POSE SELECTION FOR THE KINEMATIC CALIBRATION OF A PROTOTYPED 4 DEGREES OF FREEDOM MANIPULATOR

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ABSTRACT

This paper explores the experimental pose selection for the kinematic calibration of the constraining linkage of a 4 degrees of freedom parallel manipulator. The aim is to select a set of poses out of a larger pool of possible poses which improve the calibration accuracy. Five different criteria have been suggested in the literature and are investigated in this article. The results show that the pose selection criteria did not significantly improve the calibration of this parallel manipulator. There was very little difference between applying the criteria and not applying it and some of the results using the criteria were worse than not using any criteria.

Keywords: Pose selection, kinematic calibration

1 INTRODUCTION

The selection of measured poses to be used for calibration has an effect on the accuracy of the results obtained through the calibration. The experimental measurements contain inaccuracies and noise. These inaccuracies will limit the ability of the calibration algorithm to correctly identify the kinematic parameters.

Five pose selection criteria are discussed in this paper and they are based on analysis of the augmented identification Jacobian matrix. The augmented identification Jacobian matrix, **J**, relates the vector of end effector pose errors, **y**, to the vector of independent kinematic parameters errors, d**p,** for several different poses. This is given by:

$$\mathbf{y} = \mathbf{J}\, \mathrm{d}\mathbf{p} \tag{1}$$

A numerical approach is used to form the augmented identification Jacobian matrix as described by (Zhuang and Roth, 1996). The rank of the augmented identification Jacobian matrix is the number of independent parameters that can be used for calibration when a sufficient number of poses are used. The rank can be calculated by determining the number of nonzero singular values after completing a singular value decomposition of the augmented identification Jacobian matrix. The singular values provide a measure of the ability of the calibration algorithm to identify the parameters. The poses used to calculate the augmented identification Jacobian matrix affect the results of the singular values. This means that the singular values can be used to select a set of poses which will result in the best calibration possible.

The first pose selection criterion, O_1, is the geometric mean of all of the singular values (Borm and Menq, 1989), and is given by:

$$O_1 = \frac{(\sigma_1 \sigma_2 \ldots \sigma_m)^{1/m}}{\sqrt{m}} \tag{2}$$

In equation (1), σ_i, $i = 1, \ldots, m$ is a singular value from singular value decomposition of the augmented identification Jacobian matrix and m is the rank, or number of nonzero singular values of **J**. The aim of this index is to maximize the product of all of the singular values.

The second pose selection criterion, O_2, is the inverse condition number (Driels and Pathre, 1990). It is given by:

$$O_2 = \frac{\sigma_{\min}}{\sigma_{\max}} \tag{3}$$

This criterion maximizes the ratio of the smallest singular value, $\sigma_{\min}$, compared to the largest singular value, $\sigma_{\max}$. This criterion tends to make all of the singular values more uniform so the parameters are closer to being equally easy to identify through calibration. To improve the poses selected, this criterion is maximized.

The third pose selection criterion, O_3, is the minimum singular value (Nahvi and Hollerbach, 1996), and is given by:

$$O_3 = \sigma_{\min} \tag{4}$$

This criterion uses only the minimum singular value which means that an improvement to some of the other singular values which could result in a better calibration will not be considered if it does not increase the magnitude of the minimum singular value. To improve the poses selected, this criterion is maximized.

The fourth pose selection criterion, O_4, is known as the noise amplification index (Nahvi and Hollerbach, 1996). This criterion is given by:

$$O_4 = \frac{\sigma_{\min}^2}{\sigma_{\max}} \tag{5}$$

This criterion is the ratio of the minimum singular value squared to the maximum singular value. This criterion is similar to criterion O_2 as only the maximum and minimum singular values are considered. This criterion is maximized.

The fifth pose selection criterion, O_5, (Sun and Hollerbach, 2008) is given by:

$$O_5 = \frac{1}{\frac{1}{\sigma_1} + \frac{1}{\sigma_2} + \ldots + \frac{1}{\sigma_m}} \tag{6}$$

This criterion is the harmonic mean of the singular values divided by the number of singular values. Similar to O_1, this criterion takes into account all of the singular values and is maximized.

