# Approximate Static Balancing of a Planar Parallel Cable-Driven Mechanism

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**Abstract**

In parallel cable-driven mechanisms, the unilaterality of force transmission requires a minimum level of tension in cables to preserve their geometry. As a result, the driving electrical motors need to produce continuous torques (and power) to maintain the cable tensions. This paper proposes to use non-linear springs in parallel with the motors in order to maintain the minimum tension, leaving to the motors the application of the additional forces, i.e., those forces needed to produce accelerations and balance external forces applied at the end effector. We suggest to model the non-linear springs behavior with $n$-order polynomials. The polynomial coefficients are computed through the solution of a quadratic program, which minimizes the resultant of the spring tensions on the end-effector while maintaining these tensions above a given threshold. Our method is illustrated by its application to a two-degree-of-freedom planar parallel cable-driven manipulator.

**Keywords:** parallel cable-driven mechanisms, static balancing, quadratic programming.

## Équilibrage statique approximatif d'un mécanisme parallèle plan à entraînement par câbles

**Résumé**

L'unilatéralité de la transmission des forces dans les mécanismes parallèles à entraînement par câbles (MPEC) impose un niveau minimum de tension dans les câbles pour conserver leur géométrie. Afin de maintenir cette tension, les moteurs électriques d'un MPEC doivent fournir un couple et une puissance constants. Cet article propose d'utiliser des ressorts non-linéaires fixés en parallèle avec les moteurs pour produire cette tension minimum, laissant aux moteurs l'application de toute charge additionnelle (p. ex., pour vaincre l'inertie ou des forces extérieures appliquées à l'effecteur). Ainsi, dans le but de restreindre au minimum la dépense énergétique des moteurs, nous suggérons de déterminer les coefficients du polynôme associé au comportement des ressorts pour à la fois maintenir les câbles en tension et approximer l'équilibrage statique du mécanisme sur son espace de travail. Il est montré que ce problème peut être résolu par la programmation quadratique. À titre d'exemple, la méthode est appliquée à un MPEC plan à deux degrés de liberté.

**Mots-clé:** mécanismes parallèles entraînés par câbles, équilibrage statique, programmation quadratique.

## 1 INTRODUCTION

Nowadays, cables are used as mechanical links in devices requiring flexibility and/or where space is limited. Over the last decade, cables have been increasingly used in robotics. Most notably, this has led to design of a new family of mechanisms: parallel cable-driven mechanisms (PCDMs). Conventional parallel manipulators take advantage of the possibility of fixing their active joints to the base, thereby removing the motors themselves from the payload of the robot. The payload, (i.e., the inertia and the weight) can be further decreased by replacing the conventional rigid links with cables, that is, by turning the parallel manipulator into a PCDM. This brings new advantages, such as larger accelerations allowed through the lower inertia of the system, larger workspace, modularity (a PCDM can be easily deployed) and the low-cost of the equipment. However, there are drawbacks related to the use of cables, the main one being the unilaterality of force transmission through cables; they can only pull and not push. Moreover, this latter constraint leads to the necessity of having $n + 1$ cables to generate $n$-dof [1, 2]. Other problems are the possible interference between the cables over end-effector trajectories, and also lower stiffness of the mechanism.

Another drawback of PCDMs, comes from the necessity to keep all cables taut at any moment in order to preserve the geometry of the mechanism. Common practice is to control the motors in order to optimize the tensions distribution for all the cables while ensuring a minimal cable tension, thus preventing cable sag [3, 4]. However, preserving a minimum level of tension at all time requires continuous power from the motors, even when no external wrench is applied on the moving platform. Hence, it is suitable and intuitive to formulate the following question: could this energy expenditure be provided by a passive conservative mechanical system to avoid energy dissipation and oversizing of the electrical motors?

This paper proposes an approximate solution to this problem, which consists in adding nonlinear springs at the (fixed) actuated joints, in parallel with the electrical motors. Hence, the method presented in this work relies on a purely mechanical system to maintain the tension in the cables. The nonlinear springs are selected so as to preserve a minimum level of tension in each cable, when no external wrench is applied to the moving platform, so that the motors need only to balance out the external loads. Hence, we want the cables to remain taut when no external load is applied, even when the motors are turned off. In doing so, we must, however, preserve the neutral static equilibrium of the mechanism over its workspace, to prevent the motors from having to work against the springs when displacement is required. Now, this last requirement cannot be fulfilled with a passive spring system based on the actuated-joint positions, at least, not in general. As a result, our aim is to *approach* neutral static equilibrium over the manipulator workspace.

