COMPARISON OF REDUNDANCY RESOLUTION METHODS FOR A CABLE-SUSPENDED SPATIAL PARALLEL ROBOT

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ABSTRACT

In this paper a comparison of selected algorithms for the redundancy resolution of a cable-suspended parallel robot is presented. All m degree-of-freedom (DOF) cable robots need at least $m+1$ cables to be fully constrained because the cables can only apply forces in a single direction. In practice, many cable robots actually possess a degree of redundancy greater than one to achieve satisfactory properties. This intrinsic abundance of control inputs for a desired task such as lifting a defined payload dictates that an underdetermined system of equations must be solved to decide the optimal cable tensions. Based on the common norm-minimization algorithms introduced in the literature, this paper proposes to analyze the impact of the selected optimization on the size of the achievable workspace for a specific robot. Furthermore, the variation of the cable tensions throughout this achievable workspace will be quantified. First, a broad introduction on different aspects of the state-of-the-art for cable robots pertaining to this issue will be discussed. Then, a general kinematic model will be presented allowing to define the wrench matrix of the robot. The different types of wrench spaces that one typically define for these robots will then be discussed as well as the typical techniques to deal with redundancy. Finally, a numerical example including different methods will be presented and results discussed.

Keywords: cable robot; redundancy; norms.

COMPARAISON DE MÉTHODES DE RÉSOLUTION DE LA REDONDANCE D'UN ROBOT PARALLÈLE SPATIAL SUSPENDU À CÂBLES

RÉSUMÉ

Cet article présente une comparaison de divers algorithmes de résolution de la redondance d'un robot parallèle à câble de type suspendu. Tout robot à câbles à m degrés de liberté nécessite au minimum $m+1$ câbles pour être pleinement contraint car les câbles ne peuvent appliquer de forces que dans une seule direction. Le degré de cette redondance intrinsèque est souvent plus grand que un en pratique pour des raisons de performance. Cette surabondance de paramètres de contrôle pour la tâche à accomplir comme soulever une charge requiert la résolution d'un système d'équations sous-déterminé. En utilisant les techniques classiques de la littérature, cet article propose d'analyser l'impact des paramètres de l'algorithme choisi sur la taille de l'espace de travail que le robot peut réellement atteindre, ainsi que les tensions des câbles qui sont nécessaires. Dans un premier temps, une introduction générale est proposée soulignant les résultats récents en lien avec cette problématique. Un modèle cinématique est ensuite proposé. Celui-ci permet de définir les différents espaces de torseurs en lien avec les robots à câbles ainsi que les techniques classiques de résolution de la redondance. Finalement, un exemple sera présenté et des résultats numériques discutés.

Mots-clés : robot à cables ; redondance ; normes.

1. INTRODUCTION

Cable-driven parallel robots (cable robots for short), introduced in the scientific literature around 30 years ago [1], are machines in which the end-effector is a mobile platform connected to the ground by a series of cables of variable lengths. Adjusting these lengths allows for controlling the pose of this platform inside the workspace of the robot. Relying on cables instead of more traditional machine elements such as rigid links and joints to transmit power from the actuators to the end-effector is an attractive alternative for a parallel robot designer because the resulting design is very light and can have a huge workspace. Both aspects are typically unusual with parallel robots, at least conjointly. However, cable robots also have limitations. First of which, since cables can only pull and not push on an object, a certain degree of redundancy is required to control the m degrees of freedom (DOF) of the end-effector. It can be shown that at least $m+1$ unidirectional inputs are required to fully control a m-DOF system but it should be noted that gravity can act as one of such input. This specificity of cable robots is closely related to the issue of grasping or manipulation with robotic hands where fingers can only push on the seized objects and similar issues arise [2]. Thus, if one needs to fully control or fully constrain the pose of a spatial end-effector with six DOF, at least seven cables are required. However, looking at actual spatial cable robots, the most common architectures seem to have not seven but eight cables. A typical reason for this choice is to improve the performance in terms of achievable workspace. Indeed, real cables have minimal and maximal tension limits which must be taken into account to avoid losing control of the end-effector. This force requirement becomes easier to satisfy with more than seven cables. This is also why typical m-DOF cable-suspended robots which could in theory have only m cables usually have more. Nevertheless, multiplying the number of cables in a cable robot also brings some issues. Besides the added cost and complexity, the most pressing issue is the potential interference between the cables. Solutions to increase the number of cables without adding actuators exist, e.g. rely on differential mechanisms to couple the forces and motions of different cables as proposed in [3–5] or to achieve a specific complex trajectory [6]. However, these techniques do not solve the issue of interfering cables. One possible solution proposed in the literature is to reconfigure the robot, namely for the end-effector of the robot to travel from one part of its workspace to the other, one must detach some its anchor points on the ground and manually relocate them somewhere else on the supporting structure, see [7] for an example. Another related technique recently introduced is to attach the actuators of the cable robot onto mobile robots. These mobile robots, taking form as rovers or drones, can automatically reconfigure the cables to avoid interferences and improve selected performance indexes [8, 9]. The actuation redundancy of cable robots imply that, to balance a defined external wrench acting on the end-effector, there is an infinite number of cable tensions that are possible. Solving this redundancy becomes then a critical issue for a user to be able to exploit the full workspace of the robot, as will be shown in this paper.

