Motion Recovery after Joint Failure in Parallel Manipulators

Leila Notash

Department of Mechanical and Materials Engineering, Queen's University, Kingston, ON, notash@me.queensu.ca

Abstract
In this paper, the failure of parallel manipulators is examined. The failure modes of manipulators are discussed. Methodologies for investigating the effect of joint failures on the motion performance of manipulators are presented, and the criteria for full and partial recovery from these failures are established. The proposed methodology is applied and simulated for planar parallel manipulators as a case study.

Keywords: Parallel manipulators; failure analysis; motion recovery

1 Introduction

In parallel manipulators the mobile platform (end effector) is connected to the base by several legs/branches/wires, Figure 1. Considering the actuation, parallel manipulators could be categorized as solid-link manipulators and wire-actuated manipulators. In the solid-link parallel manipulators, the legs consist of the kinematic chains of links with actuated (active) and passive joints. In the wire/cable-actuated manipulators (also referred to as the wire-suspended or cable-driven manipulators), the legs are replaced by wires and the motion of mobile platform is controlled by changing the length of wires.

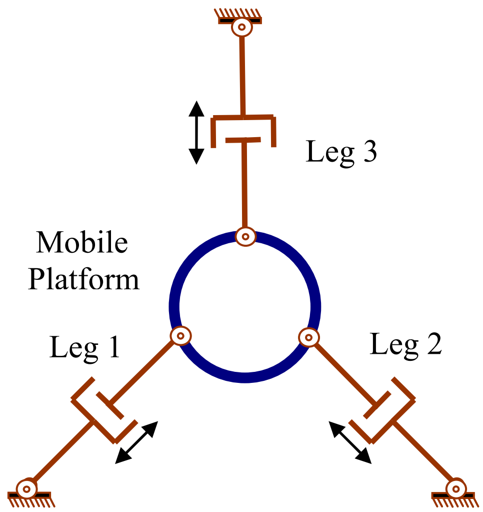

Figure 1. Planar parallel manipulator.

Parallel manipulators could be designed for high load capacity and dynamic characteristics; and low mass, cost and power consumption. Therefore, their potential applications include both the terrestrial applications, such as manufacturing, entertainment, medical and service sectors; and the space applications. For some of these applications, fail-safe manipulators are crucial, e.g., when the device is used in surgery or in high speed operation. For tasks in hazardous environments and space/remote operations, human access to the manipulator could be very difficult, dangerous or impossible, while in some applications the downtime needs to be minimized.

Failure analysis of serial manipulators has received more attention compared to parallel manipulators. A procedure for minimizing the jump in the norm of joint velocity vector of serial manipulators after joint failure was presented in [1]. In [2] the relative manipulativity index was used to investigate the Jacobian matrices of manipulators fault tolerant to joint failures. In [3] the failure mode and effect analysis was performed to study the failure modes of parallel manipulators with their effects on the degree of freedom (DOF), actuation and constraint. Redundancy types, such as redundant DOF, redundant sensing, redundant actuation and redundant legs/branches, have been suggested for fault tolerant designs. The effect of redundancy in joint displacement sensing for parallel manipulators has been investigated to reduce the number of forward displacement solutions/assembly modes [4-6]; to allow the fixtureless calibration of manipulators [7]; and to facilitate the joint sensor fault detection, isolation and recovery [8]. Redundancy in actuation has been proposed to reduce the uncertainty/singularity configurations of parallel manipulators [6, 9]. In [10] the task space was partitioned into major and secondary tasks in order to complete the major task and optimize a secondary goal such as actuator fault tolerance. The reduced motion of parallel manipulators due to active joint jam and the design modification to compensate for the accuracy degradation were investigated in [11, 12]. In [13] the effect of the active joint failures on the force/moment capabilities of parallel manipulators was investigated. The method for recovering the lost force/moment of the mobile platform was based on the projection of the lost joint force/torque onto the orthogonal complement of the null space of the transposed Jacobian matrix after failure. Taking into account the different failure modes of parallel manipulators, methodologies for the fault tolerance of these manipulators are required to compensate for their performance degradation after each failure type.

In this article, the failure of parallel manipulators is studied. Failure modes of parallel manipulators are discussed in Section 2. A methodology for recovering the lost motion due to the failure of joints/actuators is presented in Section 3. The procedure is based on the methodology proposed in [13] for recovering the lost joint force/moment because of the duality between the force/moment and motion characteristics of parallel and serial manipulators. The kinematics of planar parallel manipulators and simulation results for the loss of joint motion are reported in Section 4. The article concludes with Section 5.

2 Manipulator Failure Modes

In parallel manipulators, the mobile platform is connected to the base by a number of legs, e.g., refer to Figure 1. In general, each leg is a kinematic chain of links connected by joints. Because of the closed-loop configuration, not all of the joints of parallel manipulators are actuated, i.e., some of the joints are passive. For non-redundant actuation, considering the one degree of freedom joints such as revolute or prismatic joints, the number of active joints is equal to the DOF of the manipulator (Figure 1, with active prismatic joints). To form a kinematically redundant leg, one or more redundant active joints could be added to the leg. In this case, the number of actuators would be larger than the dimension of task space, e.g., refer to Figures 2(a) and 2(b) which respectively depict manipulators with kinematic redundancy in each leg and a combination of redundant leg and kinematic redundancy. It is also possible to form redundantly actuated parallel manipulators by actuating one or more passive joints, e.g. refer to Figure 2(c).

Parallel manipulators could fail because of the failure of their components (e.g., links, active joints and passive joints), subsystems (legs/branches and end effector) and systems (mechanical, electrical, software and controller). If any of these failures affect the performance of manipulator such that the task cannot be completed as desired, then the manipulator is considered failed. Considering the mechanical system, parallel manipulators could fail because of the failure of a link (link breakage or undesired flexibility of link) and/or failure of a joint (joint breakage, joint jam, sensor failure, actuator failure and transmission failure). These failures could result in the loss of DOF, loss of actuation, and loss of motion constraint; in addition to loss of information, please refer to [3] for detailed discussion.

From the kinematics point of view, the failure of a joint occurs if the joint is broken, if the joint is jammed (its displacement remains constant), or if the displacement/velocity/acceleration of joint is not at the desired level. When a joint breaks the manipulator loses the corresponding leg and the constraints imposed by that leg. When the back-drivable active joint of a leg fails and reduces to a passive joint, the mobile platform loses a constraint unless the leg is redundantly actuated such as the manipulator of Figure 2(c). Because of the closed-loop configuration of parallel manipulators, the change in the motion of any joints (active or passive) would alter the motion capability of these manipulators.

