TYPE SYNTHESIS OF INPUT-OUTPUT DECOUPLED PARALLEL MANIPULATORS

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Abstract

An I-O (input-output) decoupled parallel manipulator is a parallel manipulator in which each DOF of its output is controlled by one actuated joint independently. Unlike general parallel manipulators for which the set of I-O equations is highly coupled, the forward displacement analysis (also forward kinematics or direct kinematics) of I-O decoupled parallel manipulators is easy to solve. I-O decoupled parallel manipulators are suitable for fast parallel manipulator design from the kinematic point of view. In this paper, three classes of I-O decoupled parallel manipulators with 2 to 4 DOF are proposed using a geometric approach. These parallel manipulators are used to generate 2T (2-DOF planar translation), 2T1R (2-DOF planar translation in conjunction with 1-DOF rotation about axes parallel to a given constant direction) or 3T1R (3-DOF spatial translation in conjunction with 1-DOF rotation about axes parallel to a given constant direction) motions.

1 Introduction

Generally speaking, the set of I-O (input-output) equations of a PM (parallel manipulator, Fig. 1) is highly coupled and usually difficult to solve. The singularity-free trajectory planning of general PMs is also very complicated [1]. To meet the need to develop high-speed robots, PMs for which the set of I-O equations is easy to solve have been proposed [2, 3, 4, 5, 6, 7, 8, 9]. Among these

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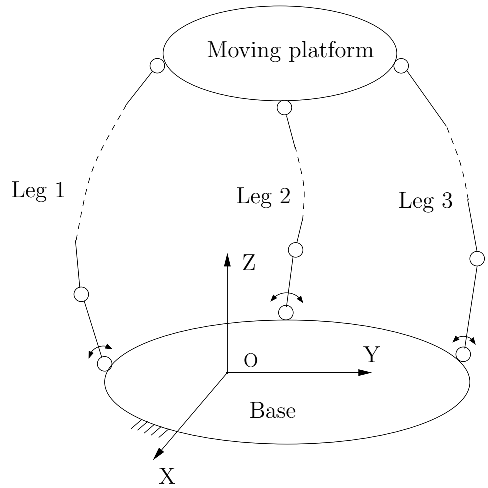


Figure 1: A parallel manipulator.

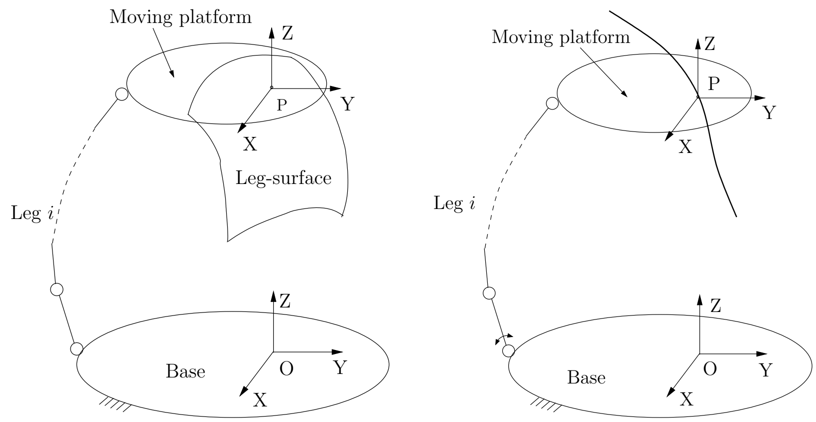
PMs, I-O decoupled PMs are promising [3, 8]. A PM is said to be I-O decoupled if each DOF of its output is controlled by one actuated joint independently. As pointed out in [10], the non-linearity of PMs is one of the reasons which prevents end-users such as those working in the field of machine tools from better understanding and adopting PMs. The research on I-O decoupled PMs may also help to remove such a burden. In [3], many types of I-O decoupled 3T-PMs[1] were proposed. In [8], an I-O decoupled 1T2R-PM (PM generating one translation and two DOFs of rotation) was proposed.