As can be plainly seen, the design of a cable robot is a complex challenge with many conflicting issues. To alleviate the burden for potential designers, software assisted design solutions have been released, e.g. [10], but in most cases one has to rely on general numerical tools. Furthermore, the previously mentioned issues are only but a few of the important aspects and depending on the application at hand other characteristics may be of critical importance such as stiffness [11–15], vibration [16], accuracy [17, 18], or trajectory planning [19, 20].

2. MODELING

2.1. Kinematics

The kinematic of a general spatial cable robot is illustrated in Fig. 1. A set of n cables of lengths l_i ($i = 1, \ldots, n$) connect points G_i on the ground where the actuated winches or routing pulleys are located to points M_i on the mobile platform. Please note that from now on n is the number of cables of the robot and

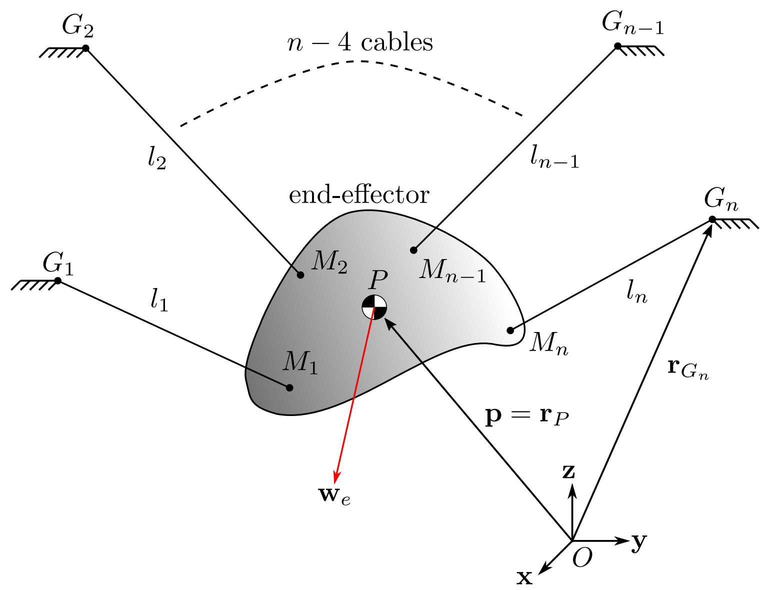


Fig. 1. Spatial cable robot kinematic model.

not its number of DOF as in the abstract and introduction. Since a spatial cable robot is modeled here, the number of DOF is six. The pose of the mobile platform is defined by the position vector $\mathbf{p}$ and an orientation described by rotation matrix $\mathbf{Q}$. This position vector $\mathbf{p}$ is defined going from a fixed point O to which the inertial frame is attached to the reference point P on the mobile platform. The positions of each of the other points of the robot with respect to O are defined by vectors $\mathbf{r}_{G_i}$ from O to G_i ($i = 1, \ldots, n$), see Fig. 1 for an example. An external wrench $\mathbf{w}_e$ is applied to the mobile platform and without any loss of generality, the application point of this external wrench is assumed to be P. This wrench $\mathbf{w}_e$ is the sum in P of the effects of all externally applied wrenches at any points of the platform.

