The Modeling of Coordinate Measuring Machines Kinematic Errors from the Geometric Errors of Guideways

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Abstract

The purpose of this paper is to propose a means to calculate the kinematic errors (commonly called motion errors) of each machine joint from the knowledge of the guideway geometric errors. Until now, the machine modeling has generally been based on the use of kinematic errors of machines for each machine joints. The proposed model establishes a closer causal relationship between the physical imperfections and their impact on the machine accuracy.

By establishing the functions between the guideways geometric errors and the kinematic errors of the machine, a better understanding of the machine behavior and its root causes will be gained. The passage from kinematic to geometric errors also represents the transition from analysis of effects to analysis of causes. This conversion is similar to the shift from a volumetric error approach to a kinematic error approach which occurred a few decades ago.

The geometric errors of the guideways are modeled as straightness errors and are represented as spline functions. A system of linear equations converts from geometric errors of the individual guideways to the kinematic error of the joint. The solution is reached through an iterative procedure. The approach is partially validated via numerical simulations.

Sommaire

L'objectif de cet article est de proposer une approche de modélisation des articulations prismatiques des machines à mesurer tridimensionnelles à partir des écarts géométriques des glissières. Ceci contraste avec l'approche usuelle dans ce domaine qui consiste à modéliser les écarts cinématiques articulaires (communément appelés écarts de mouvement). La méthode proposée est causale et permet une modélisation plus en amont du système mécanique permettant ainsi une avancée importante vers la machine virtuelle.

Ce lien causal permet une plus grande compréhension du comportement de la machine et de ses causes. Ce passage de la modélisation d'écarts cinématiques à celle des écarts géométriques représente une transition de l'analyse des effets à l'analyse des causes. Un passage similaire ayant eu lieu il y a plus de deux décennies alors de la modélisation des écarts volumétriques à la modélisation des écarts cinématiques.

Les écarts géométriques des glissières sont représentés par des écarts de rectitude selon des fonctions splines. Un système d'équations linéaires utilisant des transformations homogènes et l'hypothèse des petits angles permet le calcul des écarts cinématiques à partir des écarts géométriques des glissières. Une procédure itérative utilisant les transformations homogènes non linéarisées pour vérifier la solution linéarisée permet l'obtention d'une solution numérique pratiquement exacte.

Introduction

Major causes of kinematic errors on a coordinate measuring machine (CMM) are the form errors of the guideways due to manufacturing limitations [1]. In our analysis, the form errors will be represented as straightness errors. Yet there is no established relationship between the straightness errors of the guideway surfaces and the volumetric accuracy of a CMM in the literature. It is anticipated that a key advantage of this approach is to allow physics based modeling of the error sources such as thermal and elastic effects since it then becomes possible to directly use information on the deformation of these structural elements. This approach is in sharp contrast with many thermal modeling methods which use regression techniques to correlate measured temperatures with volumetric errors (positional errors at the probe attachment point).

In a CMM the joints are constructed using a number air bearing pads gliding on nominally straight surfaces. In a bridge type CMM structure as is analyzed in this study, the X-axis structure (the bridge) of the CMM is supported by seven different air-bearings. The bearing surface is finite and so a certain amount of low pass surface defect filtering takes place. This aspect is not considered here.

The relationship between the motion of the air bearing pad and the straightness of the guideway surface has been analyzed previously [2]. The results indicate that the air-bearing motion error is closely related to the straightness error of the guideway as opposed to its waviness and surface finish.

There are commercial applications of CMM error compensation which takes into account the guidance errors. However these guidance errors are defined as the deviation of the carriage from a reference line, and the surface deviation as a factor per se is not taken into consideration [3].

Modelling Methodology

A three-dimensional schematic of the bridge-type CMM of FXYZ structure to be analyzed is presented in Figure 1.