This paper proposes three new classes of I-O decoupled PMs. In Section 2, I-O decoupled PMs generating 2T (2-DOF planar translation) are generated using a geometric approach. In Section 3, I-O decoupled PMs generating 2T1R (2-DOF planar translation in conjunction with 1-DOF rotation about axes parallel to a given constant direction) or 3T1R (3-DOF spatial translation in conjunction with 1-DOF rotation about axes parallel to a given constant direction) motions are proposed. Finally, conclusions are drawn.

2 Generation of I-O decoupled 2T-PMs

A 2T-PM is a PM generating 2-DOF planar translation. An I-O decoupled 2T-PM is a 2T-PM in which the translations of the moving platform along each of two orthogonal directions is controlled

[1] In the literature, a 3T-PM is often denoted by TPM.



(a) A surface. (b) A curve.

Figure 2: Leg-surface for 2T-PMs.

by one actuated joint independently. Here and throughout this paper, the set of two orthogonal directions is denoted by the X-direction and Y-direction.

In this section, I-O decoupled 2T-PMs are generated using the geometric approach to the generation of I-O decoupled 3T-PMs proposed in [3].

2.1 Geometric interpretation of the forward displacement analysis of 2T-PMs

When the actuated joint, if any, of a given leg of a 2T-PM is locked and the orientation of the moving platform is constant, the moving platform will translate along a surface (Fig. 2(a)) in the case of a leg without actuated joint or a curve (Fig. 2(b)) in the case of a leg with one actuated joint. For brevity, the above surface or curve is called the leg-surface (Fig. 2). Thus, the forward displacement analysis of the 2T-PM can be described geometrically as follows: it consists in finding the intersection of two (for 2-legged PMs) or three (for 3-legged PMs) leg-surfaces.

2.2 Geometric characteristics of I-O decoupled 2T-PMs

2.2.1 Types of leg-surface of I-O decoupled 2T-PMs

It is apparent that for an I-O decoupled 2T-PM, each of its leg-surfaces must allow a translation along a straight line parallel to a given constant direction. This requires that each leg-surface of the I-O decoupled 2T-PM must be a straight line, a plane or a cylinder.

2.2.2 Combination of leg-surfaces of I-O decoupled 2T-PMs

For an I-O decoupled 2T-PM, some leg-surfaces allow a translation along the Y-direction, while the other leg-surfaces allow a translation along at least the X-direction. In addition, the translation along either X-direction or Y-direction is restrained by one leg-surface.

The combinations of leg-surfaces fall into the following five cases (see Fig. 3).

(a) Two leg-surfaces are both straight lines and perpendicular to each other (PA and PB in Fig. 3(a)).

(b) One leg-surface is a straight line while the other leg-surface is a plane. The line is perpendicular to the plane (see PB and PACD in Fig. 3(b)).

(c) One leg-surface is a straight line while the other leg-surface is a cylinder. The line is perpendicular to the axis of the cylinder (Fig. 3(c)).

(d) Three leg-surfaces are three planes. The three planes are orthogonal and one of them is constant (see PACD, PBED and PAFB in Fig. 3(d)).

(e) Two leg-surfaces are planes and the third one is a cylinder. The two planes are perpendicular to each other and one of them is constant. The axis of the cylinder is parallel to the constant plane PAFB and perpendicular to the other plane PBED (Fig. 3(e)).