In summary, in this paper, we aim at determining the behavior of nonlinear springs that would maintain the minimum required tension in the cables, while approximating static balancing of the mechanism over its workspace. This requirement forces us to develop a different approach than those used in [5, 6, 7, 8] for the exact static balancing of planar mechanisms, [9, 10, 11, 12, 13] for the exact static balancing of spatial mechanisms and [14, 15] for partial static balancing, where only some directions of motion are balanced.

For simplicity reasons, we concentrate our work on a three-cable two-dof planar parallel cable-driven mechanism (PPCDM). Thus, the second section of this paper presents the mathematical definition of the design problem in question. The third section shows results obtained through

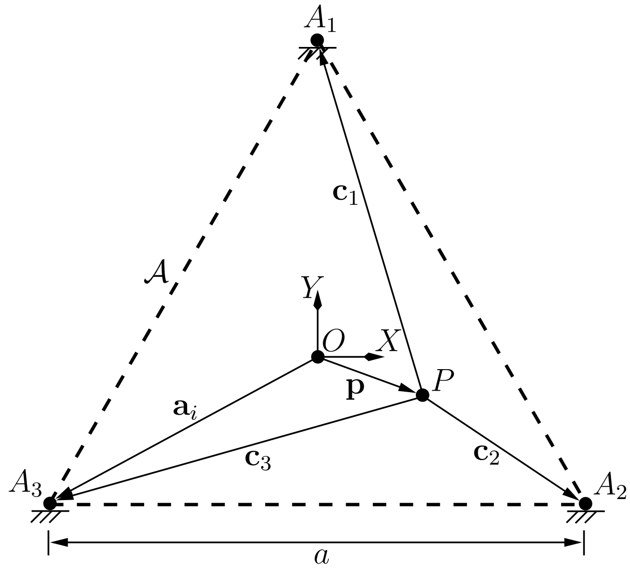


Figure 1: Kinematic modeling of the PPCDM.

the numerical solution of the ensuing optimization problem. Finally, section 4 presents some conclusions drawn from the results of section 3.

## 2 MATHEMATICAL PROBLEM DEFINITION

Consider the two-dof PCDM actuated by three motors through their three corresponding cables, as shown in Fig. 1. In this figure, vector $\mathbf{a}_i$ represents the position of the actuated reel $A_i$ of cable $i$ in the base frame, vector $\mathbf{p}$ represents the position of reference point $P$ of the end effector from the origin point $O$; vector $\mathbf{c}_i \equiv \mathbf{a}_i - \mathbf{p}$ points from $P$ to $A_i$, its magnitude being the length $c_i$ of the $i^{\text{th}}$ cable. Moreover, the workspace of the mechanism is $\mathcal{A}$, and the length of its bounding edges is $a$. Notice that the moving platform is a point, i.e., all the cables are attached at $P$.

Thus, at any moving-platform pose over the manipulator workspace, we want the motors to exert torques only to balance out external wrenches, not to keep the cables taut. Therefore, the torques exerted by the non-linear springs attached in parallel with the motors to the shaft of each reel should maintain the PCDM in static equilibrium at any point in its workspace, when no external wrench is applied.

Unfortunately, in general, achieving perfect static balancing (i.e., exact neutral equilibrium over the workspace) of a PCDM with base-fixed springs is impossible. Nevertheless, one may seek and find a set of springs that bring the PCDM "closest" possible to static equilibrium. We leave to later the definition of "closeness to static equilibrium".

In general, the static equilibrium conditions of a generic PCDM may be formulated as

$$\mathbf{W}(\mathbf{t}_M + \mathbf{t}_S) = \mathbf{w}_P, \tag{1}$$

where $\mathbf{W}$ is the cable-wrench matrix; $\mathbf{t}_M$ is the array of cable tensions due to the motors; $\mathbf{t}_S$ is the array of cable tensions due to their associated non-linear springs; and $\mathbf{w}_P$ is the external wrench applied onto the moving platform.

Here, we seek static balancing when no external wrench is applied and for no applied motor torques. However, since we can only approximate static balancing over the PCDM workspace, we have, in general, $\mathbf{W}\mathbf{t}_S \neq \mathbf{0}_n$ over the workspace.