3 Recovering Lost Velocity

For parallel manipulators, the relation between the $n\times 1$ active joint velocity vector, $\dot{\mathbf{q}}$, and the $m\times 1$ mobile platform twist (velocity vector), $\mathbf{V}$, is given as

$$\dot{\mathbf{q}} = -\mathbf{J}\mathbf{V} \tag{1}$$

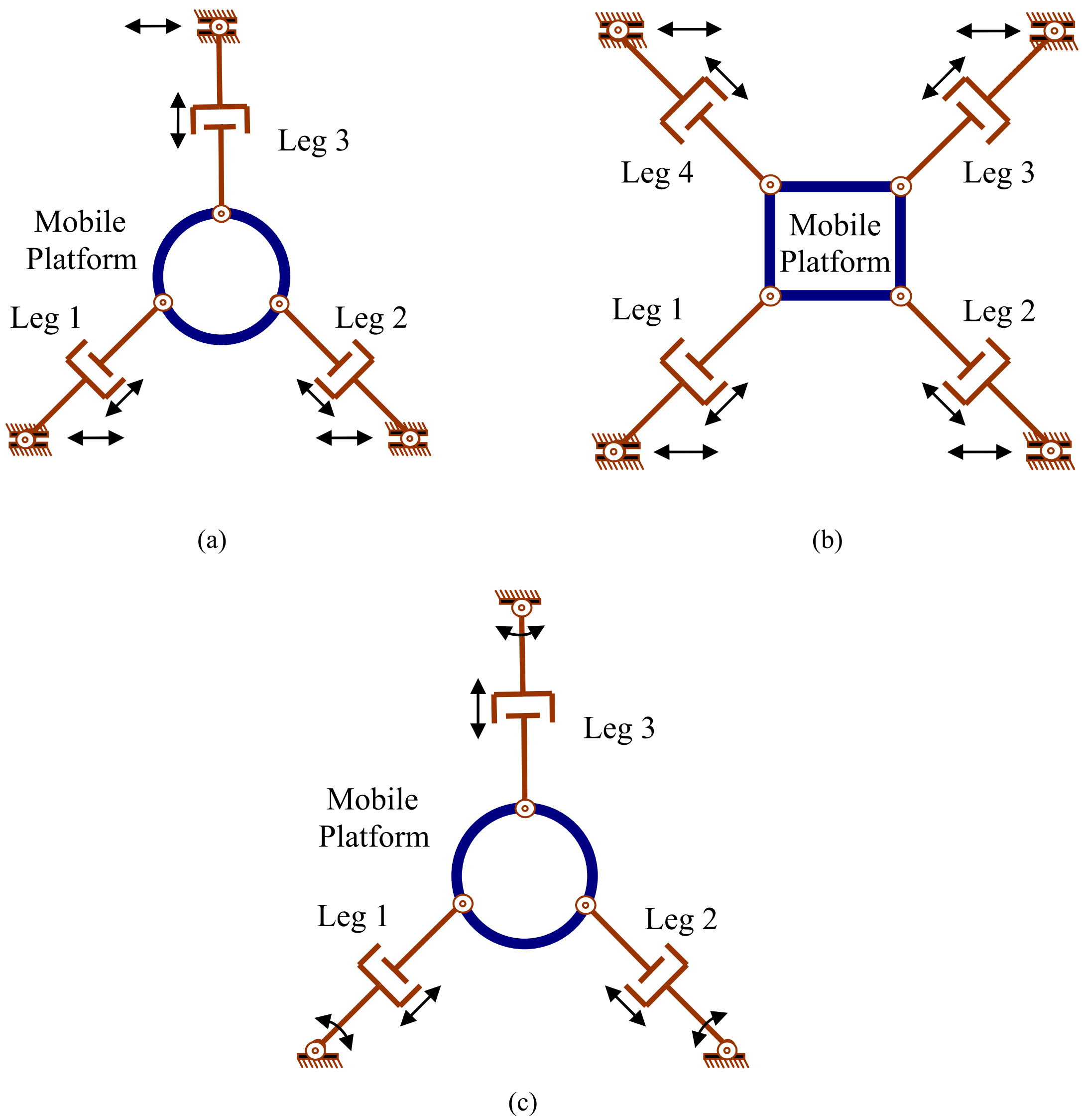

Figure 2. Planar parallel manipulators: (a) redundant active joint; (b) redundant leg and redundant active joint; and (c) actuating passive joints.

where $m \leq 6$ depending on the dimension of task space, e.g., $m = 3$ for planar motion. The Jacobian matrix of manipulator, **J**, is an $n \times m$ matrix; with $n = m$ for non-redundant manipulators and $n > m$ for redundant manipulators. The twist of mobile platform, **V**, is formulated for a given rate of change of the active joint displacements, $\dot{\mathbf{q}}$, using the generalized inverse of **J**, $\mathbf{J}^{\#}$, as

$$\mathbf{V} = -\mathbf{J}^{\#}\dot{\mathbf{q}} \tag{2}$$

Hence, to provide the required platform twist, **V** should belong to the range space of $\mathbf{J}^{\#}$. In addition, each leg of manipulator should allow the mobile platform twist **V**.

Considering leg i, the $l \times 1$ velocity vector of its active and passive joints, ${}^{i}\dot{\mathbf{q}} = {}^{i}[\dot{q}_1\ \dot{q}_2\ \dots\ \dot{q}_{l-1}\ \dot{q}_l]^T$, and the platform velocity vector, **V**, are related by the $m \times l$ Jacobian matrix of the leg, ${}^{i}\mathbf{J}$, as

$$\mathbf{V} = {}^i\mathbf{J}\,{}^i\dot{\mathbf{q}} = \left[{}^i\mathbf{J}_1\ {}^i\mathbf{J}_2 \cdots {}^i\mathbf{J}_h \cdots {}^i\mathbf{J}_{l-1}\ {}^i\mathbf{J}_l\right]{}^i\dot{\mathbf{q}} = \sum_{k=1}^{l} {}^i\mathbf{J}_k\ {}^i\dot{q}_k \qquad (3)$$

where each column of ${}^i\mathbf{J}$, ${}^i\mathbf{J}_k$, is a screw representing the axis of the corresponding joint of leg i; and $l \geq m$. Therefore, to provide the mobile platform velocity, $\mathbf{V}$ should be in the range space of all ${}^i\mathbf{J}$, for i = 1, …, n_l, where n_l is the number of legs.