2.3 A procedure to the type synthesis of I-O decoupled 2T-PMs

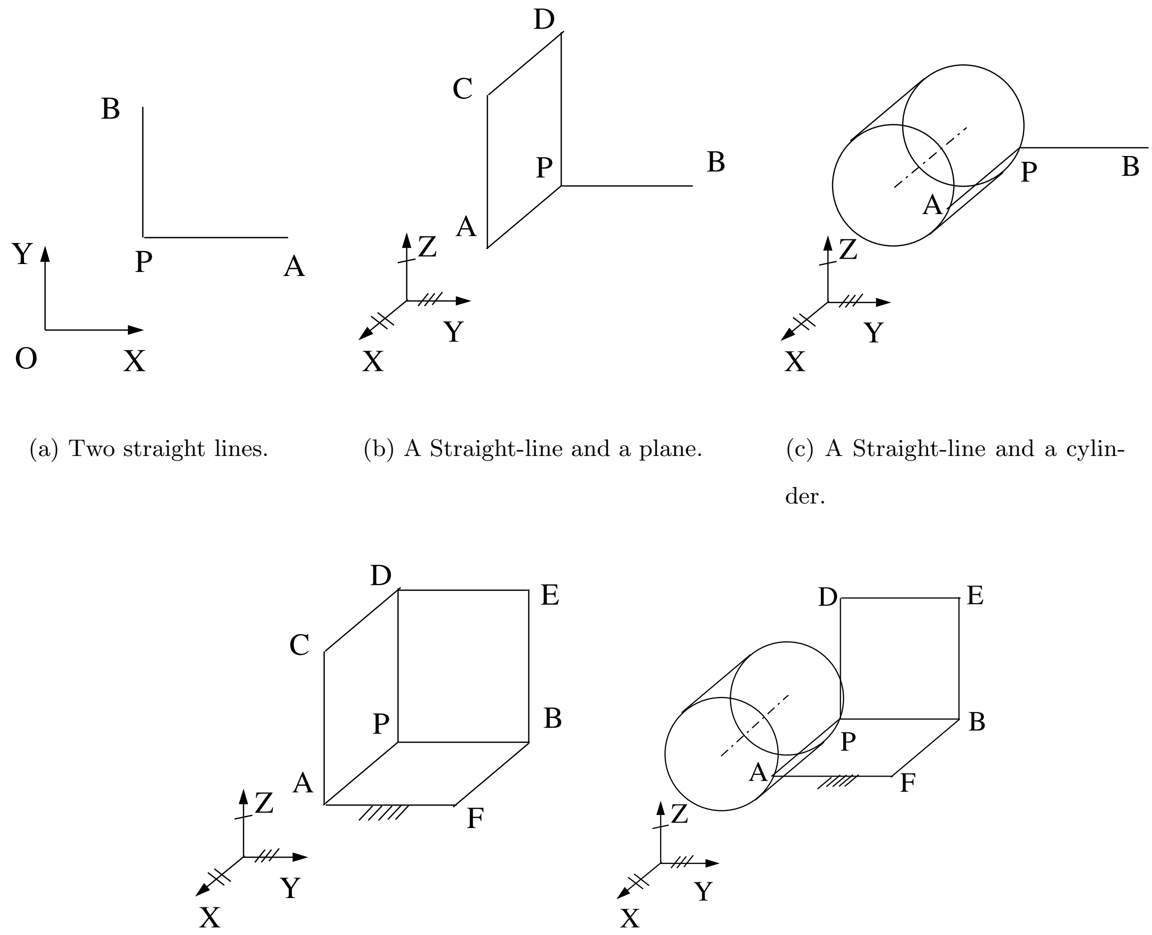
Based on the geometric characteristics of I-O decoupled 2T-PMs, types of I-O decoupled 2T-PMs can be obtained in two steps. The first step is the type synthesis of legs for I-O decoupled 2T-PMs (Section 2.3.1). The second step is the generation of types of I-O decoupled 2T-PMs by assembling a set of legs together (Section 2.3.2).

2.3.1 Type synthesis of legs for I-O decoupled 2T-PMs

Type synthesis of legs for I-O decoupled 2T-PMs can be performed in three sub-steps.

Step 1a To determine the leg-surface-chain.

A leg-surface-chain is a chain composed of unactuated joints needed to generate the specified leg-surface. For a leg with a straight line leg-surface, the leg-surface-chain should be a chain composed of one P (prismatic) joint. For a leg with a planar leg-surface, the leg-surface-chain



(a) Two straight lines. (b) A Straight-line and a plane. (c) A Straight-line and a cylinder.

(d) Three planes. (e) Two planes and a cylinder.

Figure 3: Combinations of leg-surfaces of I-O decoupled 2T-PMs.

should be a PP, $\bar{R}\bar{R}\bar{R}$, $P^{\perp}\bar{R}\bar{R}$, $\bar{R}P^{\perp}\bar{R}$ and $\bar{R}\bar{R}P_{\perp}$ chain. Here and throughout this paper, R, P and C are used to denote a revolute joint, a prismatic joint and a cylindrical joint respectively. The axes (or directions) of joints with a same leg denoted by letters with the same overbars, $\bar{R}$, $\bar{P}$ and $\acute{R}$, are parallel while the axes (or directions) of joints denoted by letters with different overbars are not parallel. $P^{\perp}$ ($P_{\perp}$) denotes a P joint the direction of which is perpendicular to the axis of its adjacent joint in the same leg which is close to the moving platform (base). $\underline{R}$ and $\underline{P}$ are used to denote the actuated R joint and actuated R joint respectively. For a leg with a cylindrical leg-surface, the leg-surface-chain should be an $\bar{R}\bar{R}P$, $\bar{R}\bar{P}\bar{R}$ or $\bar{R}\bar{R}P$ chain.