Therefore, *closeness to equilibrium* at a given pose may be measured using some norm of the resultant wrench of the spring tensions on the moving platform. For PCDMs that generate both rotations and translations of their moving platforms, the norm has to include weights that take into account the dimensionally non-homogeneous nature of their associated wrenches.

### 2.1 Minimizing the Euclidean-norm of the resultant force $\mathbf{f}_r$

Here, we consider a two-dof planar mechanism that positions a point in the plane. Therefore, the associated wrenches are pure two-dimensional forces, and we can simply use the Euclidean-norm. Thus, our goal is to minimize the scalar $\mathbf{t}_S^T\mathbf{W}^T\mathbf{W}\mathbf{t}_S$ over the manipulator workspace. This last expression is dependent upon the end-effector pose $\mathbf{p}$. Hence, taking into account every possible pose requires the integration of this function over the workspace of the mechanism.

In order to achieve the force-minimization, we use non-linear springs at the winches $A_i$ of the PPCDM. Hence, we model the $i^{\text{th}}$ spring as

$$t_i = \sum_{j=1}^{k} s_{j-1} c_i^{k-j} = f_i(c_i) \tag{2}$$

where $t_i$ is the tension in cable $i$ due to the spring $i$, $s_{j-1}$ represents coefficients of polynomial and $c_i$ is the length of the $i^{\text{th}}$ cable. Hence, $t_i$ is given by a polynomial of degree $k-1$. For the sake of conciseness, we may rewrite eq. (2) as

$$t_i = \boldsymbol{\gamma}_i^T \mathbf{s}, \tag{3}$$

where $\boldsymbol{\gamma}_i \equiv [\ c_i^{k-1} \quad c_i^{k-2} \quad \cdots \quad 1\ ]^T$ and $\mathbf{s} \equiv [\ s_0 \quad s_1 \quad \cdots \quad s_{k-1}\ ]^T$.

Since the mechanism is symmetric about axes containing the heights of triangle $A_1A_2A_3$, we have

$$t(c) = t_1(c) = t_2(c) = t_3(c), \tag{4}$$

namely, we use the same spring for all winches. It is noted that the scalar $c$ in eq. (4) represents an arbitrary length of a cable. The resultant force exerted by the cables on the end-effector may be computed as

$$\mathbf{f}_r = t(c_1)\frac{\mathbf{c}_1}{c_1} + t(c_2)\frac{\mathbf{c}_2}{c_2} + t(c_3)\frac{\mathbf{c}_3}{c_3}. \tag{5}$$

In general, a function is said “convex” when it follows these conditions (see [16]):

$$f(x) \leq \theta f(x_1) + (1-\theta) f(x_2), \quad 0 \leq \theta \leq 1, \quad x \in [x_1, x_2]. \tag{6}$$

Since the square of a strictly positive convex function yields a convex function with the same minimum, we set out to minimize $||\mathbf{f}_r||_2^2$, the square of the Euclidean-norm of $\mathbf{f}_r$, over $\mathcal{A}$, and not $||\mathbf{f}_r||_2$ (which are convex functions):

$$||\mathbf{f}_r||_2^2 = \mathbf{f}_r^T\mathbf{f}_r = \left(\boldsymbol{\gamma}_1^T\mathbf{s}\frac{\mathbf{c}_1}{c_1} + \boldsymbol{\gamma}_2^T\mathbf{s}\frac{\mathbf{c}_2}{c_2} + \boldsymbol{\gamma}_3^T\mathbf{s}\frac{\mathbf{c}_3}{c_3}\right)^T \left(\boldsymbol{\gamma}_1^T\mathbf{s}\frac{\mathbf{c}_1}{c_1} + \boldsymbol{\gamma}_2^T\mathbf{s}\frac{\mathbf{c}_2}{c_2} + \boldsymbol{\gamma}_3^T\mathbf{s}\frac{\mathbf{c}_3}{c_3}\right), \tag{7}$$

$$||\mathbf{f}_r||_2^2 = \mathbf{s}^T\mathbf{C}^T\mathbf{C}\mathbf{s}, \tag{8}$$

where

$$\mathbf{C} \equiv \frac{\mathbf{c}_1}{c_1}\boldsymbol{\gamma}_1^T + \frac{\mathbf{c}_2}{c_2}\boldsymbol{\gamma}_2^T + \frac{\mathbf{c}_3}{c_3}\boldsymbol{\gamma}_3^T. \tag{9}$$

Since $c_i = \sqrt{(\mathbf{a}_i - \mathbf{p})^T(\mathbf{a}_i - \mathbf{p})}$, we may write $\mathbf{C} = \mathbf{C}(\mathbf{p})$.