This paper discusses the use of pose selection criteria to select an optimal set of poses to be used for calibration, which will result in the most accurate identification of the kinematic parameters. Multiple criteria have been proposed. The pose selection discussed in this paper attempts to choose an optimal set of poses out of a larger set of measurements already taken. Section 2 discusses three pose selection algorithms which are used in this paper. The pose selection is applied to the experimental calibration of a 4 degrees of freedom parallel manipulator which is described in Section 3. The results of the pose selection are given in Section 4.

2 SELECTION ALGORITHMS

In this work, the poses are selected out of a larger pool of previously measured poses. Three different searches to find optimal sets of poses were applied to this problem: an elimination search, a random search, and an exhaustive search. The elimination search removes one pose at a time and is discussed in Section 2.1. The random search continually guesses a random set of poses to be used and is discussed in Section 2.2. The exhaustive search tries every possible combination of poses resulting in an optimal solution and is discussed in Section 2.3. The criterion will be influenced by all of the factors which affect the singular values of the augmented identification Jacobian matrix, including the design of manipulator, the kinematic model used, the length and angular units of parameters and the poses measured.

2.1 ELIMINATION SEARCH

The elimination search provides a simple and effective method to determine the subset of poses to be used for calibration. The criterion is initially calculated with the entire pool of possible poses. One pose is eliminated at each of the iterations until the desired number of poses remains in the subset. In order to determine which pose should be removed at a given iteration, all of the poses are systematically removed and then reinserted back into the possible set one at a time and the criterion is calculated for every case. The pose which when removed results in the maximum value of the criterion, is permanently removed from the set of possible poses and the next iteration begins. The results of this method are repeatable and tend to converge to a good solution out of the possible combinations. There is the possibility with this method that the solution is not the most optimal combination of the desired number of poses. This shortcoming is offset by the fact that the evaluation time required for this is significantly faster than an exhaustive search which tests every possible combination of poses. This allows for shorter computational time and larger data sets can be used with the same computational power.

2.2 RANDOM SEARCH

A random search is a very simple search method that can quickly yield good results but also has numerous drawbacks. The goal of the pose selection is to select a limited number of poses out of a larger pool which results in a favourable pose selection criterion. At each iteration, this method randomly selects the set of poses to be evaluated and the pose selection criterion is calculated. After the desired number of iterations, the set of poses with the highest value for the pose selection criterion is used for calibration. If a minimum acceptable value of the criterion is known then the algorithm can stop once this value has been reached. One of the drawbacks is that there is no way to know if the result is the optimal solution if that is desired. If the aim is to find a set of poses that has a high enough pose selection criterion value then this algorithm will work well. It is very difficult to determine what value of the criterion is suitable. Additionally, because of the random nature of the approach there is the chance that it will never choose a set of poses that are good enough or the optimal solution.

2.3 EXHAUSTIVE SEARCH

An exhaustive search finds the combination of poses which results in the highest value of the pose selection criterion. The exhaustive search does this by evaluating the pose selection criterion value for every possible combination of poses. The advantage of this search is that the best possible solution is found. This makes it easy to compare the sets of poses found using different criteria. This method is very computationally intensive so the pool of possible poses must be relatively small in order for the results to be obtained in a reasonable amount of time without the use of supercomputing facilities.

3 MANIPULATOR MODEL AND EXPERIMENTAL CALIBRATION

The solid model and simplified diagram for the 4 degrees of freedom manipulator is pictured in Figure 1(a) and 1(b) respectively. The end effector can translate in all three directions and have pitch rotation, i.e., roll and yaw rotations are restricted. This linkage contains an ideally planar 4-bar (parallelogram) mechanism. In order for the 4-bar mechanism to exhibit any motion, the axis of the revolute joints must be parallel without considering any deformation. It is assumed that no deformation can occur in the kinematic model, so only the case where the 4-bar mechanism is modelled as ideally planar is considered.

For the consecutive joints of manipulator which are not parallel, the Denavit-Hartenberg (DH) convention is used (Paul, 1981). In order for a small change in the task space to result in a small change in the parameters, the DH convention cannot be used for consecutively parallel or near parallel joints. In this case the modified DH parameters are used (Hayati, 1983).

The manipulator has relative joint encoders on joints one, two, four and five (J1, J2, J4 and J5 in Figure 1(b). An encoder on joint three, J3, is not required because the angle of joint three can be calculated based on the link lengths of the planar 4-bar mechanism for each position of joint two. This allows the pose of the end effector (mobile platform) to be calculated using the encoder values from the constraining linkage without considering the wire mechanism.