Step 1b To determine the actuated joint

The actuated joint may be a $\underline{\text{P}}$ or $\underline{\text{R}}$ joint.

Step 1c To determine the other unactuated joints.

In any leg for a 2T-PM involving an $\underline{\text{R}}$ joint, there must be an unactuated R joint with its axis parallel to the axis of the $\underline{\text{R}}$ joint if the leg is composed of 3 to 5 R and/or P joints. Otherwise, the orientation of the moving platform of the PM will change [3, 11]. In addition, inactive joints may also be used. An inactive joint is a joint whose joint variable is constant during the motion of a mechanism. The introduction of inactive joints in an overconstrained mechanism (see for example Figs. 4(a), 4(b), 4(c), 7(a), 8(a) and 9(a)) will reduce the number of overconstraints (also redundant constraints) of a mechanism (see for example Figs. 4(a), 4(b), 4(d), 7(b), 8(b) and 9(b)). For example, using the general mobility criteria (called Chebychev-Grübler-Kutzbach criteria), one can obtain that the DOF of the 2T-PM shown in Fig. 4(c) is -1 which is in fact not the case. The 2T-PM shown in Fig. 4(c) doesn't satisfy the general mobility criteria and is thus an overconstrained mechanism. When introducing an inactive joint to each leg of the 2T-PM shown in Fig. 4(c), the 2T-PM shown in Fig. 4(d) is obtained. In this mechanism, the three R joints on the moving platform are all inactive. One can verify that the 2T-PM shown in Fig. 4(d) satisfies the general mobility criteria and is not overconstrained. Since inactive joints make no contribution to the relative motion of a mechanism, some people may neglect the difference between a mechanism involving inactive joints (Fig. 4(d)) and its corresponding mechanism without inactive joints (Fig. 4(c)).

Using the above procedure, all the types of legs for I-O decoupled 2T-PMs can be obtained (Table 1). Due to the large amount of legs for nT-PMs involving inactive joints, only the legs for 2T-PMs without inactive joints are listed in Table 1.

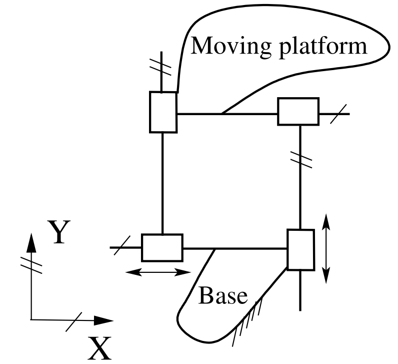
2.3.2 Generation of types of I-O decoupled 2T-PMs

I-O decoupled 2T-PMs can be obtained by assembling two or three legs obtained in Section 2.3.1 in one of the ways shown in Fig. 3. In addition, all the rotational DOF of the moving platform of 2T-PMs should be restrained by all of its legs. Figure 4 shows some of the I-O decoupled 2T-PMs we obtained.

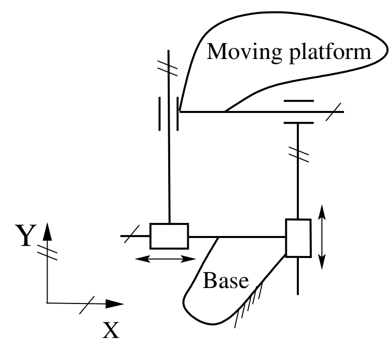
It is noted that (1) some I-O decoupled 2T-PMs, such as those involving No. 18–23 legs in Table 1, cannot be obtained from the I-O decoupled 3T-PMs [3] by blocking one of its actuated joints and (2) 2T-PMs which are not I-O decoupled can also be obtained using the above approach. In the

Table 1: Types of legs for I-O decoupled nT1R-PMs.