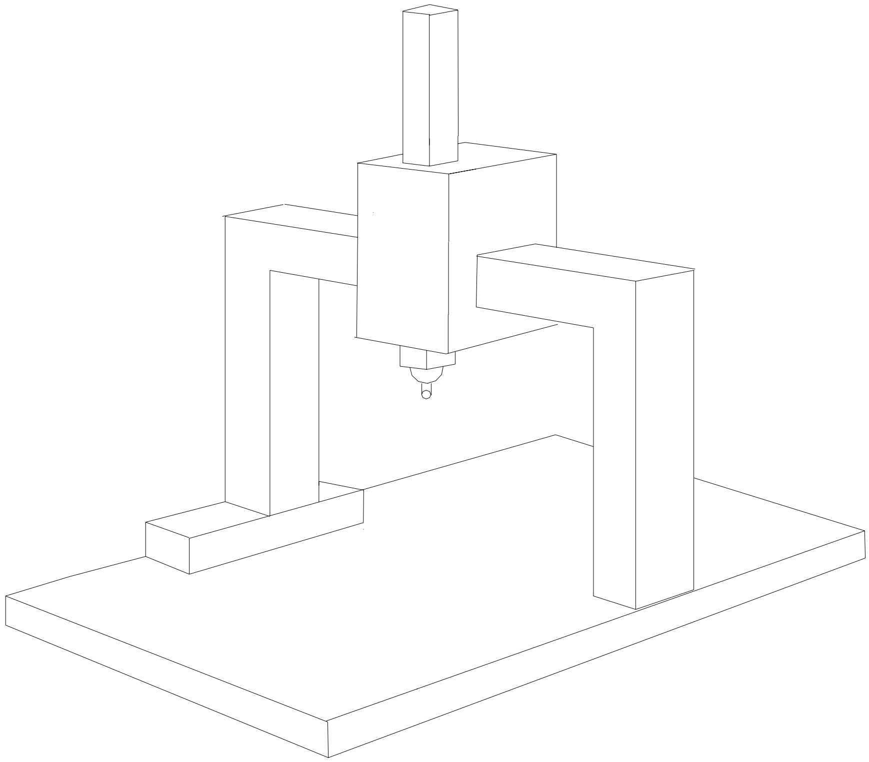

Figure 1 Three dimensional schematic of a bridge type CMM

More representative drawings of the machine are shown in Figure 2. Physically, errors of seven air pads and of the linear scale of the machine determine the resulting motion error (which we will call the joint kinematic errors) of the X-axis carriage. However as a simplifying hypothesis, the lateral pads counterparts of lateral pads A and B are eliminated. This will be compensated during the calculations, finding the lateral error as $e=e_1-e_2$, where e_1 and e_2 are individual errors of the lateral surfaces on which pads are moving. In Figure 2, A, B, C, D, and E indicate the pads. For the sake of simplicity, the surface deviations of pads A-B and C-D, which follow the same path, are taken as different functions.

