extend the search space beyond the specified initial bounds. However, the user must still provide reasonable initial bounds based on the physical characteristics of the problem analyzed.

The GCPP approach as described in [5] is used to automatically define the initial bounds of the design variables for Level 1 based on the geometric definition of the desired precision points. The results obtained in Level 1 are used to decide design variables bounds of Level 2 and those obtained in Level 2 are used for Level 3. The upper and lower bounds of the design variables for Level 2 and Level 3 are obtained according to,

$$D_{i,j+1} = D_{ij}\,(1 \pm X\%) \tag{16}$$

In addition, to the definition of the "new" bounds, the population and generation numbers are also increased. In this work, $X = 20$ was used. The desired accuracy at each subsequent level is defined such that $a_3 < a_2 < a_1$, where a_2 is the accuracy at Level 2, and a_3 is the accuracy at Level 3.

4 Six-bar Mechanism Synthesis Results

The presented methodology is applied to the synthesis of a six-bar mechanism capable of generating dwells with prescribe timing relative to the motion of the input link (crank). A coupler curve consisting of 18-precision points with two circular arcs is used for the synthesis of the six-bar mechanism. The precision points on the circular arcs (the ones producing the dwells) are to be traced in correlation with the input crank angles. The input data of the coupler curve is shown in Table 1. Note that precision points 1, 2, 9, 10, 11, and 18 are located on the dwell-producing circular arcs.

The precision points on the circular arcs that produce the dwells are to be traced in correlation to the input crank angles. Dwells of 40° and 30° at the output link are required when the crank rotates from 160° to 200° and from -15° to 15°. The pictorial representation of the dwell relationship is shown in Figure 3. When the coupler point of the mechanism passes through the dwell portion of the coupler curve, the corresponding position of the output-link should be within the prescribed accuracy constraints. The change of angle at the output link is 15^{o}. As discussed in [30], the symmetry of this problem enables simplification of the optimization process. However, for general mechanism design, this is not always the case. Because the authors wish to use this same technique on a variety of mechanisms from 4- to n-bar, this characteristic of the symmetry was ignored.

The test cases considered for the synthesis of six-bar mechanisms use DE as the optimization tool and GCPP for initial definition of the limits on the design variables. The successive optimization will be introduced and discussed, and the results for each case will be presented and compared.

Table 1: Input Data for Tests A & B

Precision Position	x- coordinate	y-coordinate	Crank Angle
1	-0.5424	2.3708	0
2	0.2202	2.9871	15
3	0.9761	3.4633	40
4	1.0618	3.6380	60
5	0.8835	3.7226	80
6	0.5629	3.7156	100
7	0.1744	3.6128	120
8	-0.2338	3.4206	140
9	-0.6315	3.1536	160
10	-1.0000	2.8284	180
11	-1.3251	2.4600	200
12	-1.5922	2.0622	220
13	-1.7844	1.6539	240
14	-1.8872	1.2654	260
15	-1.8942	0.9448	280
16	-1.8096	0.7665	300
17	-1.6349	0.8522	320
18	-1.1587	1.6081	345

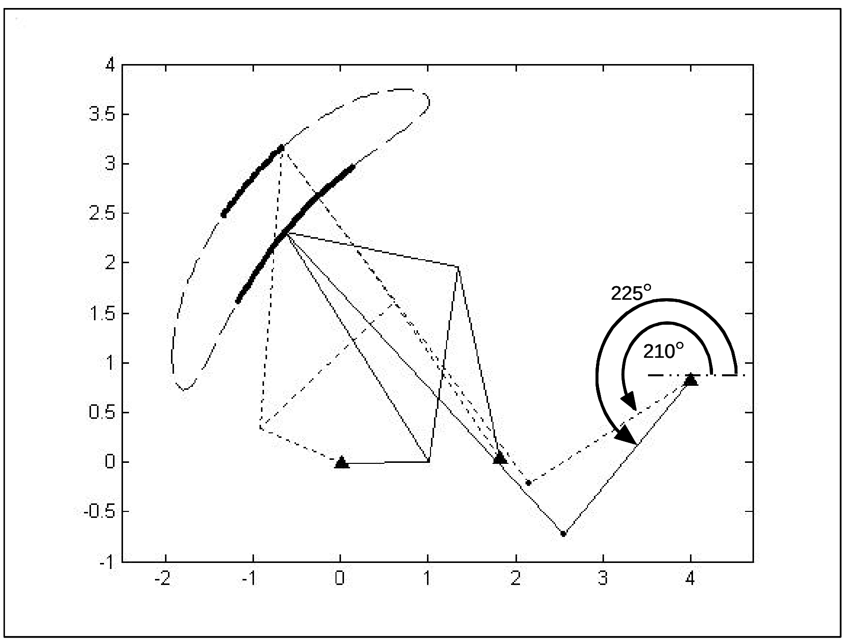

Figure 3: Coupler curve and Dwells Pictorial Representation

Test A. The optimization process is performed in two stages: stage one considers the synthesis of a four-bar mechanism, and the second stage extends the four-bar to a six-bar. The set of the 18-precision points along with the respective crank angles are shown in Table 1. All the precision points were considered during the optimization process. The results of the optimization for each stage are given in Table 2. The synthesized mechanism and coupler curves generated during the optimization process are shown in Figure 4.