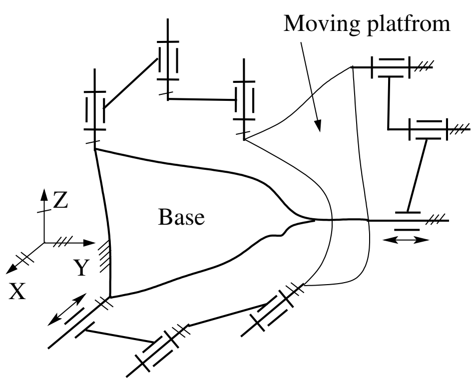
n	No	Type	Leg-surface
2	1	$\underline{\mathrm{P}}\mathrm{P}$	Straight line
	2	$\underline{\bar{\mathrm{R}}}\bar{\mathrm{R}}\mathrm{P}^{\perp}$	
	3	PP	Plane
	4	$\bar{\mathrm{R}}\bar{\mathrm{R}}\bar{\mathrm{R}}$	
	5	$\mathrm{P}^{\perp}\bar{\mathrm{R}}\bar{\mathrm{R}}$	
	6	$\bar{\mathrm{R}}\mathrm{P}^{\perp}\bar{\mathrm{R}}$	
	7	$\bar{\mathrm{R}}\bar{\mathrm{R}}\mathrm{P}_{\perp}$	
2–3	8	$\underline{\mathrm{P}}\mathrm{PP}$	
	9	$\underline{\mathrm{P}}\bar{\mathrm{R}}\bar{\mathrm{R}}\bar{\mathrm{R}}$	
	10	$\underline{\mathrm{P}}\mathrm{P}^{\perp}\bar{\mathrm{R}}\bar{\mathrm{R}}$	
	11	$\underline{\mathrm{P}}\bar{\mathrm{R}}\mathrm{P}^{\perp}\bar{\mathrm{R}}$	
	12	$\underline{\mathrm{P}}\bar{\mathrm{R}}\bar{\mathrm{R}}\mathrm{P}_{\perp}$	
	13	$\underline{\acute{\mathrm{R}}}\acute{\mathrm{R}}\mathrm{PP}$	
	14	$\underline{\acute{\mathrm{R}}}\acute{\mathrm{R}}\bar{\mathrm{R}}\bar{\mathrm{R}}\bar{\mathrm{R}}$	
	15	$\underline{\acute{\mathrm{R}}}\acute{\mathrm{R}}\mathrm{P}^{\perp}\bar{\mathrm{R}}\bar{\mathrm{R}}$	
	16	$\underline{\acute{\mathrm{R}}}\acute{\mathrm{R}}\bar{\mathrm{R}}\mathrm{P}^{\perp}\bar{\mathrm{R}}$	
	17	$\underline{\acute{\mathrm{R}}}\acute{\mathrm{R}}\bar{\mathrm{R}}\bar{\mathrm{R}}\mathrm{P}_{\perp}$	
	18	$\underline{\mathrm{P}}\mathrm{P}\bar{\mathrm{R}}\bar{\mathrm{R}}$	Cylinder
	19	$\underline{\mathrm{P}}\bar{\mathrm{R}}\bar{\mathrm{P}}\bar{\mathrm{R}}$	
	20	$\underline{\mathrm{P}}\bar{\mathrm{R}}\bar{\mathrm{R}}\mathrm{P}$	
	21	$\underline{\acute{\mathrm{R}}}\acute{\mathrm{R}}\mathrm{P}\bar{\mathrm{R}}\bar{\mathrm{R}}$	
	22	$\underline{\acute{\mathrm{R}}}\acute{\mathrm{R}}\bar{\mathrm{R}}\bar{\mathrm{P}}\bar{\mathrm{R}}$	
	23	$\underline{\acute{\mathrm{R}}}\acute{\mathrm{R}}\bar{\mathrm{R}}\bar{\mathrm{R}}\mathrm{P}$	



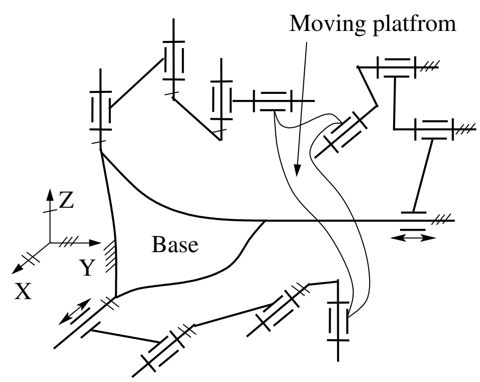
(a) 2-$\underline{P}$P 2T-PM.