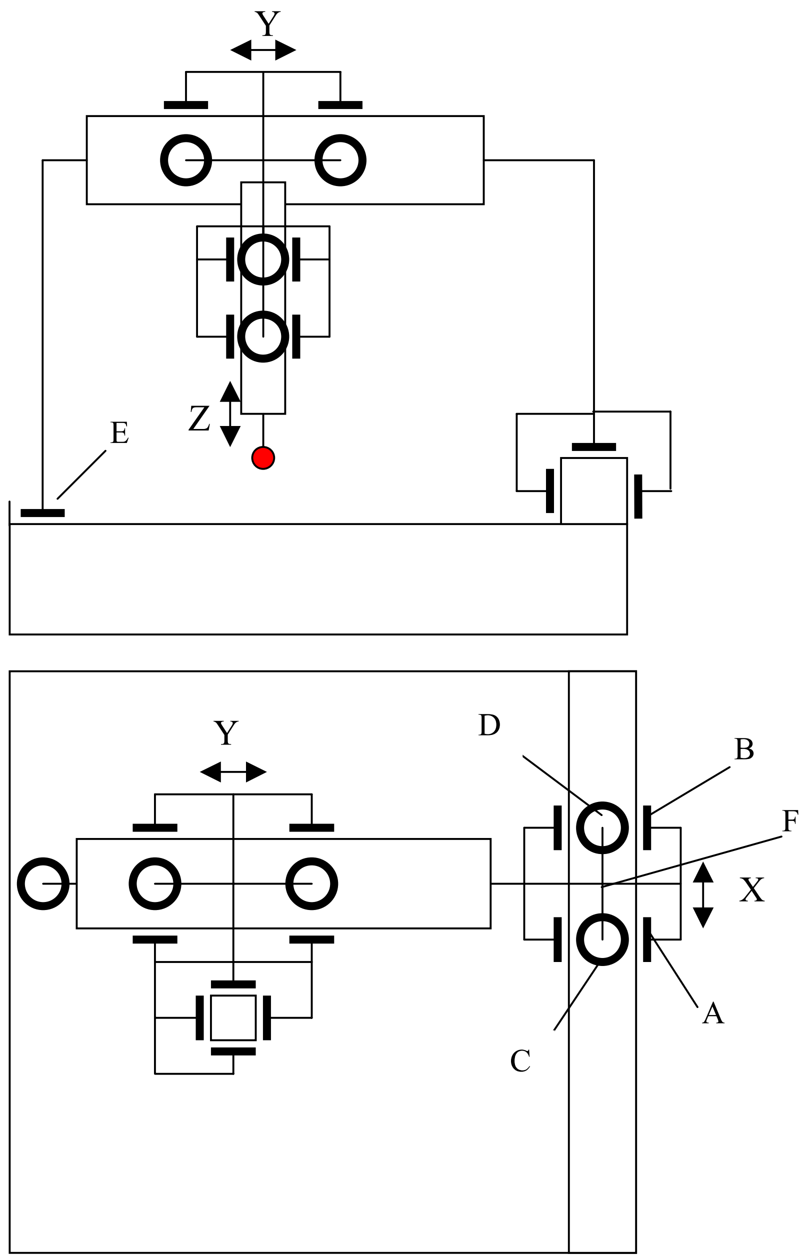

Figure 2 Front and top view of the CMM air bearing arrangement

In our model each air bearing pad is simulated as a contact point. This assumption stems from the geometrical properties of the air bearing pad. As presented in Figure 3, the air pad bears its load through a ball joint which acts as a point constraint [4].