Because we are to minimize the resultant-force magnitude over the manipulator workspace $\mathcal{A}$, we define the objective function as

$$f(\mathbf{s}) \equiv \frac{1}{2}\int_{\mathcal{A}} \mathbf{s}^T\mathbf{C}(\mathbf{p})^T\mathbf{C}(\mathbf{p})\mathbf{s}dA, \tag{10}$$

where $dA$ is an infinitely small element of area. This objective function is to be minimized over $\mathbf{s}$, our set of design parameters representing the polynomial coefficients of $t(c)$. However, because our first goal is to achieve a minimal level of tension in each cable when no external wrench is applied, we must submit $f(\mathbf{s})$ to additional constraints to ensure positive values.

## 2.2 The associated constraints

The use of cables instead of rigid links limits the forces that can be applied through each leg of the parallel manipulator. This phenomenon comes from the unilaterality of force transmission in cables: they can only pull on the moving platform. Moreover, since we want to limit sagging in cables, the tensions should always remain above a given threshold $t_{min}$, and this, for all cables at any pose of the moving platform within the workspace.

Since the foregoing constraint, $t_i > t_{min}$, applies to the joint force, it is more easily expressed over the range of joint displacement, rather than the moving-platform workspace. This gives the constraints

$$t_i(c_i) \geq t_{min},\ c_i \in [0, a],\ i = 1, 2, 3 \tag{11}$$

where $a$ is the maximum length of any cable over the workspace.

In this way, we avoid to evaluate, at each end-effector pose, the cable lengths to verify whether all tensions lie above the minimum value $t_{min}$. Moreover, since we use the same springs for all winches in the mechanism, it is possible to evaluate this constraint only for one cable.

Finally, we discretize the interval $[0, a]$ into $q$ constraints, that is, rather than verifying the constraint $t(c) \geq t_{min}$ over the whole interval, we verify it $q$ equally spaced cable lengths. The resulting set of constraints is written as

$$\mathbf{G}\mathbf{s} \geq \mathbf{1}_q t_{min} \tag{12}$$

where matrix $\mathbf{G}$ is a $q \times k$ Vandermonde matrix (with reversed order of its columns due to the application of the MATLAB® convention for polynomials), whose $q^{\text{th}}$ row is written as:

$$\boldsymbol{\gamma}_q \equiv [\ c_q^{k-1} \quad c_q^{k-2} \quad \cdots \quad c_q \quad 1\ ]^T, \tag{13}$$

$\mathbf{1}_q \equiv [\ 1 \quad 1 \quad \cdots \quad 1\ ]^T \in \mathbb{R}^q$, and $t_{min}$ is the minimum allowed tension in all cables over the entire workspace.

Equation (12) is linear regardless of the number $q$ of different evaluated lengths of the cable of maximal length $a$. Moreover, notice that the first row of $\mathbf{G}$ is $\boldsymbol{\gamma}_1 = [\ 0 \quad 0 \quad \cdots \quad 0 \quad 1\ ]^T$, at the value $c = 0$, because we consider the positive part of the null value ($(0^+)^0 = 1$).

Notice also that these constraints act only at discrete cable-lengths. Therefore, in theory, they do not constrain the tensions above $t_{min}$ over the interval $[0, a]$. In practice, however, the solutions they allow fulfill the inequality over the whole interval $[0, a]$, provided $q >> k$, i.e., provided we have many more constraints than the degree of $t(c)$.

### 2.3 The Resulting Quadratic Program

We have now defined an objective function and its associated constraints that allow approximate static balancing of the PPCDM while ensuring a minimum level of tension in its cables.