(b) 2-$\underline{P}$C 2T-PM.



(c) $\bar{R}\bar{R}\bar{R}$-$\underline{P}\bar{R}\bar{R}\bar{R}$-$\underline{P}\bar{R}\bar{R}\bar{R}$ 2T-PM.



(d) $\bar{R}\bar{R}\bar{R}\acute{R}$-$\underline{P}\bar{R}\bar{R}\bar{R}\acute{R}$-$\underline{P}\bar{R}\bar{R}\bar{R}\acute{R}$ 2T-PM.

Figure 4: I-O decoupled 2T-PMs.

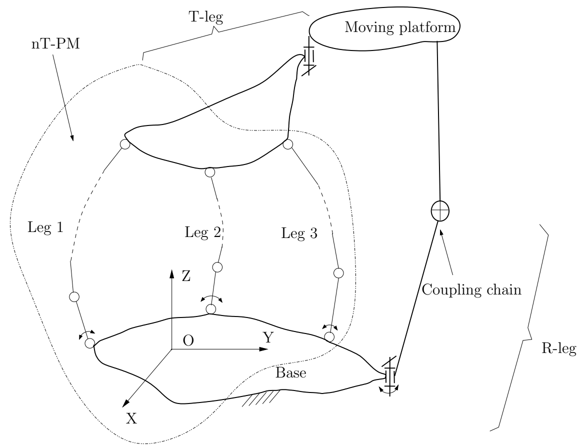


Figure 5: Construction of an nT1R-PM.

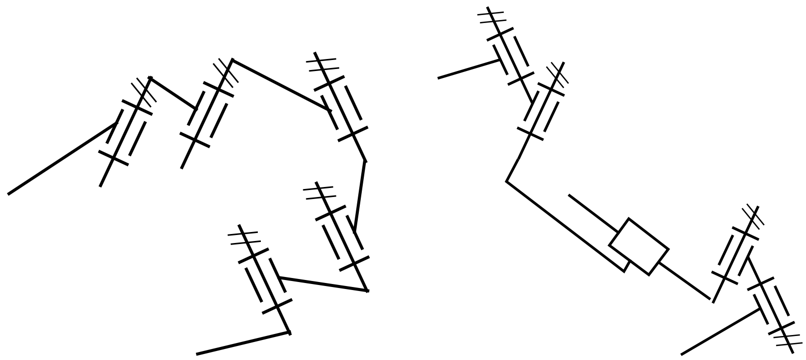
latter case, the combinations of leg-surfaces include the following cases: (a) two straight lines; (b) one straight line and one surface; and (c) one constant plane and two surfaces. The combinations of leg-surfaces should have isolated common points in order to guarantee the actuated joints work properly.

3 Generation of I-O decoupled nT1R-PMs

An nT1R-PM is a PM generating nT1R motion. The output of an nT1R-PM is represented by n (n=2, 3) translations of a point on the moving platform along n orthogonal directions as well as the rotation of the moving platform about axes in a constant direction. An I-O decoupled nT1R-PM is an nT1R-PM in which each DOF of its output is controlled independently by one actuated joint.

An I-O decoupled nT1R-PM is composed of two legs (Fig. 5). One leg, called T-leg, is composed of an I-O decoupled nT-PM and an R joint. The other leg, called R-leg, is a composed of an $\underline{\text{R}}$ joint and a coupling chain. The axis of the $\underline{\text{R}}$ joint in the R-leg is parallel to the axis of the R joint in the T-leg. The coupling chain guarantees that the rotation of the moving platform is controlled by the actuated joint in the R-leg no matter what the inputs in the T-leg are. For clarity, a coupling chain is denoted by $\oplus$ throughout this paper except for Fig. 6.