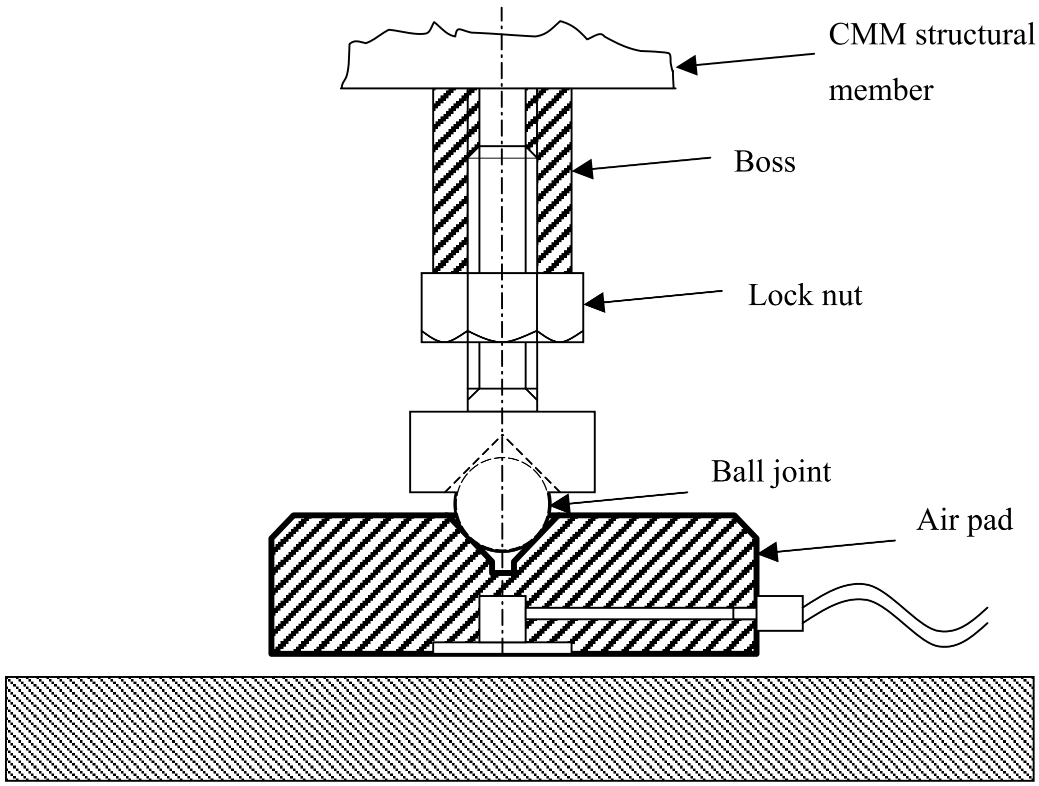

Figure 3 Air bearing system

Other important hypotheses in the methodology are that machine elements are rigid (rigid body hypothesis) and that the guideways geometric errors are small so that the small angle approximation can be applied later on in the analysis.

The joint kinematic errors of the X-axis will be determined from individual geometric guideway errors. It will be assumed that the errors occur at the center of stiffness of the carriage, which may be defined as the point at which, when a force is applied to the system, no net angular motion results [5].To this end, we utilize a homogeneous transformation matrix to first describe the position of a bearing point as a function of the carriage motion error. Hence, the true position of a contact point may be expressed in as:

$$\begin{bmatrix} x_A + \delta x_A \\ y_A + \delta y_A \\ z_A + \delta z_A \\ 1 \end{bmatrix} = T * \begin{bmatrix} x_A \\ y_A \\ z_A \\ 1 \end{bmatrix} \tag{1}$$

where T represents the homogeneous transformation matrix introduced in [6], which may also be expressed in the partitioned form:

$$T = \left[\begin{array}{c:c} R & p \\ \hdashline 000 & 1 \end{array}\right]$$

Where the 3×3 submatrix **R** is a rotation matrix and the 3×1 column vector **p** is a translation vector. If we express **R** and **p** by their complete representations:

$$\begin{bmatrix} x_A+\delta x_A \\ y_A+\delta y_A \\ z_A+\delta z_A \\ 1 \end{bmatrix} = \begin{bmatrix} c(\varepsilon_y)c(\varepsilon_z) & c(\varepsilon_z)s(\varepsilon_y)s(\varepsilon_x)-s(\varepsilon_z)c(\varepsilon_x) & c(\varepsilon_z)s(\varepsilon_y)c(\varepsilon_x)+s(\varepsilon_z)s(\varepsilon_x) & \delta_x(x) \\ s(\varepsilon_z)c(\varepsilon_y) & s(\varepsilon_z)s(\varepsilon_y)s(\varepsilon_x)+c(\varepsilon_z)c(\varepsilon_x) & s(\varepsilon_z)s(\varepsilon_y)c(\varepsilon_x)-c(\varepsilon_z)s(\varepsilon_x) & \delta_y(x) \\ -s(\varepsilon_y) & c(\varepsilon_y)s(\varepsilon_x) & c(\varepsilon_y)c(\varepsilon_x) & \delta_z(x) \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_A \\ y_A \\ z_A \\ 1 \end{bmatrix} \quad (2)$$

where c= cos, s= sin and $\varepsilon_x = \varepsilon_x(x)$, $\varepsilon_y = \varepsilon_y(x)$, $\varepsilon_z = \varepsilon_z(x)$ and x_A, y_A and z_A are the structural position coordinates (drawing values) of the machine X axis contact point A in stiffness center reference frame of the machine X axis, ($\delta x_A, \delta y_A, \delta z_A$) are the contact point A position variations due to the guideways form errors and $\delta_x(x), \delta_y(x), \delta_z(x)$, $\varepsilon_x(x)$, $\varepsilon_y(x)$, $\varepsilon_z(x)$ are the X-axis center of stiffness joint kinematic errors (motion errors) also resulting from the guideway errors.