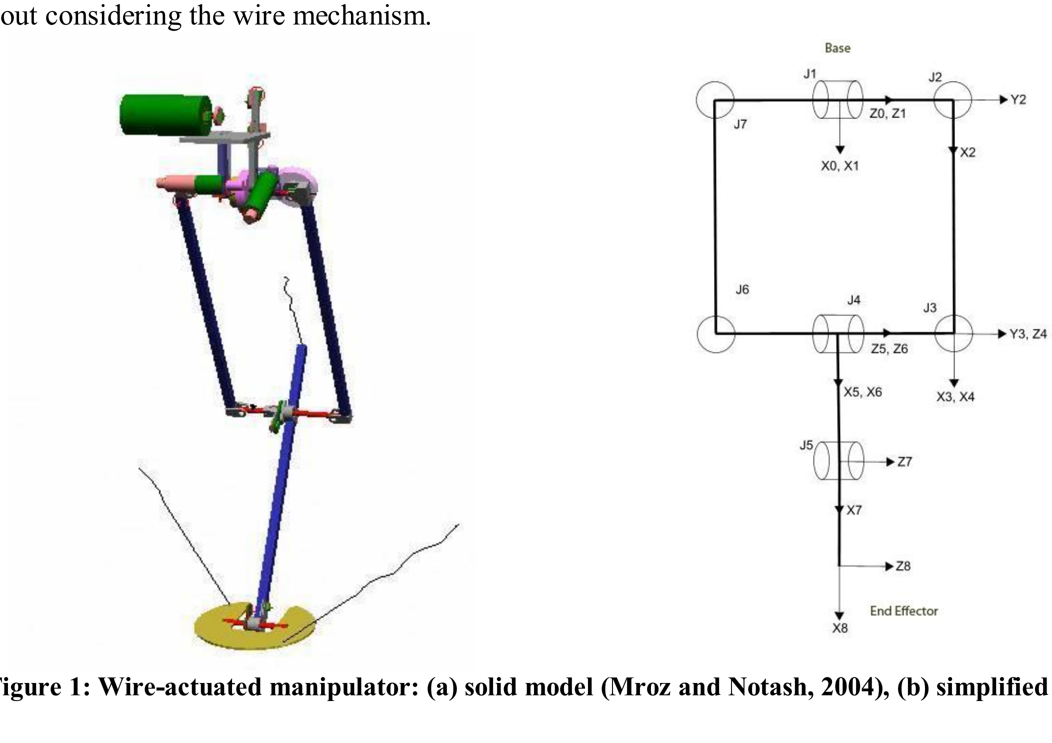

Figure 1: Wire-actuated manipulator: (a) solid model (Mroz and Notash, 2004), (b) simplified diagram.

A kinematic model was created using the modified DH parameters with 18 kinematic parameters as described in (Horne and Notash, 2008). The 4-bar closed-loop mechanism is treated as a planar parallelogram. No misalignment is included between the axis of joint one (and joint four) and the axis of the corresponding link of the mechanism, on which the joint is mounted.

The pose of the end effector was tracked using a Polaris tracking system (from Northern Digital Inc.). To track the pose, three reflective spheres (markers) were placed on the end effector and their positions were measured by the Polaris system. The position of the markers on the end effector is known so the end effector pose can be calculated. The root-mean-square (RMS) volumetric acceptance criterion for the Polaris is reported to be 0.350 mm (based on a single marker stepped through over 1200 positions in the defined workspace, using the mean of 30 samples at each position, at 20˚C) (User's Guide, 2002).

A calibration routine was written to geometrically calibrate this manipulator. This routine calculates the error between the pose measured with the external motion tracking system and the pose calculated with the kinematic model using the joint encoder readings. Along with geometric parameters, the model also contains joint offsets for the encoders. These encoder offset parameters are necessary because relative encoders are used on the manipulator. This means that all of the measurements are based on the position when the controller is initialized. During the data collection, the first measurement is taken in approximately at the zero configuration of manipulator so the value of the calibrated offset should be small. The total pose error is then given by:

$$\sum_{j=1}^{n} \mathbf{e}_j^{T}\mathbf{e}_j \tag{7}$$

where n is the total number of measured poses and $\mathbf{e}_j$ is a vector of the translational and rotational error. A nonlinear minimization technique is then used to minimize the total error by adjusting the geometric parameters.