Because of the closed-loop configuration of parallel manipulators, the constraints imposed by the legs would limit the motion capability of these manipulators as a result of the failure of a joint (active or passive). When joint h is failed its motion would be different than the desired value. The failure of a joint could be because of jamming (zero velocity and acceleration) or because of having a different motion than the desired value (different velocity and acceleration). These will be discussed in the following subsection.

3.1 Different Joint Velocity

When joint h (active or passive) on leg i is failed its velocity (rotational velocity for revolute joints and translational velocity for prismatic joints) ${}^i\dot{q}_{ch}$ will be different (instantaneously or permanently) than the desired value ${}^i\dot{q}_h$. If joint h is jammed its displacement will remain constant and the joint velocity will reduce to zero, ${}^i\dot{q}_{ch} = 0$. If joint h has constant velocity, lower or higher velocity than is required then ${}^i\dot{q}_{ch} \neq 0$. The failed joint will not result in the failure of manipulator if ${}^i\dot{q}_{ch}$ does not affect the motion of mobile platform.

When the velocity of joint h, ${}^i\dot{q}_{ch}$, is different than the required value ${}^i\dot{q}_h$ the velocity equation for leg i is

$$\mathbf{V}_f = {}^i\mathbf{J}\,{}^i\dot{\mathbf{q}}_f = \sum_{k=1}^{l} {}^i\mathbf{J}_k\ {}^i\dot{q}_k - {}^i\mathbf{J}_h({}^i\dot{q}_h - {}^i\dot{q}_{ch}) \qquad (4)$$

with ${}^i\dot{\mathbf{q}}_f = {}^i[\dot{q}_1\ \dot{q}_2 \ldots \dot{q}_{ch} \ldots \dot{q}_{l-1}\ \dot{q}_l]^T$. The change in the velocity of mobile platform will be

$$\Delta\mathbf{V}_f = \mathbf{V} - \mathbf{V}_f = {}^i\mathbf{J}({}^i\dot{\mathbf{q}} - {}^i\dot{\mathbf{q}}_f) = {}^i\mathbf{J}_h({}^i\dot{q}_h - {}^i\dot{q}_{ch}) \qquad (5)$$

where ${}^i\dot{\mathbf{q}} - {}^i\dot{\mathbf{q}}_f = {}^i[0\,0 \ldots (\dot{q}_h - \dot{q}_{ch}) \ldots 0\,0]^T$ is the lost motion due to failure of joint h. If at this pose the required velocity from the failed joint h is ${}^i\dot{q}_{ch}$, i.e., ${}^i\dot{q}_h = {}^i\dot{q}_{ch}$, there will be no change in the velocity of the manipulator. However, if the required velocity of joint h is different, i.e., ${}^i\dot{q}_h \neq {}^i\dot{q}_{ch}$, then the manipulator would be considered as failed unless the leg has a redundant joint to provide the lost motion ${}^i\mathbf{J}_h({}^i\dot{q}_h - {}^i\dot{q}_{ch})$ due to failure of joint h.

The jamming of a joint (${}^i\dot{q}_{ch} = 0$) decreases the DOF of the corresponding leg, and hence, the degree of freedom of the manipulator will be reduced if leg i does not have redundant DOF. Similarly, when the velocity of a joint is different than the desired value or has reached the limit the performance of manipulator will be degraded unless the leg has a redundant DOF. The full recovery is feasible only if the lost motion of the mobile platform is in the range space of the Jacobian matrix corresponding to the remaining (healthy) joints of that leg. Therefore, in general, to allow for the full recovery of lost motion, the leg with a failed joint should have a redundant joint. If the failed joint is an active joint the redundant joint could be an active one or could be kept locked prior to the joint failure and activated after the failure. If the jammed joint is a passive joint, similar to the active joint failure case, for the full recovery of lost motion, a redundant active joint on that leg will be required. To fully control the motion of leg, in general, a redundant passive joint would be acceptable if it could be locked prior to failure.

3.1.1 Correctional input from healthy joints

When joint h has a different velocity, the correctional velocity $\Delta^i\dot{\mathbf{q}}_{corr} = {}^i[\Delta\dot{q}_1\ \Delta\dot{q}_2\ \ldots 0 \ldots \Delta\dot{q}_{l-1}\ \Delta\dot{q}_l]^T$, to be provided by the remaining joints of leg i, will compensate for the lost twist partially or completely, where in $\Delta^i\dot{\mathbf{q}}_{corr}$ entry h is replaced by a zero. Then, the recovered velocity of the platform will be

$$\mathbf{V}_r = {}^i\mathbf{J}\,{}^i\dot{\mathbf{q}}_f + {}^i\mathbf{J}\Delta^i\dot{\mathbf{q}}_{corr} = {}^i\mathbf{J}\,{}^i\dot{\mathbf{q}}_f + {}^i\mathbf{J}_f\,\Delta^i\dot{\mathbf{q}}_{corr} \tag{6}$$

where in the reduced Jacobian matrix of leg i, ${}^i\mathbf{J}_f$, column h is replaced by a zero vector

$${}^i\mathbf{J}_f = \left[{}^i\mathbf{J}_1\ {}^i\mathbf{J}_2 \cdots \mathbf{0} \cdots {}^i\mathbf{J}_{l-1}\ {}^i\mathbf{J}_l\right] \tag{7}$$

The change in the platform velocity after applying the correctional velocity will be

$$\Delta\mathbf{V}_r = \mathbf{V} - \mathbf{V}_r = {}^i\mathbf{J}({}^i\dot{\mathbf{q}} - {}^i\dot{\mathbf{q}}_f) - {}^i\mathbf{J}_f\,\Delta^i\dot{\mathbf{q}}_{corr} \tag{8}$$

To fully recover the lost twist, $\Delta\mathbf{V}_r = \mathbf{0}$, the correctional velocity from the healthy joints will be

$$\Delta^i\dot{\mathbf{q}}_{corr} = {}^i\mathbf{J}_f^{\#}\,{}^i\mathbf{J}_h({}^i\dot{q}_h - {}^i\dot{q}_{ch}) = {}^i\mathbf{J}_f^{\#}\,{}^i\mathbf{J}({}^i\dot{\mathbf{q}} - {}^i\dot{\mathbf{q}}_f) \tag{9}$$