For five air bearings and the linear actuator, a particular line from equation (1) is selected according to the physical influence of the contact point. Here it is assumed that contact points are on surfaces nominally parallel to one of the reference frame axis so that there is no need to perform vector projections. In consequence, for pads A and B (see Figure 2), one has to select the second line, for pads C, D and E the third line, and for the linear actuator (represented by constraint F) the first line of the equation system. As a result, we obtain the following system of equations which define the total carriage motion error:

$$
\begin{aligned}
&y_A+\delta y_A = s(\varepsilon_z)c(\varepsilon_y)\ x_A+(s(\varepsilon)_z s(\varepsilon_y)s(\varepsilon_x)+c(\varepsilon_z)c(\varepsilon_x))\ y_A+(s(\varepsilon_z)s(\varepsilon_y)c(\varepsilon_x)-c(\varepsilon_z)s(\varepsilon_x))\ z_A+\delta y\\
&y_B+\delta y_B = s(\varepsilon_z)c(\varepsilon_y)\ x_B+(s(\varepsilon_z)s(\varepsilon_y)s(\varepsilon_x)+c(\varepsilon_z)c(\varepsilon_x))\ y_B+(s(\varepsilon_z)s(\varepsilon_y)c(\varepsilon_x)-c(\varepsilon_z)s(\varepsilon_x))\ z_B+\delta y\\
&z_C+\delta z_C = -s(\varepsilon_y)\ x_C+c(\varepsilon_y)s(\varepsilon_x)\ y_C+c(\varepsilon_y)c(\varepsilon_x)\ z_C+\delta z\\
&z_D+\delta z_D = -s(\varepsilon_y)\ x_D+c(\varepsilon_y)s(\varepsilon_x)\ y_D+c(\varepsilon_y)c(\varepsilon_x)\ z_D+\delta z\\
&z_E+\delta z_E = -s(\varepsilon_y)\ x_E+c(\varepsilon_y)s(\varepsilon_x)\ y_E+c(\varepsilon_y)c(\varepsilon_x)\ z_E+\delta z\\
&x_F+\delta x_F = c(\varepsilon_z)c(\varepsilon_y)\ x_F+(c(\varepsilon_z)s(\varepsilon_y)s(\varepsilon_x)-s(\varepsilon_z)c(\varepsilon_x))y_F+(c(\varepsilon_z)s(\varepsilon_y)c(\varepsilon_x)+s(\varepsilon_z)s(\varepsilon_x))\ z_F+\delta x
\end{aligned}
\quad . \tag{3}
$$

Using the small angle approximation, this system may be reduced to

$$
\begin{aligned}
&\delta y_A = \varepsilon_z * x_A - \varepsilon_x * z_A + \delta y\\
&\delta y_B = \varepsilon_z * x_B - \varepsilon_x * z_B + \delta y\\
&\delta z_C = -\varepsilon_y * x_C + \varepsilon_x * y_C + \delta z\\
&\delta z_D = -\varepsilon_y * x_D + \varepsilon_x * y_D + \delta z\\
&\delta z_E = -\varepsilon_y * x_E + \varepsilon_x * y_E + \delta z\\
&\delta x_C = -\varepsilon_z * y_F + \varepsilon_y * z_F + \delta x
\end{aligned}
\tag{4}
$$

which may be expressed in the matrix form

$$
\begin{bmatrix}
0 & 1 & 0 & -z_A & 0 & x_A\\
0 & 1 & 0 & -z_B & 0 & x_B\\
0 & 0 & 1 & y_C & -x_C & 0\\
0 & 0 & 1 & y_D & -x_D & 0\\
0 & 0 & 1 & y_E & -x_E & 0\\
1 & 0 & 0 & 0 & z_F & -y_F
\end{bmatrix}
\begin{bmatrix}\delta x\\ \delta y\\ \delta z\\ \alpha\\ \beta\\ \gamma\end{bmatrix}
=
\begin{bmatrix}\delta y_A\\ \delta y_B\\ \delta z_C\\ \delta z_D\\ \delta z_E\\ \delta x_F\end{bmatrix}. \tag{5}
$$

This may be expressed as

$$\mathrm{J}\ \boldsymbol{\tau} = \boldsymbol{\Delta} \tag{6}$$

for which a solution may be found as follows:

$$\boldsymbol{\tau} = \mathrm{J}^{-1}\ \boldsymbol{\Delta} \tag{7}$$

Thus, we obtain the joint kinematic errors of the X axis carriage of the machine. This system being a linearization of the phenomenon, an iterative procedure is used to progressively converge close to the exact solution.