Since $\mathbf{s}$ is constant over the manipulator workspace, it can be factored out of the integral in eq. (10). As a result, eq. (10) is a quadratic function of $\mathbf{s}$, which we may rewrite as

$$f(\mathbf{s}) \equiv \frac{1}{2}\mathbf{s}^T\mathbf{P}\mathbf{s}, \tag{14}$$

where

$$\mathbf{P} \equiv \int_{\mathcal{A}} \mathbf{C}^T\mathbf{C}dA. \tag{15}$$

In our case, the workspace is triangular, and eq. (15) can be written as

$$\mathbf{P} \equiv 6\int_0^{\frac{a}{2}} \int_{-\frac{\sqrt{3}a}{6}}^{-\frac{\sqrt{3}x}{3}} \mathbf{C}^T\mathbf{C}dydx. \tag{16}$$

In summary, we have defined the objective function (eq. (14)) which allows us to approach static balancing of the PPCDM over its workspace, and the associated constraints (eq. (12)) which maintain the tension in each cable above the minimum at anytime:

$$\min_{\mathbf{s}} \ \tfrac{1}{2}\mathbf{s}^T\mathbf{P}\mathbf{s} \quad \text{subject to} \quad -\mathbf{G}\mathbf{s} \leq -\mathbf{1}_q t_{min}. \tag{17}$$

Apparently, the objective function is quadratic, the inequality constraints are affine and convex, while there are no equality constraints. As a result, the optimization problem of eq. (17) is a quadratic program (QP) [16], which is well-known to be a convex problem and provided that $\mathbf{P}$ is symmetric positive-definite.

In general, a convex optimization problem is one of the form:

$$\min_{\mathbf{x}} \ f_0(\mathbf{x}) \quad \text{subject to} \quad f_i(\mathbf{x}) \leq 0,\ i = 1, \ldots, n \quad \text{and} \quad \mathbf{A}\mathbf{x} = \mathbf{b} \tag{18}$$

where $f_0, \ldots, f_n$ must be convex functions and the equality constraint functions must be affine. Our optimization problem follows all these conditions, thus it can be defined as a convex optimization problem.

Moreover, from eq. (16), it is apparent that $\mathbf{P}$ is symmetric positive semidefinite. Showing that this matrix is symmetric positive definite in general is more difficult, and remains to be done. Nevertheless, observations show that it is symmetric positive definite for $1 \leq k \leq 10$, thus making the

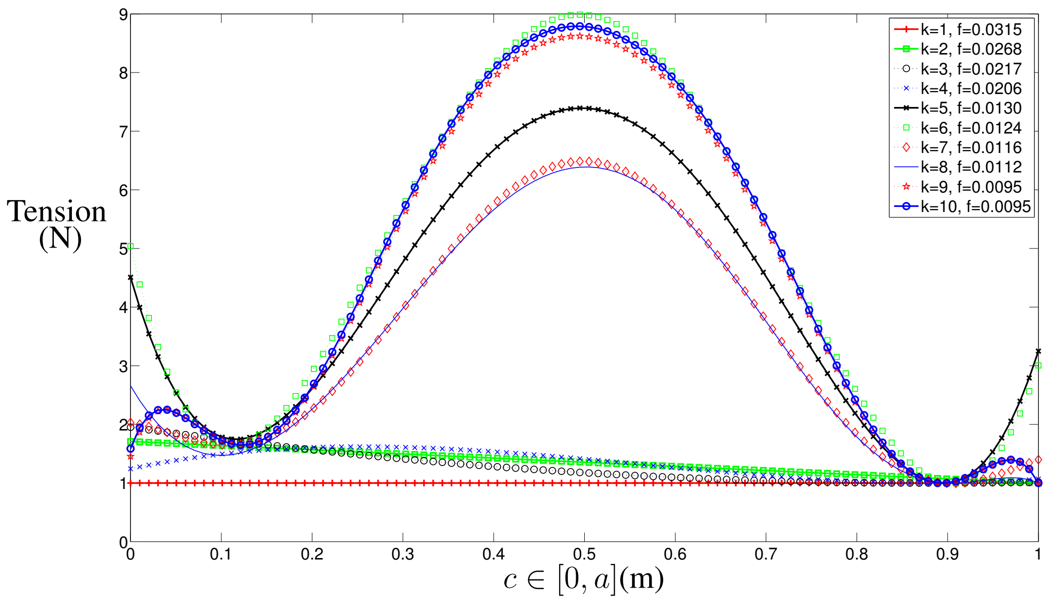


Figure 2: Tension in the cables in function of $k-1$ order of springs function.

optimization problem a quadratic program in these cases. In the next section, the quadratic program of eq. (17) is solved, using the *quadprog* command which pertains to the standard MATLAB® optimization toolbox, to determine the optimum non-linear spring function of degree $(k-1)$.