The relationship between the input winding velocities and output end-effector twist is defined by:

$$\dot{\mathbf{l}} = \mathbf{J}\dot{\mathbf{t}} \tag{1}$$

where $\dot{\mathbf{l}} = [\dot{l}_1 \ \ldots \ \dot{l}_n]^T$ is the time derivative vector of the cable lengths, $\dot{\mathbf{t}} = [\dot{\mathbf{p}}^T \ \omega^T]^T$ is the output velocity vector (twist) combining the linear ($\dot{\mathbf{p}}$) and angular (ω) velocities of the end-effector at point P. The matrix $\mathbf{J}$ is the Jacobian of the cable robot and its expression is rather simple, namely:

$$\mathbf{J}^T = \begin{bmatrix} \mathbf{e}_1 & \ldots & \mathbf{e}_n \\ \mathbf{r}_{M_1} \times \mathbf{e}_1 & \ldots & \mathbf{r}_{M_n} \times \mathbf{e}_n \end{bmatrix} \tag{2}$$

for $i = 1, \ldots, n$ and where:

$$\mathbf{e}_i = \frac{\mathbf{r}_{G_i} - \mathbf{r}_{M_i}}{||\mathbf{r}_{G_i} - \mathbf{r}_{M_i}||} \tag{3}$$

is a unit vector along the i^{th} cable while $\mathbf{r}_{M_i}$ is the position of point M_i, i.e.:

$$\mathbf{r}_{M_i} = \mathbf{p} + \mathbf{Q}\mathbf{r}^0_{M_1}. \tag{4}$$

with $\mathbf{r}^0_{M_i}$ the same vector in the reference pose where $\mathbf{Q}$ is the identity matrix and $\mathbf{p} = \mathbf{0}$.

Considering the principle of virtual work, one can then express the relationship between the cable tensions and the balanced external wrench, assuming a static equilibrium, as:

$$\mathbf{W}\mathbf{f} = -\mathbf{w}_e \quad \Rightarrow \quad \mathbf{J}^T\mathbf{f} = -\mathbf{w}_e \tag{5}$$

where $\mathbf{f} = [f_1 \ \dots \ f_n]^T$ is the vector of the individual forces f_i exerted by each cable onto the platform.

The inverse kinematic problem, namely establishing the cable lengths for a known pose of the mobile platform defined by both $\mathbf{p}$ and $\mathbf{Q}$ is straightforward, i.e.:

$$l_i = ||\mathbf{r}_{M_i} - \mathbf{r}_{G_i}|| = ||\mathbf{p} + \mathbf{Q}\mathbf{r}^0_{M_1} - \mathbf{r}_{G_i}|| \quad \forall i. \tag{6}$$

On the other hand, the direct kinematic problem is more complicated to solve but luckily, not often required.

2.2. Wrench Spaces

Once the kinematic model of the cable robot has been established, one can have a closer look to the wrenches this robot is capable of producing. To this aim and acknowledging that applied forces on the mobile platform cannot switch sign, a first property was defined in the literature as wrench closure [21]. The wrench-closure workspace (WCW) of a cable robot is the part of the $SE(3)$ displacement group where this robot can generate any arbitrary wrench if sufficient cable tensions can be generated, i.e.:

$$\mathscr{W}_C = \{(\mathbf{p},\mathbf{Q})\} \text{ such as } \forall \mathbf{w} \in \mathbb{R}^6,\ \exists \mathbf{f} \geq \mathbf{0} \mid \mathbf{w} = \mathbf{W}(\mathbf{p},\mathbf{Q})\mathbf{f} \tag{7}$$

where $\mathbf{f} \geq \mathbf{0}$ means that all components of $\mathbf{f}$ must be non-negative. Wrench-closure for cable robots has many connections to traditional machine design [22] and force-closure, as introduced by Reuleaux.

The concept of wrench-closure workspace was subsequently extended to encompass more stringent limits on the cable tensions by defining the wrench-feasible workspace (WFW). The latter assumes that a required wrench set $\mathscr{W}_R$ has been defined as the specific wrenches that the robot must be able to create. The wrench-feasible workspace is then:

$$\mathscr{W}_F = \{(\mathbf{p},\mathbf{Q})\} \text{ such as } \forall \mathbf{w} \in \mathscr{W}_R,\ \exists \mathbf{f} \mid \mathbf{f}_{min} \leq \mathbf{f} \leq \mathbf{f}_{max} \text{ and } \mathbf{w} = \mathbf{W}(\mathbf{p},\mathbf{Q})\mathbf{f} \tag{8}$$

where similarly to the definition of the wrench-closed workspace, $\mathbf{f}_{min} \leq \mathbf{f} \leq \mathbf{f}_{max}$ means that all components of $\mathbf{f}$ are within a range defined by a minimal and maximal value. The definition of the wrench-feasible workspace was introduced in [23] and it was subsequently shown in [24] that the volume of the wrench space satisfying the feasibility constraints (namely the minimal and maximal allowable tensions) is a mathematical element called a zonotope. As such, it possesses certain properties, e.g. it is a convex n-dimensional polyhedron with point symmetry, that allows for its efficient determination and also to check whether it contains $\mathscr{W}_R$ for the most common forms of the latter (a point or hypersphere for instance) [24].