Table 2: Stage 1 Results for Four-bar Synthesis

	Pop No	Iterations	Evals	Total Error	Acc
Level 1	150	113	17100	0.0540	0.100
Level 2	170	30	5270	0.0204	0.050
Level 3	200	98	19800	7.69e-004	0.010
Level 4	250	36	9250	1.94e-004	0.005

Test B. The optimization process was performed in one stage considering the direct synthesis of a six-bar mechanism without first synthesizing a four-bar mechanism. The set of the 18-precision points along with the respective crank angles are shown in Table 1. All the precision points were considered during the optimization process. The results of the optimization are given in Table 3. The synthesized mechanism and coupler curves generated during the optimization process are shown in Figure 5.

5 Conclusions

In this manuscript, we presented the synthesis of six-bar mechanisms for specified dwell. In this work, a recently developed evolutionary algorithm called Differential Evolution (DE) was used along with a novel technique for defining the initial bounds of the design variables called Geometric Centroid of Precision Points (GCPP). Combination of these two components enables using the unique features of DE to enable the automatic search beyond the initially defined design variable bounds in an effort to reach a global optimum. Successive optimizations enabled accuracy improvement. Two penalty functions were employed: one that emphasizes the deviation from the precision points and penalizes based on the magnitude of the deviation, and the second one that penalizes any constraint violation irrespective of the magnitude of violation. The developed methodology was successfully applied to the synthesis of six-bar mechanisms considering two different approaches for the synthesis. The results presented verify the validity of the developed methodology.

6 Acknowledgements

The authors would like to thank Dr. Storn for making the DE code used in this study freely available and for his input on DE related questions, and Mr. Peter Trogos of MathPros Inc. for MATLAB related support.

Table 3: Direct Synthesis of Six-bar Mechanism

	Pop No	Iterations	Funct Evals	Total Error	Accuracy
Level 1	150	99	15000	0.0526	0.100
Level 2	170	53	9180	0.0203	0.050
Level 3	200	225	45225	0.0011	0.010
Level 4	250	95	24000	2.71e-004	0.005

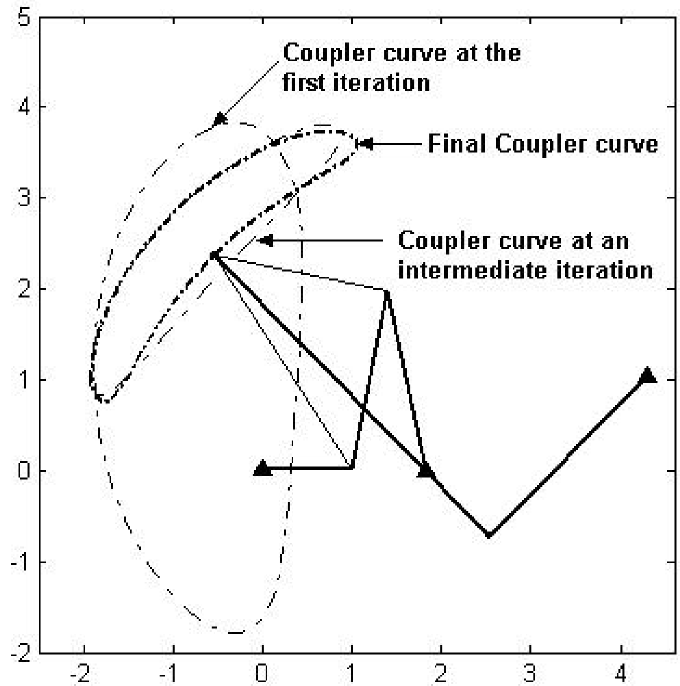

Figure 4: Six-Bar Mechanism Iteration History Using Two Stage Synthesis

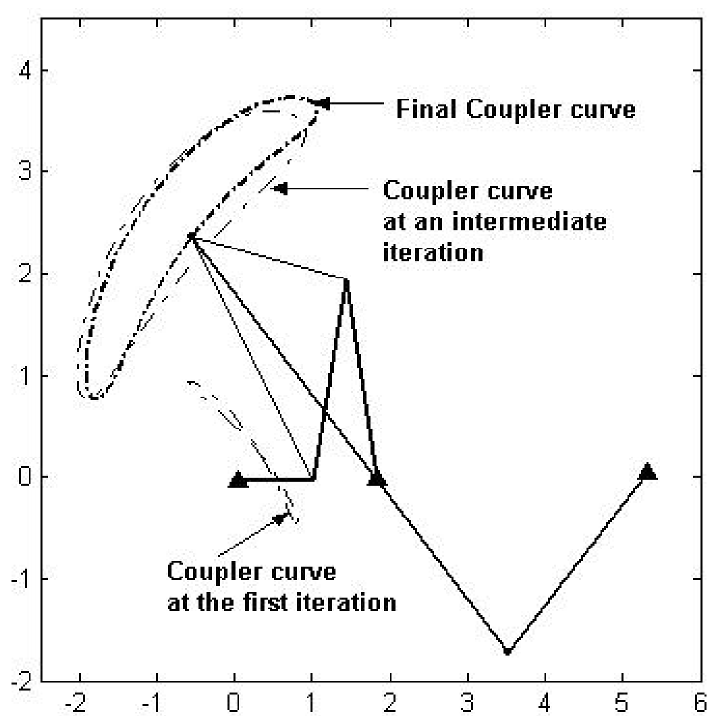

Figure 5: Six-Bar Mechanism Iteration History Using Direct Synthesis

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