Prior to calibration some of the collected poses were eliminated in order to achieve better calibration results. Along with each measured pose, the Polaris system outputs a value that is a measure of how well the actual marker measurements fit the marker positions defined for that rigid body (NDI Polaris and Aurora Combined Application Programmers' Interface Guide, 2002). A smaller value indicates that there is less error in the measured pose. During experimentation, these values ranged from approximately 0.10 mm to 1.50 mm with an average value of 0.23 mm. This error parameter was used in order to eliminate data which were not measured accurately by the Polaris system. A cut-off value of 0.30 mm was established in order to eliminate the poses which were not measured accurately by the tracking system so the data with values larger than 0.30 were eliminated, i.e., a smaller value is desired.

4 RESULTS AND DISCUSSION

The calculation of the pose selection criteria requires the augmented identification Jacobian matrix to be formed, as described in Section 1, followed by a singular value decomposition of this matrix. These singular values are then used to calculate the appropriate pose selection criterion. This makes the calculation of each of the criterion computationally expensive. In order to obtain results in a reasonable amount of time, two different cases were established. The first case was to select 15 poses to be used for calibration out of a pool of 20. For the exhaustive search, this case took an average of 4.6 hours to complete. For this case, the criterion results, poses selected, and calibration results are given in Section 4.1. Along with the exhaustive search this section also contains the results using the random search and elimination search to provide a comparison. The results were also obtained by selecting a set of poses randomly and then calculating the criteria and calibration. In order to provide a baseline to compare the effectiveness of the searches, this was done multiple times and the results were averaged.

A second case was established which selected 50 poses out of 100 possible poses. The exhaustive search was not feasible based on the computational time so it was not included in these results. Selecting 50 out of 100 poses was done to limit the computation time required for the elimination search. The average computation time for this case to complete with the elimination search was 3.3 hours. The results for this case also contain the random search for comparison. The criterion values and calibration results are presented in Section 4.2. The list of poses selected was not included because of the large number of poses selected for each method.

The experimental measurements were taken so that the poses were distributed throughout the workspace. Three dimensional plots were made using the pools of 20 and 100 poses. These plots show that while the poses are not completely uniformly distributed in space, they are in general well dispersed throughout the workspace.

The results in this article were obtained on a 2GHz computer with 700 Mb of RAM. The computation time included in the results is given to provide a magnitude of the computation load and not an exact measurement of the speed of each algorithm. The searches were not done under identical conditions, and because of this there are significant differences between the times for the same search using a different criterion. The actual calculation of the criterion should be a very small part of the computational load so these deviations should be attributed to other processes which may have been running on the computer during the calculation.

In general, six parameters can be identified based on each measured pose. Although this manipulator is designed as a 4 degrees of freedom manipulator, six parameters can still be identified from each measurement. The kinematic model contains 18 independent parameters. More than three poses should be used in order to minimize the effect of measurement inaccuracy and noise on the calibration result.

4.1 15 POSES SELECTED FROM 20

The first pose selection case being examined is selecting 15 poses out of a pool of 20 poses. In this section the criterion results, poses selected, and calibration results are presented.

4.1.1 CRITERION RESULTS

The results for the criteria evaluated with all 20 poses and random sets of 15 poses are presented in Table 1. In order to provide a suitable baseline, ten sets of 15 poses were randomly chosen from the 20 and the criteria were calculated. The average, standard deviation, maximum and minimum values of the criteria are included to describe the 10 sets of poses (trials). It is important to note that with a random sample of poses from the available pool, the average value of the pose selection criteria decreased with the selection of fewer poses compared to the full set of 20 poses.

Table 1: Baseline Criterion Results (15 out of 20 Poses).