Another type of workspace of interest for cable robot is the static workspace $\mathscr{W}_S$ (SW), namely the part of $SE(3)$ where the robot can be in static equilibrium considering that a certain mass of its end-effector (possibly combined to a payload, e.g. [25]) and practical limits on the cable tensions are met, i.e.:

$$\mathscr{W}_S = \{(\mathbf{p},\mathbf{Q})\} \text{ such as } \exists \mathbf{f} \mid \mathbf{f}_{min} \leq \mathbf{f} \leq \mathbf{f}_{max} \text{ and } m\mathbf{g} = \mathbf{W}(\mathbf{p},\mathbf{Q})\mathbf{f} \tag{9}$$

where m is the mass the robot must be able to lift (not to be confused with the number of DOF as in Section 1) and $\mathbf{g}$ is the gravitational wrench at P. The latter definition is of particular interest with cable-suspended robots. These cable robots rely on the gravitational pull to produce force and motion in the downward direction and generally, do not have cables attached below the platform. Since the magnitude of this downward pull cannot be changed, cable-suspended robots have an empty wrench-closure workspace. They are however quite interesting in practice since the cables are restricted to the upper part of the workspace allowing for a space free of cables under the mobile platform.

2.3. Redundancy Resolution

Irrespectively of the type of workspace considered, it has to be noticed that the matrix $\mathbf{W}(\mathbf{p},\mathbf{Q})$ in Eqs. 7-9 has 6 lines for a spatial robot and typically $n > 6$ columns. Therefore, solving these equations for a specific wrench $\mathbf{w}$ is an underdetermined problem. It is well known that these problems can have zero, one, or infinitely many solutions. The latter case being the most interesting since it corresponds to the useful part of the workspace of the cable robot. The easiest solution to deal with such an underdetermined system of equations is to compute the Moore-Penrose inverse of matrix $\mathbf{W}$ which yields the solution with a minimal 2-norm (Euclidean), i.e. the optimal cable tensions are:

$$\mathbf{f} = \mathbf{W}^I\mathbf{w} = \mathbf{W}^T\left(\mathbf{W}\mathbf{W}^T\right)^{-1}\mathbf{w} \tag{10}$$

for a given $\mathbf{w}$ and assuming that $\mathbf{W}$ is of rank 6. While this solution is very elegant and fast to compute, it has some limitations. Indeed, while solving the previous equation produces a solution it is does not take into account the repartition of the cable tensions and while the overall norm of vector $\mathbf{f}$ is minimized, nothing can be said for the individual cable tensions which means that significant unbalance between them may exist. It is also possible that the solution computed using the 2-norm has components above the threshold of the maximal cable tensions or below that of the minimal tensions. A solution to this issue is that rather than relying on the 2-norm as an optimality criterion, one can use other norms. Of particular interest is the infinity norm which is the maximum of the absolute values of the components of $\mathbf{f}$. Solving an underdetermined system of equations by minimizing the infinity norm is referred to as finding the minimal effort solution. An attractive specificity of the minimal effort solution is that algorithms exist in the literature to compute its value in a finite number of steps, most famously Cadzow's [26]. The application and characteristics of the infinity norm control of redundant robotic systems have been extensively investigated in [27–29] where continuity issues were pointed out as well as means to mitigate them.

A comparison between the infinity, Euclidean, and 4-norm algorithms for a specific trajectory of a planar cable robot with a redundancy of one was presented in [30] where a non-iterative formulation of the 4-norm solution was also proposed as a simpler and continuous alternative of the minimum effort solution. Noticeably, in [30], the p-norm that is minimized is not the one of the vector $\mathbf{f}$ directly but of $\mathbf{f}-\mathbf{f}_m$ where $\mathbf{f}_m$ is the vector of the middle values between the minimal and maximal allowable tensions. This change aimed at producing better balanced cable tensions and critically, avoiding these tensions to go below the required minimal values.