The generalized inverse of ${}^i\mathbf{J}_f$ is

$${}^i\mathbf{J}_f^{\#} = {}^i\mathbf{J}_f^T\left({}^i\mathbf{J}_f\,{}^i\mathbf{J}_f^T\right)^{-1} \tag{10}$$

if ${}^i\mathbf{J}_f$ has full row-rank, i.e., $\mathbf{V}$ belongs to the range space of ${}^i\mathbf{J}_f$, $\mathbf{V} \in \Re({}^i\mathbf{J}_f)$, and the vector of joint velocities for leg i is physically consistent (all entries have the same dimension, e.g., radians/second). Then

$$\Delta\mathbf{V}_r = \mathbf{V} - \mathbf{V}_r = \left(\mathbf{I} - {}^i\mathbf{J}_f\,{}^i\mathbf{J}_f^{\#}\right){}^i\mathbf{J}({}^i\dot{\mathbf{q}} - {}^i\dot{\mathbf{q}}_f) = \mathbf{0} \tag{11}$$

If the vector of joint velocities is not physically consistent, e.g., leg i has a combination of revolute and prismatic joints, when ${}^i\mathbf{J}_f$ has full row-rank a weighting metric would be required for calculating the generalized inverse of ${}^i\mathbf{J}_f$ [14]. Then

$${}^i\mathbf{J}_f^{\#} = \mathbf{W}_{\dot{q}}\,{}^i\mathbf{J}_f^T\left({}^i\mathbf{J}_f\,\mathbf{W}_{\dot{q}}\,{}^i\mathbf{J}_f^T\right)^{-1} \tag{12}$$

The weighting metric $\mathbf{W}_{\dot{q}}$ is chosen such that ${}^i\dot{\mathbf{q}}^T(\mathbf{W}_{\dot{q}}^{-1}\,{}^i\dot{\mathbf{q}})$ becomes physically consistent, e.g., to minimize/maximize the kinetic energy of leg i.

When ${}^i\mathbf{J}_f$ does not have full row-rank, ${}^i\mathbf{J}_f^{\#}$ and ${}^i\mathbf{J}_f\,{}^i\mathbf{J}_f^{\#}$ may be calculated after removing the zero columns of ${}^i\mathbf{J}_f$. Then, $\Delta^i\dot{\mathbf{q}}_{corr}$ calculated using equation (9) would correspond to the healthy joints only.

3.1.2 Multi-joint failure

The method could be easily extended to the case that the velocities of g joints of leg i are different than the required values. In this case, g columns of ${}^i\mathbf{J}_f$, corresponding to the joints with different velocities, will be zero. The lost platform twist will be $\sum {}^i\mathbf{J}_h({}^i\dot{q}_h - {}^i\dot{q}_{ch}) = {}^i\mathbf{J}({}^i\dot{\mathbf{q}} - {}^i\dot{\mathbf{q}}_f)$ and the correctional velocity will be

$$\Delta^i\dot{\mathbf{q}}_{corr} = {}^i\mathbf{J}_f^{\#}\sum {}^i\mathbf{J}_h({}^i\dot{q}_h - {}^i\dot{q}_{ch}) = {}^i\mathbf{J}_f^{\#}\,{}^i\mathbf{J}({}^i\dot{\mathbf{q}} - {}^i\dot{\mathbf{q}}_f) \tag{13}$$

where the summation is taken over the failed joints. The mobile platform velocity corresponding to the correctional velocity will be zero when $\sum {}^i\mathbf{J}_h({}^i\dot{q}_h - {}^i\dot{q}_{ch}) = \mathbf{0}$, i.e., when the combined motion of these g

joints does not affect the motion of platform (self-motion of leg i), or when $\sum {}^i\mathbf{J}_h({}^i\dot{q}_h - {}^i\dot{q}_{ch})$ is in the null space of ${}^i\mathbf{J}_f^{\#}$ which is the same as in the null space of ${}^i\mathbf{J}_f\,{}^i\mathbf{J}_f^{\#}$.

3.1.3 Criteria for full and partial recovery

The deviation in the platform velocity after applying the correctional velocity from the healthy joints of leg i will be zero, i.e., $\Delta\mathbf{V}_r = \left(\mathbf{I} - {}^i\mathbf{J}_f\,{}^i\mathbf{J}_f^{\#}\right){}^i\mathbf{J}({}^i\dot{\mathbf{q}} - {}^i\dot{\mathbf{q}}_f) = \mathbf{0}$, when $\mathbf{V}$ belongs to the range space of ${}^i\mathbf{J}_f$, $\mathbf{V} \in \Re({}^i\mathbf{J}_f)$. The lost motion that cannot be retrieved could be characterized considering the range space of $\mathbf{I} - {}^i\mathbf{J}_f\,{}^i\mathbf{J}_f^{\#}$. Hence, the condition for partial recovery of the lost motion after the failure of joint h, (or failure of g joints), i.e., when $\Delta\mathbf{V}_f$ belongs to the orthogonal complement of the range space of ${}^i\mathbf{J}_f$, $\Delta\mathbf{V}_f \in \Re({}^i\mathbf{J}_f)^{\perp}$, is

$$\mathbf{V}_{\Re^{\perp}} = \left(\mathbf{I} - {}^i\mathbf{J}_f\ {}^i\mathbf{J}_f^{\#}\right)\mathbf{V} \neq \mathbf{0} \tag{14}$$

When one or more entries of $\mathbf{V}_{\Re^{\perp}}$ are zero the corresponding components of the mobile platform twist could be completely recovered.

For the lost motion to be entirely recovered, all the components of the twist $\mathbf{V} = {}^i\mathbf{J}\,{}^i\dot{\mathbf{q}}$ projected onto the orthogonal complement of the range space of ${}^i\mathbf{J}_f$ should be zero. That is, the condition for full recovery of the lost motion is

$$\mathbf{V}_{\Re^{\perp}} = \left(\mathbf{I} - {}^i\mathbf{J}_f\ {}^i\mathbf{J}_f^{\#}\right)\mathbf{V} = \mathbf{0} \tag{15}$$

provided that the overall joint velocities of leg i ${}^i\dot{\mathbf{q}}_f + \Delta{}^i\dot{\mathbf{q}}_{corr}$ will not surpass the limit of the remaining joints. Otherwise, the procedure could be repeated for the joint corresponding to the entry of ${}^i\dot{\mathbf{q}}_f + \Delta{}^i\dot{\mathbf{q}}_{corr}$ that reaches/exceeds the limit.