	Full Set of 20	Random Set of 15 out of 20, 10 trials			
Criterion	Value	Average	Standard Deviation	Maximum	Minimum
O_1	6.531E–01	4.863E–01	2.796E–02	5.173E–01	4.225E–01
O_2	8.327E–05	6.459E–05	1.661E–05	8.780E–05	3.521E–05
O_3	8.709E–03	6.143E–03	1.527E–03	8.151E–03	3.658E–03
O_4	7.252E–07	4.914E–07	1.620E–07	6.861E–07	1.326E–07
O_5	7.145E–03	4.724E–03	1.327E–03	6.555E–03	2.483E–03

The results of the criteria calculated from the poses selected using the elimination search is given in Table 2. The initial criteria values are the results when all 20 poses are included. The final criteria values are the results after the five poses have been eliminated. With the removal of the poses, three of the investigated five criteria decreased. The criteria O_2 and O_4 however increased. All five of the final criteria results are higher than the average results from randomly selected sets listed in Table 1. This improvement shows the algorithm works as expected. These results executed in an average time of approximately 39 seconds.

Table 2: Criterion Results Using an Elimination Search (15 out of 20 Poses).

Criterion	Initial Criterion Value	Final Criterion Value	Computation Time
O_1	6.531E–01	5.495E–01	39 s
O_2	8.327E–05	1.079E–04	40 s
O_3	8.709E–03	8.383E–03	38 s
O_4	7.252E–07	9.032E–07	40 s
O_5	7.145E–03	6.759E–03	37 s

The criteria results for the random search are given in Tables 3 and 4. Table 3 contains the results after 10 iterations of the random search and Table 4 after 100. For both cases ten trials (data sets) were completed and the averages, standard deviations, maximums and minimums are displayed. As it would be expected, the results for all of the criteria increase as more iterations of the random search are used. Both the average and maximum results were lower for the random search compared to the corresponding final criterion value of the elimination search for all five criteria. This indicates that the elimination search achieved better results compared to the random search.

Table 3: Criterion Results Using a Random Search with 10 Iterations (15 out of 20 Poses).

	Random Set of 15 out of 20 poses, 10 trials (data sets)			
Criterion	Average	Standard Deviation	Maximum	Minimum
O_1	5.178E–01	1.107E–02	5.327E–01	4.942E–01
O_2	9.921E–05	3.672E–06	1.040E–04	9.291E–05
O_3	7.682E–03	3.792E–04	8.193E–03	6.956E–03
O_4	7.222E–07	7.455E–08	8.671E–07	6.344E–07
O_5	6.193E–03	3.074E–04	6.576E–03	5.726E–03

Table 4: Criterion Results Using a Random Search with 100 Iterations (15 out of 20 Poses).

	Random Set of 15 out of 20 poses, 10 trials (data sets)			
Criterion	Average	Standard Deviation	Maximum	Minimum
O_1	5.312E–01	7.078E–03	5.472E–01	5.234E–01
O_2	1.050E–04	1.394E–06	1.068E–04	1.019E–04
O_3	8.175E–03	6.021E–05	8.323E–03	8.113E–03
O_4	8.575E–07	1.968E–08	8.837E–07	8.154E–07
O_5	6.583E–03	8.209E–05	6.692E–03	6.410E–03

Table 5 contains the results for the criteria from the exhaustive search. The initial value of the criterion is evaluated using the first set of 15 poses. Following that all possible combinations were evaluated and the results for the largest pose selection criterion are given as the final values. The final results for each of the five criteria are the same using the exhaustive and elimination search. This means that for this case, the elimination search did choose the optimal set of poses. The average computation time for the exhaustive search was 4.6 hours which is over 400 times longer than the average time to complete the elimination search.

Table 5: Criterion Results Using an Exhaustive Search (15 out of 20 Poses).

Criterion	Initial Criterion Value	Final Criterion Value	Computation Time
O_1	4.848E–01	5.495E–01	3.52 hrs
O_2	8.331E–05	1.079E–04	3.86 hrs
O_3	6.500E–03	8.383E–03	3.98 hrs
O_4	5.415E–07	9.033E–07	5.54 hrs
O_5	5.361E–03	6.760E–03	6.07 hrs

4.1.1 SELECTED POSES

The results in this section show the five poses which were removed for each criterion. Each pose in the pool of possibilities was assigned a number from 1 to 20. The results for the random search is not included because the poses selected are not consistent each time the search is completed. For each of the five pose selection criteria, the sets of poses that are selected is based on an evaluation criterion which is based on the singular values of the augmented identification Jacobian matrix. Criteria O_1 and O_5 contain all of the singular values, O_3 is only the minimum singular value, and O_2 and O_4 contain the minimum and maximum singular values.