Finally, a general methodology to solve the force redundancy of a $n+2$ cable n-DOF cable robot was later presented in [31] in which all feasible solutions can be derived. Then, a particular solution referred to as the weighted barycenter was proposed as corresponding to the cable tensions producing the desired wrench and with the maximal distance from the allowable cable tension limits. The technique presented in that previous reference has obvious advantages: it allows to obtain all possible solutions to the problem and is also guaranteed to produce an answer within a limited number of iterations. The weighted barycenter solution also keeps the best possible safety margins from a forbidden cable tension. The only disadvantages of this technique are that the computations required by the algorithm are not trivial and it is limited to the case of a redundancy degree of 2.

3. COMPARISON

Taking as an example the suspended cable robot shown in Fig. 2, we propose to compare different strategies of solving the redundancy of the cable tensions for a fixed payload. This robot is of the cable-suspended type and its geometry corresponds to a smaller scale version of the CoGiRo system [32] with the same number of cables (eight) and a scaled down geometry. The geometrical parameters of the robot are listed in Tables 1 and 2 (in meters). A static workspace is considered here and the payload is the mass of the mobile platform approximated at a value of $m = 40$ kg, i.e.:

$$\mathscr{W}_R = \{m\mathbf{z}\} = \{40\mathbf{z}\}. \tag{11}$$

where $\mathbf{z}$ is the unit vector opposite to gravity. The cables have minimal and maximal tensions of 0 and 200 kg (all force units in the paper are in kg) respectively:

$$\begin{cases} \mathbf{t}_{min} = [0\ 0\ 0\ 0\ 0\ 0\ 0\ 0]^T \\ \mathbf{t}_{max} = [200\ 200\ 200\ 200\ 200\ 200\ 200\ 200]^T \end{cases} \tag{12}$$

and the Cartesian workspace (in meters) of the robot is defined by:

$$\mathscr{C} = \begin{cases} -4.6 < x < 4.6 \\ -1.8 < y < 1.8 \\ \ \ 0.0 < z < 2.5 \end{cases} \tag{13}$$

where $\mathbf{p} = [x\ y\ z]^T$ is the position of the robot mobile platform. The orientation of the mobile platform is assumed to be constant and no rotation is desired (the required rotation matrix $\mathbf{Q}$ is equal to the identity matrix for all positions). It is determined using the procedure presented in [24] that the static workspace $\mathscr{W}_S$ of this robot is 71.9% of $\mathscr{C}$ (discretized with 75 mm steps) for the required wrench space $\mathscr{W}_R$.

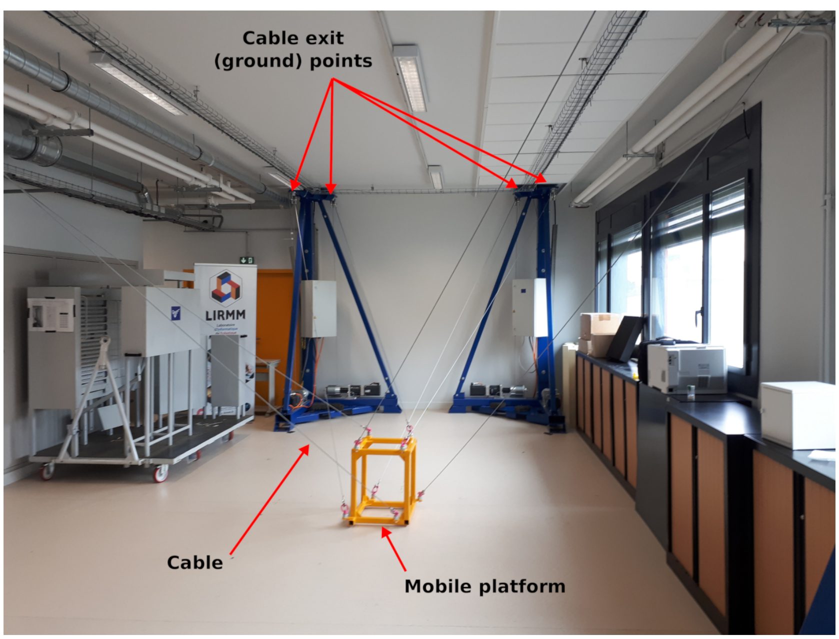


Fig. 2. The suspended cable robot considered in this work.