If the velocity of the remaining joints of leg i cannot be changed the error in the platform velocity will be calculated as

$$\Delta\mathbf{V}_f = {}^i\mathbf{J}_f\ {}^i\mathbf{J}_f^{\#} \sum {}^i\mathbf{J}_h({}^i\dot{q}_h - {}^i\dot{q}_{ch}) \tag{16}$$

3.2 Leg singularities

When a leg of parallel manipulator is at a singularity configuration the motion provided by the joints will become linearly dependent and the leg, and hence the platform, loses a degree of freedom (gains a constraint). At singularity, the rank of the $m \times l$ Jacobian matrix of the leg is less than the dimension of task space m, i.e., ${}^i\mathbf{J} = \left[{}^i\mathbf{J}_1\ {}^i\mathbf{J}_2 \cdots {}^i\mathbf{J}_h \cdots {}^i\mathbf{J}_{l-1}\ {}^i\mathbf{J}_l\right]$ does not have full row-rank.

While at a singular configuration of leg i, if the mobile platform twist projected onto the orthogonal complement of the range space of ${}^i\mathbf{J}$ results in a zero vector, i.e., $\mathbf{V}_{\Re^{\perp}} = \left(\mathbf{I} - {}^i\mathbf{J}\,{}^i\mathbf{J}^{\#}\right)\mathbf{V} = \mathbf{0}$, twist $\mathbf{V}$ could be provided by the platform. Otherwise, when $\mathbf{V}_{\Re^{\perp}} = \left(\mathbf{I} - {}^i\mathbf{J}\,{}^i\mathbf{J}^{\#}\right)\mathbf{V} \neq \mathbf{0}$ the platform cannot have the required velocity and is considered failed even though no joint is failed. Similarly, if one or more joints fail while the leg is at a singularity, depending on whether the dependent joints are the healthy ones, the non-zero columns of ${}^i\mathbf{J}_f$ may be linearly dependent, and as long as $\mathbf{V}_{\Re^{\perp}} = \left(\mathbf{I} - {}^i\mathbf{J}_f\ {}^i\mathbf{J}_f^{\#}\right)\mathbf{V} = \mathbf{0}$, the platform will have twist $\mathbf{V}$. ${}^i\mathbf{J}\,{}^i\mathbf{J}^{\#}$ (and ${}^i\mathbf{J}_f\ {}^i\mathbf{J}_f^{\#}$) may be calculated using the singular value decomposition of ${}^i\mathbf{J}$.

3.3 Implementation

To apply the proposed methodology for fault tolerance, at each pose, the mobile platform velocity $\mathbf{V}$ should be monitored to identify if the platform motion could be entirely provided by each leg i of the manipulator using $\mathbf{V}_{\Re^\perp} = \left(\mathbf{I} - {}^i\mathbf{J}\,{}^i\mathbf{J}^{\#}\right)\mathbf{V} = \mathbf{0}$, as well as by the manipulator using $\mathbf{V}_{\Re^\perp} = \left(\mathbf{I} - \mathbf{J}^{\#}\mathbf{J}\right)\mathbf{V} = \mathbf{0}$. Otherwise, the components of the mobile platform velocity should be inspected. When a joint of leg i is failed the platform velocity after failure and the error in the platform velocity are examined employing the reduced Jacobian matrix of the leg. The possibility for full or partial recovery of the platform motion is investigated by checking $\mathbf{V}_{\Re^\perp} = \left(\mathbf{I} - {}^i\mathbf{J}_f\,{}^i\mathbf{J}_f^{\#}\right)\mathbf{V}$. Once the correctional velocity from the remaining joints of the leg, $\Delta{}^i\dot{\mathbf{q}}_{corr} = {}^i\mathbf{J}_f^{\#}\,{}^i\mathbf{J}({}^i\dot{\mathbf{q}} - {}^i\dot{\mathbf{q}}_f)$, is calculated the joint velocity vector of leg i is updated using ${}^i\dot{\mathbf{q}}_f + \Delta{}^i\dot{\mathbf{q}}_{corr}$ while monitoring the velocity limits of joints. For the partial recovery case, the deviation in the retrieved motion of platform is identified employing $\Delta\mathbf{V}_r = \left(\mathbf{I} - {}^i\mathbf{J}_f\,{}^i\mathbf{J}_f^{\#}\right){}^i\mathbf{J}({}^i\dot{\mathbf{q}} - {}^i\dot{\mathbf{q}}_f)$.

4 Case Study

To model the manipulators, the attachment points of the legs to the base and to the mobile platform are, respectively, labeled as A_i and B_i. A fixed reference frame Ψ(X,Y,Z) is assigned to the base, with origin at point 0, and a moving reference frame Γ(X′,Y′,Z′) is attached to the centre of mass, point P, of the mobile platform. The position vector of point P in the base frame is $\mathbf{p} = [p_x \;\; p_y \;\; p_z]^T$. The orientation of the mobile platform with respect to the base frame Ψ(X,Y,Z) is given by Euler angles. For planar manipulators, the mobile platform orientation is represented by angle φ, Figure 3.

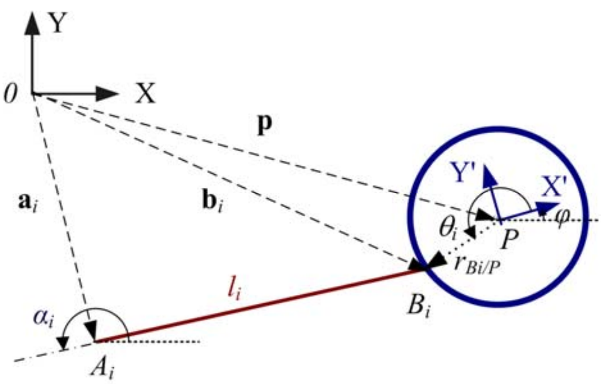

Figure 3. Parameters of planar parallel manipulators.

The methodologies presented in the previous section are valid for both planar and spatial parallel manipulators. In this section, planar parallel manipulators are used as the case study. The kinematics of planar manipulators is presented first, then the effect of joint failure is discussed and some simulation results are reported.