## 3 OPTIMIZATION RESULTS

Since our problem is convex [16], we can readily find the global solution $\mathbf{s}^*$ associated with a given degree $k-1$ of the non-linear spring function. Furthermore, let us minimize the objective function for degrees varying from $k = 1$ to $k = 10$, define the distance $a = 1$ meter and constrain the tension above $t_{min} = 1$ Newton for a number $q = 10k$ of cable lengths. This yields the results shown in Fig. 2.

### 3.1 Effect of the degree $k-1$ of the polynomial

This figure shows the optimum cable tension distributions corresponding to each degree $(k-1)$ of the spring behavior as a function of the cable length. The legend of the graph of Fig. 2 also includes the corresponding minimum values of the objective function. As expected, we notice that the minimum value of $f$ decreases as the degree of the spring polynomial function increases. Moreover, the constraints are always verified, since all tension distributions lie above $t = 1$ N. This fact confirms that using $q = 10k$ cable lengths to constrain the level of tension is sufficient.

Notice that maintaining a minimum level of tensions in the cables can be achieved with linear springs. Indeed, it is possible to design PPCDM using constant-tension springs ($k = 1$) (i.e., springs similar to those used in a carpenter tape) or linear springs ($k = 2$). However, these choices do not yield good results when it comes to approximating static balancing of the PPCDM over the whole workspace. Figures 3 and 4 show the magnitude of the resultant force applied on the end effector over the workspace $\mathcal{A}$, for a zero-order spring function and a first-order spring func-

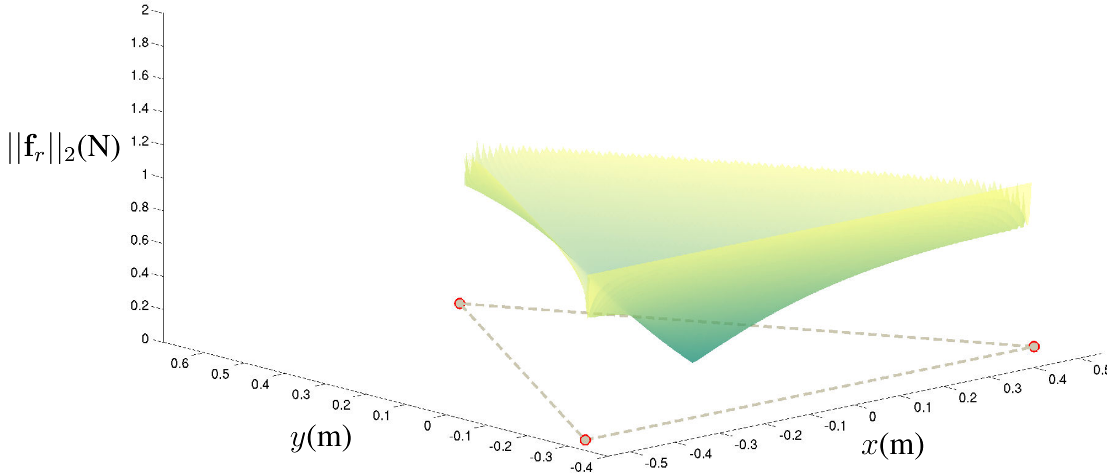


Figure 3: $||\mathbf{f}_r||_2$ over the workspace for a zero-order spring function ($k = 1$).