Point	G_1	G_2	G_3	G_4	G_5	G_6	G_7	G_8
x	-4.20	-4.60	-4.60	-4.20	4.20	4.60	4.60	4.20
y	-1.80	-1.35	1.35	1.80	1.80	1.35	-1.35	-1.80
z	3.00	3.00	3.00	3.00	3.00	3.00	3.00	3.00

Table 1. Coordinates of the anchor (cable exit) points on the ground (meter)

Point	M_1	M_2	M_3	M_4	M_5	M_6	M_7	M_8
x	0.25	-0.25	-0.25	0.25	-0.25	0.25	0.25	-0.25
y	-0.25	0.17	-0.13	0.17	0.25	-0.17	0.13	-0.17
z	0.00	0.50	0.00	0.50	0.00	0.50	0.00	0.50

Table 2. Coordinates of the connection points on the mobile platform (meter)

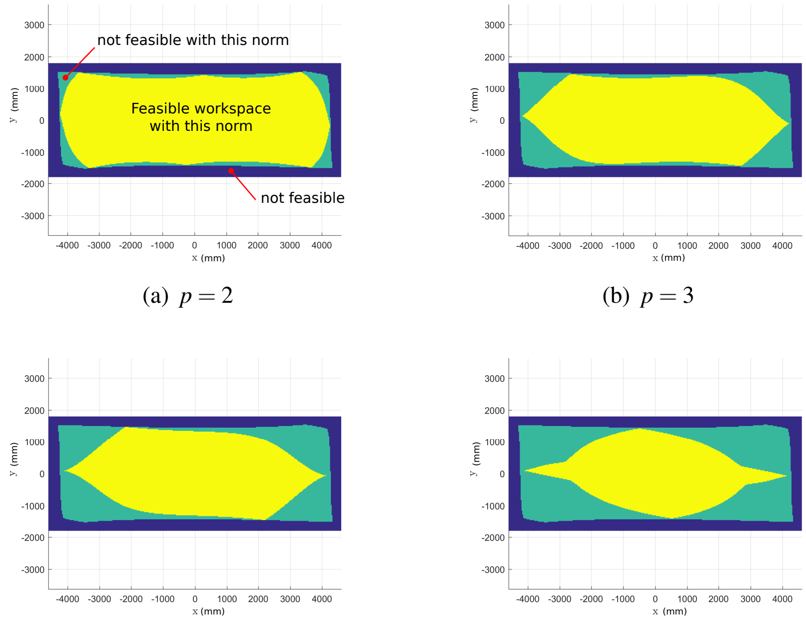
The percentages of $\mathscr{C}$ that are actually feasible by the robot depending of which unconstrained p-norm is used to compute the cable tensions balancing the required payload are listed in Table 3. These results were obtained numerically using the same discretization of $\mathscr{C}$ as before. The 2-norm solution is computed using Eq. 10 while Cadzow algorithm has been implemented to compute the one of minimal infinity-norm. All other p-norm solutions were obtained numerically using an interior-point algorithm and the results of Eq. 10 were used as initial estimates.

$\mathscr{W}_S$	2-norm	3-norm	4-norm	5-norm	6-norm	7-norm	8-norm	9-norm	10-norm	∞-norm
71.9%	60.3%	55.0%	51.6%	50.0%	48.6%	47.3%	46.4%	45.3%	44.8%	38.5%

Table 3. Feasible percentage of the Cartesian workspace using unconstrained p-norm solutions.

An illustration of the parts of the workspace yielding solutions within the tension limits, and the parts where no acceptable solutions exist, can be found in Figs. 3-5 where slices of the constant-orientation workspace (COW) are shown. The central yellow area correspond to a position where the pose of the robot is feasible with the selected norm, i.e. the resulting tensions are within the allowable range. The light blue area is the part of the static workspace where these constraints were violated by the solution found using the unconstrained p-norm. Since these areas are part of the static workspace as found by studying the wrench zonotopes of the robot, it is known that a feasible solution actually exists but the numerical unconstrained minimization of the considered p-norm yielded an unfeasible solution. The dark blue outer rim is the part of the workspace where no feasible solutions are possible. As can be clearly seen from these figures, the infinity norm is actually the worst choice for finding the cable tensions and only at the highest elevations (z close to 2.5 m) there are cases where this method finds a solution while the others fail. Despite its simplicity, the 2-norm solution performed actually quite well, only failing at the very edge of the static workspace.

