On the Use of Impulsive Bilateral Constraints to Characterize Topology Transitions

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Abstract

Topology transitions occur often during the functioning of a number of mechanical systems. An approach is presented for the dynamic analysis of topology transitions. This relies on the use of impulsive bilateral constraints. The method has been successfully applied for the analysis of the heel strike in bipedal locomotion, and for the study of impacts using an experimental robotic device.

Keywords: Variable Topology Systems, Impulsive Constraints, Bipedal Walking, Impact.

1 Introduction

Mechanical systems with time-varying topology appear frequently in natural or human-made artificial systems. The nature of topology transitions is a key characteristic in the functioning of such systems. Typical situations that occur in variable topology systems are: (1) The number of degrees of freedom of the system decreases via the development of certain new connections; (2) the constraint configuration is changing – i.e., some constraints are added and some become passive –, but the effective number of degrees of freedom may stay the same.

The topology change can generally be seen as an impulsive motion event on the characteristic time scale of the finite motion of the system. There are several possibilities to develop models, analysis and simulation approaches for variable topology systems [1]. The presented approach is based on the use of bilateral impulsive constraints, which model events where physical connections are suddenly established.

2 Impulsive Dynamics of the Topology Transition

Let us consider that t_i represents the instant when certain constraint conditions are suddenly imposed and the topology of the system changes. This event takes place in the $[t_i^-,\ t_i^+]$ interval where t_i^- and t_i^+ represent the so-called pre- and post-event instants, respectively. This interval can generally be considered "instantaneous" on the characteristic time scale of the finite motion. Such topology transition can be characterized by the following m constraint equations

$$\mathbf{A}\mathbf{v}^+ = \mathbf{0}, \tag{1}$$

where $\mathbf{v}^+$ represents the $n \times 1$ array of generalized velocities at the post-event instant t_i^+, and $\mathbf{A}$ is the $m \times n$ Jacobian matrix of the constraints. These constraints represent the post-event required topology at the velocity level, and capture the physical conditions due to a sudden change in topology.

A general approach to characterize dynamic aspects of variable topology systems can be developed based on the constraints in Eq. (1). This makes it possible to interpret a decomposition of the tangent space of the system to two subspaces defined such that they are orthogonal to each other with respect to the mass metric of the tangent space [2]. These subspaces are denoted as Space of Constrained Motion (SCM), which is related to the constraints in Eq. (1), and Space of Admissible Motion (SAM), which is the orthogonal complement to the SCM.

The decomposition can be made based on two asymmetric projector operators, $\mathbf{P}_c$ and $\mathbf{P}_a$, which can be used to project kinematic and kinetic quantities to the SCM and the SAM respectively. The total kinetic energy of the system can also be decoupled using this approach [2]. The above technique allows for a decoupled representation of the impulsive dynamic equations governing the topology transition. The dynamic equations associated with the SCM, for the general case of non-ideal constraint realization, are

$$\mathbf{M}\left(\mathbf{v}_c^+ - \mathbf{v}_c^-\right) = \mathbf{A}^T(\bar{\boldsymbol{\lambda}} + \bar{\boldsymbol{\Lambda}}), \tag{2}$$

and the equations associated with the SAM are

$$\mathbf{M}\left(\mathbf{v}_a^+ - \mathbf{v}_a^-\right) = \mathbf{P}_a^T \bar{\mathbf{f}}_N, \tag{3}$$

where $\mathbf{v}_c = \mathbf{P}_c\mathbf{v}$ and $\mathbf{v}_a = \mathbf{P}_a\mathbf{v}$, $\mathbf{M}$ is the $n \times n$ mass matrix of the system, $\bar{\mathbf{f}}_N$ represents the impulses of the generalized non-ideal forces, and $\bar{\boldsymbol{\lambda}}$ and $\bar{\boldsymbol{\Lambda}}$ represent the impulses of the generalized constraint and non-ideal forces associated with the local parameterization of the SCM. It can be shown that $\mathbf{P}_c^T\bar{\mathbf{f}}_N = \mathbf{A}^T\bar{\boldsymbol{\Lambda}}$ [2]. In the case of ideal constraint realization only $\bar{\boldsymbol{\lambda}}$ is present in Eq. (2). However, in general, the magnitudes of the elements of $\bar{\boldsymbol{\lambda}}$ are different for ideal and non-ideal realizations.

The presented formulation and concepts are applied to two situations of topology transition that can be modelled using Eq. (1). The first situation is the heel strike event in bipedal walking. Numerical analysis on the effect of various parameters on the dynamics of the heel strike will be presented. The second example includes an experimental multibody system. The case of general impact will be considered, with emphasis on the compression phase which can be modelled using bilateral constraints. Non-ideal phenomena, such as friction, are also present in this case. We will use the data obtained via performing several sets of experiments for different system configurations. Based on this, we will illustrate how the proposed concepts can be used to capture the dynamics of topology transitions.

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