Table 6 contains the poses which were not selected using the elimination search for each criterion. The poses listed in this table are given in the order which they were removed. This order provides a measure of how much affect the pose has on the criterion because at each stage the pose is removed which results in the remaining poses having a maximum value of the selection criterion. Poses 18 and 2 were eliminated based on all five criteria and pose 18 was one of the first two removed for each criterion. Pose 2 however was removed first by three of the criteria, second by another and last by O_1. Poses 6 and 9 were removed by four of the criteria, pose 16 by three, pose 4 by two, and poses 5 and 7 by only one. The sets of poses selected by the criteria were slightly different except for O_3 and O_4. These two criteria are most strongly influenced by the minimum singular value so it is logical that they might result in a common set of poses.

Table 6: Elimination Search, Poses Not Selected (15 out of 20 Poses).

Criterion	Poses Not Selected				
O_1	18	4	7	6	2
O_2	18	2	6	9	5
O_3	2	18	9	6	16
O_4	2	18	9	6	16
O_5	2	18	9	4	16

Similarly, Table 7 displays the results for the exhaustive search for each of the five criteria. These results are displayed in ascending numerical order because the poses are not removed one at a time as it is done in the elimination search. The poses selected using the exhaustive searches are identical to those obtained from the elimination searches. This shows that the elimination search works well and can arrive at the optimal set of poses. The fact that an optimal result was achieved using the elimination search in this case does not guarantee an optimal solution will be found for all cases. It was expected that the same poses were selected using both the elimination and exhaustive search because the final criteria values were the same for both the exhaustive and elimination search.

Table 7: Exhaustive Search, Poses Not Selected (15 out of 20 Poses).

Criterion	Poses Not Selected				
O_1	2	4	6	7	18
O_2	2	5	6	9	18
O_3	2	6	9	16	18
O_4	2	6	9	16	18
O_5	2	4	9	16	18

4.1.3 CALIBRATION RESULTS

This section contains calibration results for the pose selection for the case where 15 poses are selected out of a pool of 20 poses. The results in this section are based on experimental data so only the ability to reduce the root-mean-square (RMS) of the end effector pose error is discussed and not the ability to determine the actual physical values of the parameters. The actual values for the model parameters have been measured but are not known exactly so the ability to improve the parameters is not measurable.

In order to provide a baseline to evaluate the performance of the criterion, calibration results were obtained using a set of 15 poses randomly selected from the pool of 20 poses. This random selection of poses and calibration was done 10 times and results of the calibration are presented in Table 8. The average, standard deviation, maximum, and minimum of the RMS of the end effector pose error are presented to describe the results from the 10 sets of 15 poses randomly selected. The RMS of the error in translation and rotation are reported in millimeters and degrees, respectively. Initial, final and verification results are given for both the translational and rotational errors. The initial result is the error when the nominal unadjusted manipulator parameters are applied to the measured poses. The final result is the value of the error after the parameters have been adjusted to better fit the model to the calibration poses. These updated model parameters are then applied to an independent set of poses and the initial error is calculated (verification). For the results presented in Table 8, 110 poses were used for the verification. The verification provides the most important results because it is a measure of how well the updated model tracks the manipulator pose through poses which were not used for the calibration. The final calibration results describe how well the model was adjusted to match that particular set of poses. In this case, the verification is also preferable because a small number of poses were used for the calibration. The results for both the translation and rotation are as they would be expected. The initial level of error is decreased in the final calibration results. The verification results are an improvement over the initial error; and they are slightly larger than the final calibration error.

Table 8: Baseline Calibration Results (15 out of 20 Poses).

	RMS Translation (mm)			RMS Rotation (deg)		
	Initial	Final	Verification	Initial	Final	Verification
Full set (of 20)	25.36	6.24	7.08	2.77	1.18	1.45
Average	25.32	6.30	7.32	2.79	1.15	1.45
Standard Deviation	0.54	0.25	0.40	0.14	0.09	0.02
Max	26.03	6.64	8.07	3.00	1.29	1.51
Min	24.18	5.92	6.74	2.57	0.97	1.45

The calibration results for the poses selected based on the criteria are given in Table 9. The same expected trend occurs for all of the criteria where the verification error is an improvement from the initial error and slightly larger than the final error. The sets of poses selected through both the exhaustive and elimination searches were identical. As this is only one trial, no conclusions can be made about the pose selection criteria which can be extended to all situations. Along with this, it is difficult to assess improvement because the changes are small and there are changes in both the translation and rotation. All of the translation verification results show a slight improvement over the average value of the random sets of 15 poses. The rotation results are near equal although some of the selected poses result in up to 0.05 degrees additional error. Of the five calibration results, the lowest error in translation was for O_5 however it did not result in the lowest error in rotation. The criterion O_2 resulted in the lowest error in rotation but the highest error in translation for the verification.