The key issue in the type synthesis of I-O decoupled nT1R-PMs is the type synthesis of coupling chains. One class of coupling chains is the coupling chains which guarantee that the moving platform always rotates the same angle as the $\underline{\text{R}}$ joint in the R-leg. A coupling chain should satisfy two



(a) $\bar{\mathrm{R}}\bar{\mathrm{R}}\bar{\mathrm{R}}\acute{\mathrm{R}}\acute{\mathrm{R}}$ chain. (b) $\acute{\mathrm{R}}\bar{\mathrm{R}}\mathrm{P}\bar{\mathrm{R}}\acute{\mathrm{R}}$ chain.

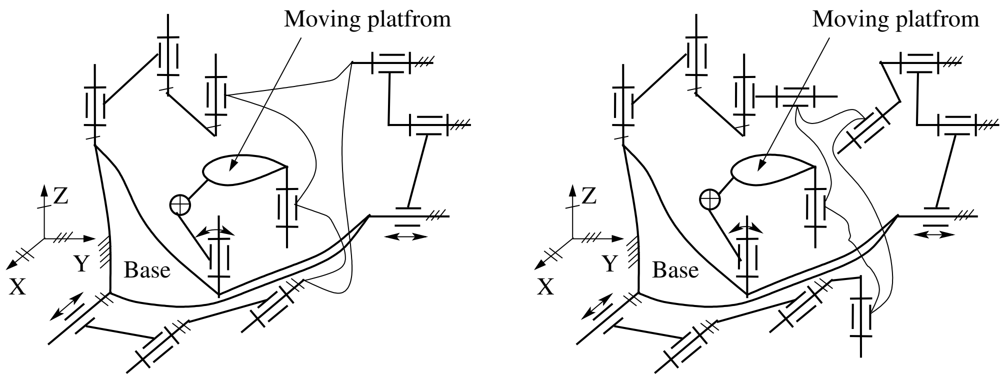
Figure 6: Some coupling chains.

conditions: (1) It should restrain the relative rotation of the moving platform with respect to the link adjacent to the $\underline{\mathrm{R}}$ joint in the R-leg, and (2) It should not restrain the translation of the moving platform. The coupling chain (Fig. 6) is actually the legs for parallel kinematic chains generating 3T motion which are composed of 3 to 5 P and/or R joints [11, 12, 13]. The coupling chain shown in Fig. 6(b) was used in a 3T1R-PM in [14].

In constructing an I-O decoupled nT1R-PM, the coupling chain should be assembled in a way such that the axis of the $\underline{\mathrm{R}}$ joint in the R-leg is not perpendicular to the line or plane which is perpendicular to the axes of all the R joints in the coupling chain.

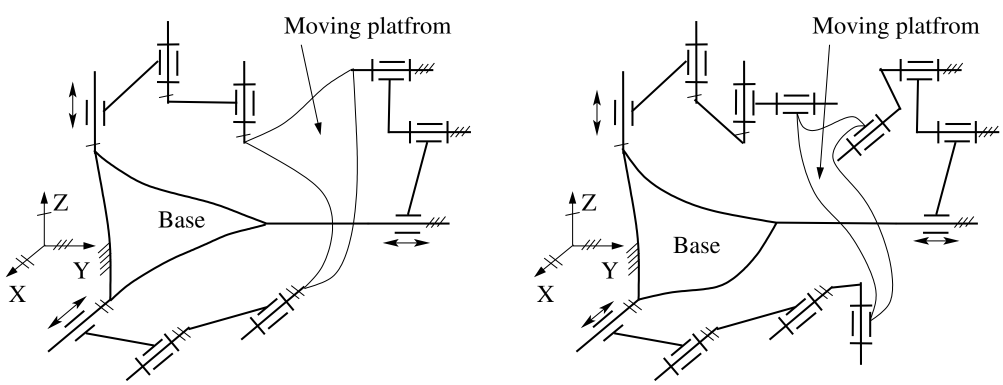
Figure 7 shows some I-O decoupled 2T1R-PMs obtained from the I-O decoupled 2T1R-PMs shown in Fig. 4. Figure 9 shows some I-O decoupled 3T1R-PMs obtained from the I-O decoupled 3T-PMs shown in Fig. 8. Figure 10 shows a CAD model of the 3T1R-PM in Fig. 9(a).