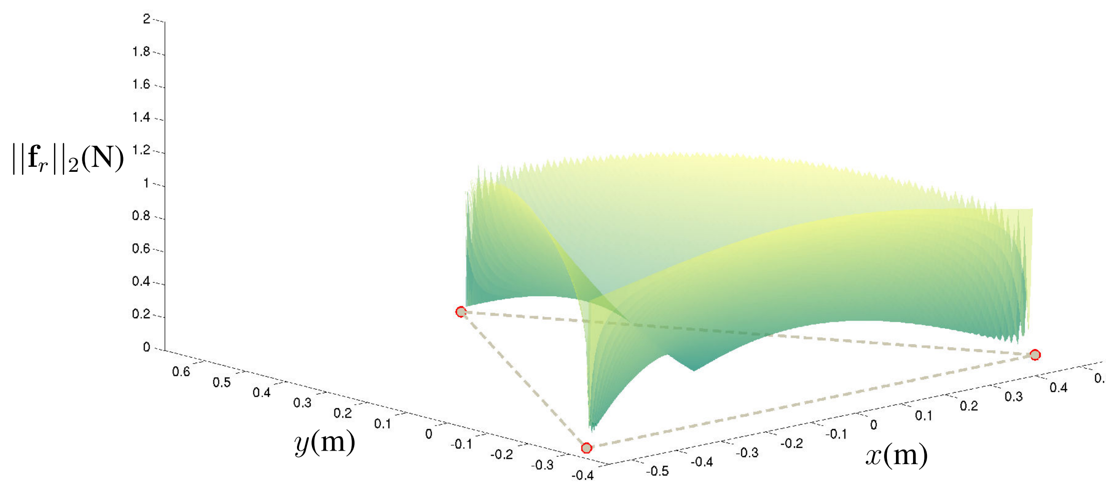


Figure 4: $||\mathbf{f}_r||_2$ over the workspace for a first-order spring function ($k = 2$).

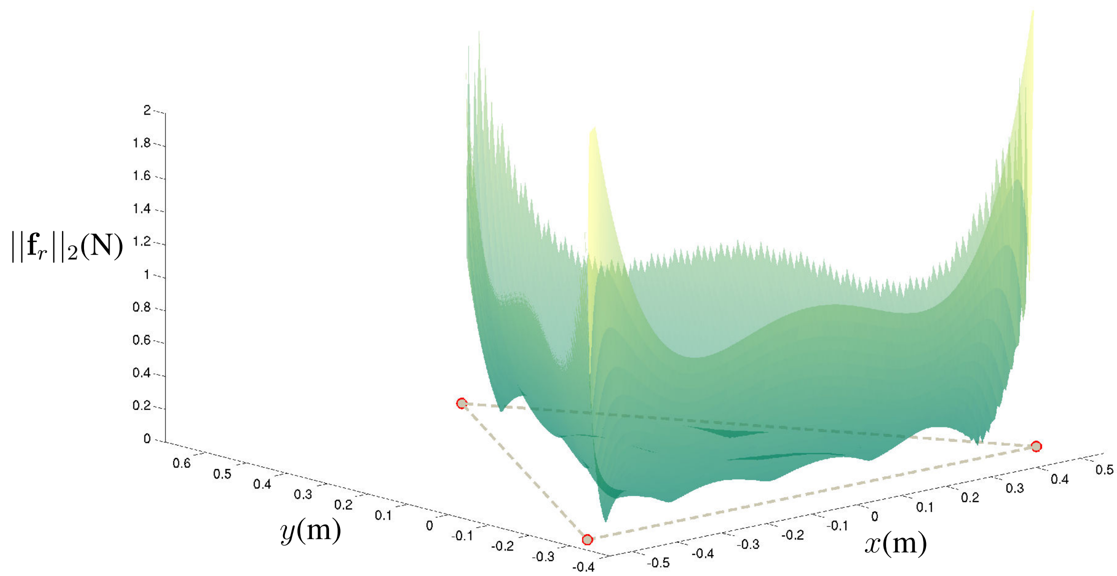


Figure 5: $||\mathbf{f}_r||_2$ over the workspace for a fourth-order spring function ($k = 5$).

tion, respectively. Hence, for these springs, the end-effector always tends to go back towards an equilibrium posture corresponding to the centroid of triangle $A_1A_2A_3$. The motors still have to provide additional energy in order to maintain an arbitrary pose in the workspace, even though the desired minimum tension in the cables is ensured by the addition of springs. In Fig. 3, only one equilibrium point ($||\mathbf{f}_r||_2 = 0$) is found inside the entire workspace. Increasing the degree of the spring function by one leads to four approximate equilibrium points, as can be seen in Fig. 4.

As could be expected, Fig. 5, which corresponds to $k = 5$, exhibits a better approximate static balancing of the PPCDM over its workspace. Indeed, the norm of the resultant force is closer to zero over a considerable portion of $\mathcal{A}$, and becomes larger near its boundaries. Thus, a fourth-order spring function could be suitable to achieve our goals since, in general, the displacements of the end-effector are far enough from the boundaries of the workspace. Nevertheless, we notice that the Euclidean-norm resultant forces close to the winches $A_i$, $i = 1, 2, 3$, are relatively high in comparison with $k = 1, 2$. Maintaining these positions could turn out to be demanding from the motors.