Simulation

The proposed method is numerically validated through simulations. First geometrically distorted guideways are represented using spline functions as shown in Figure 4. The coordinates of the contact points and of the line along which the scale error is measured are given in Table 1. The point of contact method just described was used to calculate carriage motion errors at the nominal centre of stiffness reference frame location resulting from these geometric errors and also from the scale errors. The results are shown in Figure 5.

As may be observed from the graphs and as anticipated the modelled geometric errors of the guideways directly impact on the kinematic behavior of the carriage.

Conclusion

A novel modelling approach has been introduced for establishing the relationship between the causal joint geometric errors (or guideways geometric errors) and the resulting joint kinematic errors of the carriage (motion errors). This approach is one building block in the development of virtual machines for machine tools and coordinate measuring machines. It should facilitate the use of thermal deformation as well as elastic deformation models of machine elements such as the guideways. Once the motion errors are known using the proposed approach it is then a simple matter to propagate these joint kinematic errors to the tool or probe tip using homogenous transformation matrices. The simulation so far conducted support the approach. An experimental validation is now under preparation.

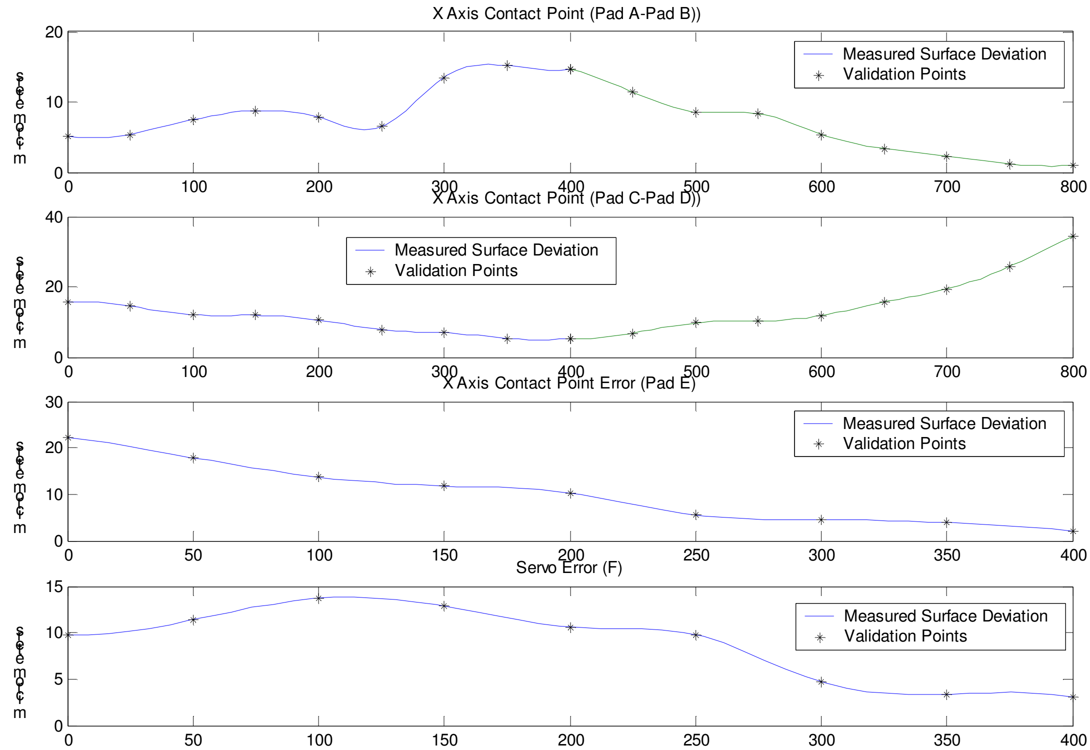

Figure 4 Simulated guideways and contact points calculated.