It is noted that when the I-O decoupled nT-PM in an nT1R-PM is replaced by an nT-PM which is not I-O decoupled, one can obtain an nT1R-PM which is partially I-O decoupled. In the nT1R-PM obtained, the rotation of the moving platform is controlled by the actuated joint in the R-leg, while the translation of the moving platform, which is denoted by the position of a point on the axis of the R joint in the T-leg, is controlled by the actuated joints in the T-leg.



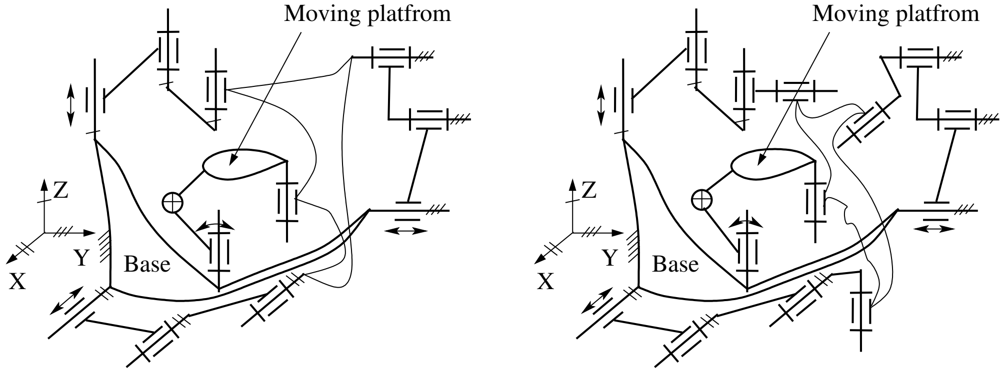
(a) $(\bar{R}\bar{R}\bar{R}\text{-}2\underline{P}\bar{R}\bar{R}\bar{R})R\text{-}\underline{R}\oplus$ 2T1R-PM. (b) $(\bar{R}\bar{R}\bar{R}\acute{R}\text{-}2\underline{P}\bar{R}\bar{R}\bar{R}\acute{R})R\text{-}\underline{R}\oplus$ 2T1R-PM.

Figure 7: I-O decoupled 2T1R-PMs.



(a) 3-$\underline{P}\bar{R}\bar{R}\bar{R}$ 3T-PM. (b) 3-$\underline{P}\bar{R}\bar{R}\bar{R}\acute{R}$ 3T-PM.

Figure 8: I-O decoupled 3T-PMs.



(a) (3-$\underline{P}\bar{R}\bar{R}\bar{R}$)R-$\underline{R}\oplus$ 3T1R-PM.   (b) (3-$\underline{P}\bar{R}\bar{R}\bar{R}$)R-$\underline{R}\oplus$ 3T1R-PM.

Figure 9: I-O decoupled 3T1R-PMs.

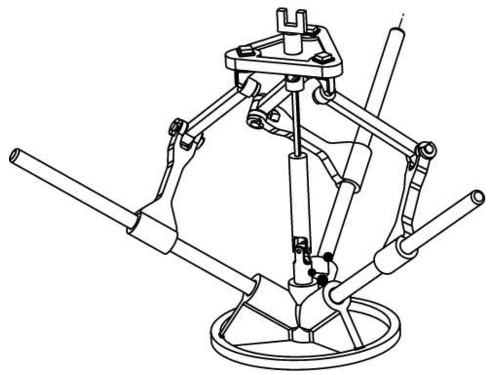


Figure 10: CAD model of a (3-$\underline{P}\bar{R}\bar{R}\bar{R}$)R-$\underline{R}\oplus$ 3T1R-PM.

4 Conclusions

I-O decoupled PMs have been obtained for generating 2T (2-DOF planar translation), 2T1R (planar translation in conjunction with 1-DOF rotation about axes along a constant direction) or 3T1R (3-DOF spatial translation in conjunction with 1-DOF rotation about axes along a constant direction) motions. For an I-O decoupled PM, each DOF of its output is controlled by one actuated joint independently.

The optimal selection of types of I-O decoupled PMs, which is under our current investigation, is still an open issue. The results of this paper contributes to the design of fast PMs and will promote the better understanding of parallel mechanisms.

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