Table 9: Elimination and Exhaustive Search Calibration Results (15 out of 20 Poses).

	RMS Translation (mm)			RMS Rotation (deg)		
Criterion	Initial	Final	Verification	Initial	Final	Verification
O_1	25.52	6.66	6.89	2.56	1.13	1.50
O_2	25.71	6.59	6.91	2.88	1.30	1.45
O_3	25.83	6.50	6.80	2.89	1.28	1.46
O_4	25.82	6.50	6.80	2.89	1.28	1.46
O_5	25.76	6.30	6.73	2.79	1.18	1.47

4.2 50 POSES SELECTED FROM 100

In general, an increase in the number of poses uniformly distributed in the workspace of manipulator used for calibration increases the accuracy of the calibration. This section discusses the results when 50 poses are selected from a pool of 100. It is expected that this will improve the calibration results compared to the previous case of 15 poses because the measurement inaccuracy and noise should have less of an effect on the result. Due to the large number of poses removed, the list of poses removed is not presented here. The exhaustive search was not investigated because of the large computational requirements.

4.2.1 CRITERION RESULTS

Table 10 shows the criteria results for the full set of 100 poses and for 50 poses randomly selected out of the 100 poses. Ten different sets of 50 poses were selected randomly and the average, standard deviation, maximum and minimum are displayed for each of the criteria. These results show a drop in criteria value by approximately 50% when half of the poses are used for all criteria except for criterion O_2, which remained nearly constant with a slight increase.

Table 10: Baseline Criterion Results (50 out of 100 Poses).

	Full Set of 100	Random Set of 50 out of 100, 10 trials			
Criterion	Value	Average	Standard Deviation	Maximum	Minimum
O_1	3.183	1.530	8.431E–02	1.661	1.382
O_2	9.479E–05	9.630E–05	1.238E–05	1.201E–04	8.034E–05
O_3	4.990E–02	2.481E–02	2.069E–03	2.822E–02	2.079E–02
O_4	4.730E–06	2.127E–06	8.396E–07	3.5318E–06	1.140E–06
O_5	4.008E–02	2.008E–02	3.560E–03	2.550E–02	1.470E–02

The criteria results for the elimination search are given in Table 11. All of the final criteria results increase compared to the average criteria from the random sets of poses. Criteria O_1 (geometric mean), O_3 (minimum singular value), and O_5 (harmonic mean divided by the number of singular values) all decreased from the full set of 100 poses. The other criteria, O_2 (inverse condition number) and O_4 (noise amplification index), both increased as the poses were removed.

Table 11: Criterion Results Using an Elimination Search (50 out of 100 Poses).

Criterion	Initial Criterion Value	Final Criterion Value	Computation Time
O_1	3.183	2.030	4.28 hrs
O_2	9.479E–05	1.756E–04	2.75 hrs
O_3	4.990E–02	4.573E–02	4.15 hrs
O_4	4.730E–06	8.009E–06	2.73 hrs
O_5	4.008E–04	3.444E–02	2.39 hrs

The criteria results for the random search after 10 and 100 iterations are given in Tables 12 and 13 respectively. Similar to the previous case, the results from the random search were not as good as the results from the elimination search for all of the criteria.

Table 12: Criterion Results Using a Random Search, 10 Iterations (50 out of 100 Poses).

	Random Set of 50 out of 100, 10 trials (data sets)			
Criterion	Average	Standard Deviation	Maximum	Minimum
O_1	1.674	4.350E–02	1.759	1.615
O_2	1.187E–04	7.212E–06	1.294E–04	1.056E–04
O_3	3.194E–02	2.546E–03	3.455E–02	2.736E–02
O_4	3.684E–06	4.878E–07	4.217E–06	2.841E–06
O_5	2.402E–02	1.363E–03	2.633E–02	2.153E–02

Table 13: Criterion Results Using a Random Search, 100 Iterations (50 out of 100 Poses).