Table 1 Locations of the bearing points and of the linear scale calibration (Dimensions in millimeters)

	A	B	C	D	E	F
x axis	-200	200	-200	200	0	0
y axis	-300	-300	-290	-290	300	-300
z axis	-500	-500	-480	-480	-510	-485

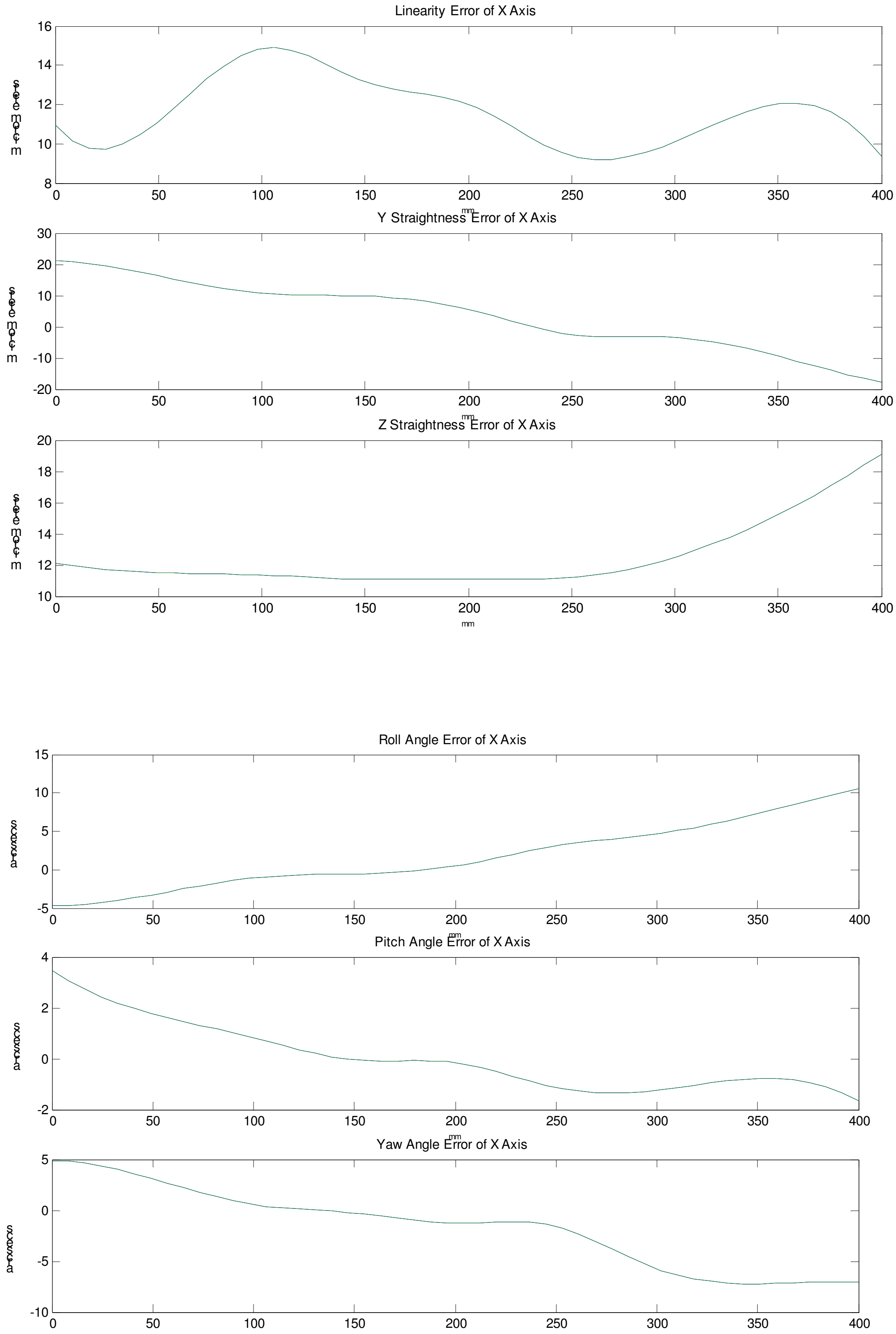

Figure 5 Joint kinematic errors (or motion errors) resulting from the guideways geometric errors.

Acknowledgements:

This work was made possible through an Individual Research Grant from the National Science and Engineering Research Council of Canada and the Auto 21 Network of Centres of Excellence of Canada.

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