4.1 Kinematic Analysis

For planar manipulators, the position vector of the base attachment point of leg i, point A_i, in the fixed frame Ψ(X,Y) is $\mathbf{a}_i = [a_{ix} \;\; a_{iy}]^T$, for i = 1, …, n_l, and the position vector of point B_i in Γ(X′,Y′) is ${}^\Gamma\mathbf{b}_i = [r_{Bi/P}\cos\theta_i \;\; r_{Bi/P}\sin\theta_i]^T$ where $r_{Bi/P}$ is the length of the line segment $\overline{PB_i}$, refer to Figure 3. The orientation of lines $\overline{PB_i}$ (angular position of points B_i on the platform) with respect to the mobile platform frame Γ(X′,Y′) are represented by angles θ_i. The position of B_i relative to the base frame, $\mathbf{b}_i = [b_{ix} \;\; b_{iy}]^T$, is calculated using the homogeneous transformation matrix relating Γ(X′,Y′) to Ψ(X,Y)

$$\mathbf{A}_{\Psi,\Gamma} = \begin{bmatrix} \cos\varphi & -\sin\varphi & p_x \\ \sin\varphi & \cos\varphi & p_y \\ 0 & 0 & 1 \end{bmatrix}. \tag{17}$$

which is in terms of the mobile platform position $\mathbf{p} = [p_x \quad p_y]^T$ and orientation φ.

For non-redundant 3 DOF planar manipulators ($m = 3$; two translations on the plane and a rotation about an axis normal to the plane), each leg should have 3 DOF ($l = 3$), e.g., include three revolute joints, or two revolute joints and one prismatic joint. For symmetric actuation, three legs ($n_l = 3$), each with one actuated joint, are required ($n = 3$). Without loss of generality, the legs of the manipulator are considered to have identical joint layouts, such as revolute-prismatic-revolute (RPR) or RRR arrangements.

With the RPR arrangement, when the prismatic joints are actuated the layout is represented as $\mathrm{R\underline{P}R}$, e.g., the manipulator of Figure 1 with $n_l = n = 3$. The velocity of active joints, ${}^i\dot{\mathbf{l}} = [\dot{l}_1 \; \dot{l}_2 \; \ldots \; \dot{l}_{n-1} \; \dot{l}_n]^T$, is related to the platform velocity, $\mathbf{V} = \begin{bmatrix} v_x & v_y & \dot{\varphi} \end{bmatrix}^T$, with the $n \times 3$ Jacobian matrix $\mathbf{J}$

$$\dot{\mathbf{l}} = -\mathbf{J}\mathbf{V} \tag{18}$$

$$\mathbf{J} = \begin{bmatrix} \cos\alpha_1 & \sin\alpha_1 & v_1 \\ \vdots & \vdots & \vdots \\ \cos\alpha_n & \sin\alpha_n & v_n \end{bmatrix} \tag{19}$$

where the direction cosines corresponding to the axis of active prismatic joint of leg i are calculated as $\cos\alpha_i = l_{ix} / l_i$ and $\sin\alpha_i = l_{iy} / l_i$, with l_i representing the joint displacement. The moment of prismatic joint axis, v_i, with respect to the origin of $\Gamma(X',Y')$, point P, formulated in $\Psi(X,Y)$ is

$$v_i = -\cos\alpha_i \left(b_{iy} - p_y\right) + \sin\alpha_i \left(b_{ix} - p_x\right). \tag{20}$$

Considering the velocity relation $\dot{\mathbf{l}} = -\mathbf{J}\mathbf{V}$ and the $n \times m$ Jacobian matrix $\mathbf{J}$, where $n \geq m$, the velocity vector of active prismatic joints is physically consistent (all entries have the same dimension of length/time). Hence, even though for the general motion the entries of the mobile platform velocity vector are not unit consistent (include rotational and translational velocities), as long as $\mathbf{J}$ has full column-rank there is no need for a weighting metric on the task space velocity. Therefore, matrix $\mathbf{J}^{\#}$ is the un-weighted generalized inverse of matrix $\mathbf{J}$ and is calculated as

$$\mathbf{J}^{\#} = \left(\mathbf{J}^T\mathbf{J}\right)^{-1}\mathbf{J}^T \tag{21}$$

To form kinematically redundant legs, an active prismatic joint is added to each leg between the base and the first revolute joint; the $\mathrm{\underline{P}R\underline{P}R}$ layout of Figures 2(a) and 2(b). When the axis of the first prismatic joint is in the X direction, the pose of mobile platform in terms of the joint displacements of leg i, ${}^i\mathbf{q} = [d_i \quad \alpha_i \quad l_i \quad \beta_i]^T$, is

$$\begin{bmatrix} p_x \\ p_x \\ \varphi \end{bmatrix} = \begin{bmatrix} a_{ix} + d_i - l_i c\alpha_i - r_{Bi/P}c(\alpha_i + \beta_i) \\ a_{iy} - l_i s\alpha_i - r_{Bi/P}s(\alpha_i + \beta_i) \\ \alpha_i + \beta_i - \theta_i \end{bmatrix} \tag{22}$$

for $i = 1, \ldots, n_l$, where $c\alpha_i = \cos\alpha_i$, $s\alpha_i = \sin\alpha_i$, $c(\alpha_i + \beta_i) = \cos(\alpha_i + \beta_i)$ and $s(\alpha_i + \beta_i) = \sin(\alpha_i + \beta_i)$.