Finally, Fig. 6 presents the best approximate static balancing obtained in this work. However, considering the high order of the spring function and the light improvement of the objective function over the fourth-order spring function, one may well prefer to use a smaller degree of the spring function. Moreover, even though the last result provides the best minimization of $||\mathbf{f}_r||_2$ over $\mathcal{A}$, the gain between the functions with $k = 4$ and $k = 5$ is more advantageous ($f_{k=5} - f_{k=4} = -0.0076$) than that between the spring functions with $k = 5$ and $k = 10$ ($f_{k=10} - f_{k=5} = -0.0035$).

Notice that we use a threshold value of $t_{min} = 1$ Newton to perform our QP optimizations. Any other positive value $t_{min}$ could have been used, which would have led to very similar results.

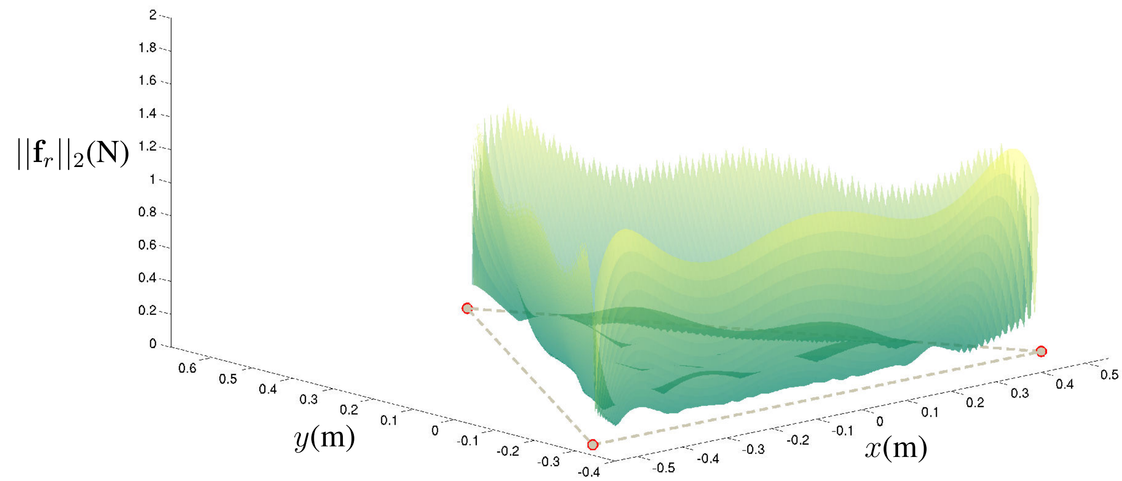


Figure 6: $||\mathbf{f}_r||_2$ over the workspace for a ninth-order spring function ($k = 10$).

## 4 Conclusions

This paper proposes the use of non-linear springs in a PCDM to maintain its cables taut and to approximate its static balancing over its entire workspace. The aim is to decrease the power required from the electrical motors to perform arbitrary displacements of the moving platform.

The definition of the mathematical problem was first presented for a two-dof PPCDM. This problem includes the minimization of the magnitude of the resultant force $\mathbf{f}_r$ exerted on the moving platform over the whole workspace when no external wrench is applied while maintaining a minimum level of tension in the cables. These requirements led to the development of a quadratic programming that can easily be solved using readily available techniques.

Thence, the QP problem was solved for s for degrees of the non-linear springs function ranging from $0$ to $9$. After analyzing the results for $t_{min} = 1$ Newton, a fourth-degree polynomial was decided to be a suitable compromise between the order of the spring function and the objective-function minimization.

Finally, in order to implement these theoretical results to approximate the static balancing of a PPCDM, future works will be directed towards the determination of a suitable mechanical system to reproduce the non-linear spring function obtained. To this end, we will evaluate applicability of the method presented by Ulrich and Kumar [6], who used specific cam shapes and linear springs to statically balance a two-dof planar serial robot. Although the baselines of their works are different—they realize exact static balancing with no constraints on the spring force—arrangements of cams and linear springs could be appropriate for our approach as well. Another means of producing the desired spring functions would be to combine a linear torsion spring with a linkage. These concepts and their implementation will be the subject of further reports.

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