	Random Set of 50 out of 100, 10 trials (data sets)			
Criterion	Average	Standard Deviation	Maximum	Minimum
O_1	1.722	2.561E–02	1.772	1.686
O_2	1.342E–04	5.159E–06	1.463E–04	1.268E–04
O_3	3.482E–02	1.090E–03	3.702E–02	3.271E–02
O_4	4.506E–06	2.604E–07	5.024E–06	4.155E–06
O_5	2.625E–02	5.925E–04	2.756E–02	2.548E–02

4.2.2 CALIBRATION RESULTS

Table 14 contains the calibration results for the full set of 100 poses. From the full set, 50 poses were randomly selected and then used for the calibration. This was done ten times and the average, standard deviation, maximum and minimum of the calibration results (RMS values of the end effector pose error) are also included in the table. An interesting observation is that the verification of the RMS end effector error in translation was lower for the average of the 50 poses compared to the full set of 100 poses. This calculation was repeated and a similar result occurred.

Table 14: Baseline Calibration Results (50 out of 100 Poses).

	RMS Translation (mm)			RMS Rotation (deg)		
	Initial	Final	Verification	Initial	Final	Verification
Full set (of 100)	25.29	7.55	8.01	3.01	1.66	1.34
Average	25.27	5.89	7.13	3.05	1.66	1.34
SD	0.46	0.66	0.32	0.22	0.34	0.03
Max	25.90	6.91	7.72	3.53	2.49	1.44
Min	24.51	4.60	6.57	2.72	1.27	1.32

Table 15 contains the calibration results from the elimination search. The RMS of the end effector pose error for all of the criteria and the average of the baseline calibration results have almost the same value. The spread for the RMS of the end effector verification error for translation was 0.66 mm and the spread of the rotation was 0.01 degrees. For all of the criteria, the verification results for translation were slightly worse than the average baseline results. This means that using the criteria resulted in a slightly worse calibration compared to randomly choosing the poses. The results for the verification RMS of the rotation error are all very close to 1.33 radians for the baseline and elimination search results for all of the criteria. Although the criteria did not perform well, the difference between the average and criteria results are very small.

Table 15: Elimination Search Calibration Results (50 out of 100 Poses).

	RMS Translation (mm)			RMS Rotation (deg)		
Criterion	Initial	Final	Verification	Initial	Final	Verification
O_1	25.38	7.36	7.88	3.11	1.72	1.33
O_2	26.26	6.69	7.41	3.05	1.81	1.32
O_3	26.03	6.69	7.22	3.19	1.86	1.33
O_4	26.26	6.69	7.41	3.05	1.81	1.32
O_5	26.04	6.26	7.49	3.03	1.61	1.32

5 CONCLUSIONS

In this paper, the effectiveness of pose selection criteria applied to the experimental kinematic calibration of a 4 degrees of freedom manipulator was examined. The pose selection criteria are used to select a set of poses which would improve the calibration accuracy. Five different criteria have been suggested in the literature and were applied to the manipulator. All of the criteria investigated were based on the singular values of the augmented identification Jacobian matrix. These criteria are: the geometric mean of the singular values, the inverse condition number, the minimum singular value, the noise amplification index, and the inverse of the sum of the reciprocals of all of the singular values.

Each of the criteria was applied to the experimental manipulator to select 15 out of 20 and 50 out of 100 poses. Three search algorithms were used: a random search which repeatedly selects a random set of poses, an elimination search which removes one pose at a time, and an exhaustive search which tests every possible combination of poses. The results showed a relatively small change to the RMS of the verification error when the criteria were used to select the poses compared to a randomly selected set of poses to provide a baseline comparison. For the case of selecting 15 poses, all three search algorithms were applied and the results for the elimination search were the same as the results for the exhaustive search. This suggests that the elimination search will yield good results. From the experimental results, it is impossible to say that any of the criteria yield better results compared to randomly selecting the poses or using the full set of poses. These results suggest that, provided that a suitable number of well distributed poses are used for calibration, the pose selection criteria do not have a significant effect on the results. The effectiveness of the pose selection criteria was also explored with a simulation of a 2 degrees of freedom planar serial manipulator (Horne and Notash, 2009). The results of this simulation indicated that using a larger number of poses, irregardless of any pose selection criteria, is preferable.

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