The twist of mobile platform, $\mathbf{V}=\begin{bmatrix}v_x & v_y & \dot{\varphi}\end{bmatrix}^T$, is related to the joint velocity vector of leg *i*, ${}^i\dot{\mathbf{q}}$, through the 3×4 Jacobian matrix of the leg, ${}^i\mathbf{J}$, as

$$\begin{bmatrix} v_x \\ v_y \\ \dot{\varphi} \end{bmatrix} = \begin{bmatrix} 1 & l_i s\alpha_i + r_{Bi/P}\, s(\alpha_i+\beta_i) & -c\alpha_i & r_{Bi/P}\, s(\alpha_i+\beta_i) \\ 0 & -l_i c\alpha_i - r_{Bi/P} c(\alpha_i+\beta_i) & -s\alpha_i & -r_{Bi/P} c(\alpha_i+\beta_i) \\ 0 & 1 & 0 & 1 \end{bmatrix} \begin{bmatrix} \dot{d}_i \\ \dot{\alpha}_i \\ \dot{l}_i \\ \dot{\beta}_i \end{bmatrix} = {}^i\mathbf{J}\,{}^i\dot{\mathbf{q}} \quad (23)$$

Matrix ${}^i\mathbf{J}$ is not square, and hence, the generalized inverse is used to solve for the joint velocities ${}^i\dot{\mathbf{q}}$

$${}^i\dot{\mathbf{q}} = {}^i\mathbf{J}^{\#}\,\mathbf{V} + \left(\mathbf{I} - {}^i\mathbf{J}^{\#}\,{}^i\mathbf{J}\right)\boldsymbol{\lambda} \quad (24)$$

where the first term on the right-hand side of equation (24), ${}^i\mathbf{J}^{\#}\,\mathbf{V}$**,** is the minimum norm or particular solution and the second term is the homogenous solution in which $\mathbf{I}-{}^i\mathbf{J}^{\#}\,{}^i\mathbf{J}$ projects the arbitrary 4×1 vector $\boldsymbol{\lambda}$ to the null space of ${}^i\mathbf{J}$.

4.2 Recovering Lost Motion

The parallel manipulator of Figure 2(a), with kinematically redundant legs, is used for the failure recovery simulation; where the task space dimension is m = 3 and the number of legs is n_l = 3, with l = 4 joints per leg and a total of n = 6 actuated joints. The coordinates of the base attachment points A_i, i = 1, …, 3, in the base frame are respectively (–2, –1.5), (2, –1.5) and (0, 1.5). The position of connection points B_i on the platform, ${}^{\Gamma}\mathbf{b}_i$, is set at a constant radius of $r_{Bi/P}$ = 0.25 meters. The angular coordinates, θ_i, i = 1, …, 3, of the leg connections to the mobile platform are respectively –150°, –30° and 90°.

For the three-leg manipulator of Figure 2(a), when the mobile platform pose is $\mathbf{p} = [0 \quad -1.5]^T$ meters and φ = –30° the joint displacements of leg 1 are ${}^i\mathbf{q} = [0 \quad -180 \quad 1.750 \quad 0]^T$, i.e., leg 1 is in the X direction. Then, the Jacobian matrix of leg 1 will be

$${}^1\mathbf{J} = \begin{bmatrix} 1.0 & 0.0 & 1.0 & 0.0 \\ 0 & 2.0 & 0.0 & 0.250 \\ 0 & 1.0 & 0 & 1.0 \end{bmatrix} \quad (25)$$

For the platform twist of $\mathbf{V} = [1 \quad 1 \quad 0]^T$, i.e., for a linear velocity of $[1 \quad 1]^T$ m/s and zero rotational velocity about the Z direction, using $\mathbf{V}={}^1\mathbf{J}\;{}^1\dot{\mathbf{q}}$, the minimum norm vector of joint velocity is

$${}^1\dot{\mathbf{q}} = [0.500 \quad 0.571 \quad 0.500 \quad -0.571]^T \quad (26)$$

When the first active joint (h = 1) of leg 1 (i = 1) is jammed, i.e., has zero velocity, there remain three joints (one active prismatic and two passive revolute joints) for a 3 DOF task

$${}^1\dot{\mathbf{q}}_f = [0 \quad 0.571 \quad 0.500 \quad -0.571]^T \quad (27)$$

and the twist of mobile platform is calculated using $\mathbf{V}_f = {}^1\mathbf{J}\;{}^1\dot{\mathbf{q}}_f$ as

$$\mathbf{V}_f = [0.500 \quad 1.000 \quad 0.0]^T \quad (28)$$

The projection of platform twist $\mathbf{V}$ onto the range space of $\mathbf{I}-{}^1\mathbf{J}_f\,{}^1\mathbf{J}_f^{\#}$ is a zero vector, which indicates that the failure of the first active joint of leg 1 could be fully recovered by the remaining joints provided the correctional velocity does not result in a joint velocity exceeding the limit.

To fully recover from the failure of joint 1, the joint velocities are adjusted such that $\dot{d}_1 = 0.500$ m/s, which results in the minimum norm solution for ${}^1\dot{\mathbf{q}}$, is set to zero. Using an identity matrix as the weighting matrix, the correctional velocity to be provided by joints 2, 3 and 4, should be

$$\Delta^i \dot{\mathbf{q}}_{corr} = {}^1\mathbf{J}_f^{\#}\ {}^1\mathbf{J}({}^1\dot{\mathbf{q}} - {}^1\dot{\mathbf{q}}_f) = {}^1\mathbf{J}_f^{\#}\ {}^1\mathbf{J}_1\ \dot{d}_1 = [0\quad -0.000\quad 0.500\quad 0.000]^T \tag{29}$$

That is, in this configuration of leg 1, because the two prismatic joints are collinear, the motion of the failed first prismatic joint is fully recovered by the second prismatic joint. Then, the overall joint velocities will be

$$^1\dot{\mathbf{q}}_f + \Delta^1\dot{\mathbf{q}}_{corr} = [0\quad 0.571\quad 1.000\quad -0.571]^T \tag{30}$$

which results in $\Delta\mathbf{V}_r = \left(\mathbf{I} - {}^1\mathbf{J}_f\ {}^1\mathbf{J}_f^{\#}\right){}^1\mathbf{J}({}^1\dot{\mathbf{q}} - {}^1\dot{\mathbf{q}}_f) = \mathbf{0}$, and hence, produces the original platform twist of

$$\mathbf{V} = {}^1\mathbf{J}({}^1\dot{\mathbf{q}}_f + {}^1\dot{\mathbf{q}}_{corr}) = [1.000\quad 1.000\quad 0.000]^T \tag{31}$$

If the velocity of the remaining joints could not be changed, the error in the platform twist would be

$$\Delta\mathbf{V}_f = {}^1\mathbf{J}_f\ {}^1\mathbf{J}_f^{\#}\ {}^1\mathbf{J}_1\ \dot{d}_1 = [0.500\quad 0.000\quad 0.000]^T \tag{32}$$

For this manipulator, different cases of joint failures were investigated. Four cases are reported in Table 1, which correspond to the mobile platform pose of $[p_x\quad p_y\quad \phi]^T = [1\quad 0.5\quad -30^\circ]^T$ and the joint displacements of $^1\mathbf{q} = [d_1\quad \alpha_1\quad l_1\quad \beta_1]^T = [0\quad -143.973^\circ\quad 3.400\quad -36.027^\circ]^T$ (length parameters are in meters). Two platform twists and the corresponding minimum norm joint velocities are reported. In Cases 1 and 3, following the failure of the corresponding prismatic joint, the required twist of the platform is fully recovered by properly adjusting the velocity of the remaining three joints while keeping the 2-norm of the vector of joint velocities as low as possible. In case 2, after the failure of both passive revolute joints, the two active prismatic joints cannot recover the lost angular velocity of the mobile platform. This is also evidenced from $\mathbf{V}_{\Re^\perp} = \left(\mathbf{I} - {}^1\mathbf{J}_f\ {}^1\mathbf{J}_f^{\#}\right)\mathbf{V} \neq \mathbf{0}$ which has a nonzero value for its third component corresponding to the angular velocity of platform. In case 4, after the failure of the first two joints ($\dot{d}_{c1} = 0$ and $\dot{\alpha}_{c1} = 0$), no component of the mobile platform twist could be fully recovered by the last two joints.

5 Conclusions

In this article, failure analysis of parallel manipulators was considered in view of their failure modes. The effect of joint failures on the motion performance of parallel manipulators was investigated. A methodology was presented for recovering the lost motion of the mobile platform due to zero or different/limited joint velocity. The proposed procedure is based on the projection of the lost joint motion onto the orthogonal complement of the null space of the reduced Jacobian matrix of the corresponding leg, in order to calculate the correctional velocity to be provided by the healthy joints of the leg.

Criteria were established for examining if the lost platform motion could be fully or partially recovered. It was presented that when the result of projecting the required mobile platform twist onto the orthogonal complement of the range space of the reduced Jacobian matrix of the leg with failed joint is a null vector the lost twist would be entirely recovered. When the resultant vector has a zero entry then the corresponding entry of the linear or angular velocity vector could be fully recovered. In these cases, the overall velocity of healthy joints should not exceed the joint limits. The criteria for assessing whether the lost platform twist will be retrieved could also be utilized to examine if the mobile platform would have the required twist in general, as well as while a leg is at singularity and has lost one or more DOF.

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Table 1 Example joint failures for leg 1 of parallel manipulator.

$\mathbf{V}=[1\quad 2\quad 0.873]^T$ $\quad {}^1\dot{\mathbf{q}}=[0.914\quad 0.409\quad 1.118\quad 0.464]^T$ (m/s and rad/s),		
	Full Recovery	**Partial Recovery**
${}^1\dot{q}_{ch}$	**Case 1:** $\dot{q}_{c1}=\dot{d}_{c1}=5$	**Case 2:** $\dot{q}_{c2}=\dot{\alpha}_{c1}=0$, $\dot{q}_{c4}=\dot{\beta}_{c1}=0$
$\mathbf{V}_{\Re^\perp}=\left(\mathbf{I}-{}^1\mathbf{J}_f\,{}^1\mathbf{J}_f^{\#}\right)\mathbf{V}$	$\mathbf{V}_{\Re^\perp}=[0\quad 0\quad 0]^T$	$\mathbf{V}_{\Re^\perp}=[0\quad 0\quad 0.873]^T$
$\mathbf{V}_f={}^1\mathbf{J}\,{}^1\dot{\mathbf{q}}_f$	$[5.086\quad 2.000\quad 0.873]^T$	$[1.818\quad 0.657\quad 0]^T$
$\Delta^i\dot{\mathbf{q}}_{corr}={}^1\mathbf{J}_f^{\#}\,{}^1\mathbf{J}({}^1\dot{\mathbf{q}}-{}^1\dot{\mathbf{q}}_f)$	$[0\quad 0.707\quad -3.305\quad -0.707]^T$	$[-2.664\quad 0\quad 2.282\quad 0]^T$
$\Delta\mathbf{V}_r=\left(\mathbf{I}-{}^1\mathbf{J}_f\,{}^1\mathbf{J}_f^{\#}\right){}^1\mathbf{J}({}^1\dot{\mathbf{q}}-{}^1\dot{\mathbf{q}}_f)$	$\Delta\mathbf{V}_r=[0\quad 0\quad 0]^T$	$\Delta\mathbf{V}_r=[0\quad 0\quad 0.873]^T$
$\mathbf{V}=[1\quad 0\quad 0.873]^T$ $\quad {}^1\dot{\mathbf{q}}=[0.455\quad -0.146\quad 0.313\quad 1.019]^T$ (m/s and rad/s)		
	Full Recovery	**Partial Recovery**
${}^1\dot{q}_{ch}$	**Case 3:** $\dot{q}_{c3}=\dot{l}_{c1}=0$	**Case 4:** $\dot{q}_{c1}=\dot{d}_{c1}=0$, $\dot{q}_{c2}=\dot{\alpha}_{c1}=0$
$\mathbf{V}_{\Re^\perp}=\left(\mathbf{I}-{}^1\mathbf{J}_f\,{}^1\mathbf{J}_f^{\#}\right)\mathbf{V}$	$\mathbf{V}_{\Re^\perp}=[0\quad 0\quad 0]^T$	$\mathbf{V}_{\Re^\perp}=[0.432\quad -0.594\quad 0.148]^T$
$\mathbf{V}_f={}^1\mathbf{J}\,{}^1\dot{\mathbf{q}}_f$	$[0.747\quad -0.184\quad 0.873]^T$	$[0.653\quad 0.439\quad 1.019]^T$
$\Delta^i\dot{\mathbf{q}}_{corr}={}^1\mathbf{J}_f^{\#}\,{}^1\mathbf{J}({}^1\dot{\mathbf{q}}-{}^1\dot{\mathbf{q}}_f)$	$[0.387\quad 0.067\quad 0\quad -0.067]^T$	$[0\quad 0\quad 0.059\quad -0.249]^T$
$\Delta\mathbf{V}_r=\left(\mathbf{I}-{}^1\mathbf{J}_f\,{}^1\mathbf{J}_f^{\#}\right){}^1\mathbf{J}({}^1\dot{\mathbf{q}}-{}^1\dot{\mathbf{q}}_f)$	$\Delta\mathbf{V}_r=[0\quad 0\quad 0]^T$	$\Delta\mathbf{V}_r=[0.299\quad -0.411\quad 0.103]^T$

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