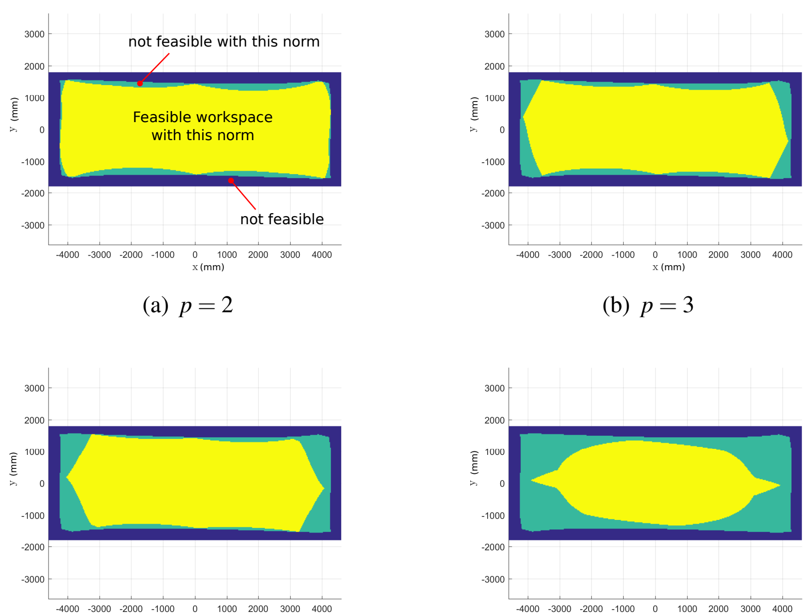
In Table 4, the results with the same procedure is presented but using a constrained optimization this time. The added constraint being that cable tensions must not go below $\mathbf{t}_{min}$, namely the zero vector in this example. Please note that since Cadzow's algorithm does not allow for adding constraint, a numerical procedure similarly to the other norms of lower order was used now for the infinity norm also. The same remark holds for the 2-norm. It is obvious from these results that the critical constraint that was violated in the previous simulations and prevented to use the full workspace was exclusively (within the numerical precision) the one corresponding to the permissible minimal tensions.



(a) $p = 2$ (b) $p = 3$

(c) $p = 4$ (d) $p = \infty$

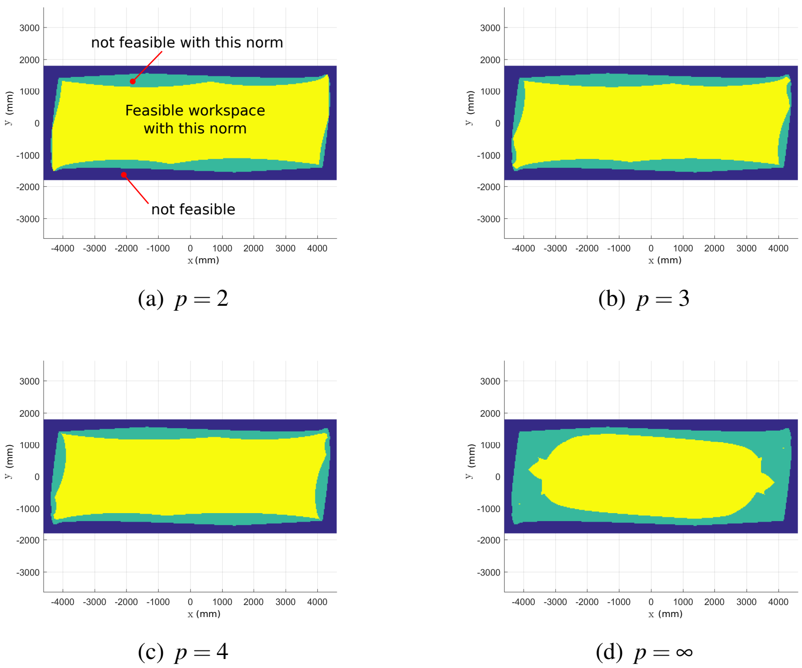
Fig. 3. Achievable zero rotation static workspaces using different unconstrained p-norm optimization algorithms at $z = 250$ mm showing the wrench-closure and wrench-feasible workspaces (WCW and WFW respectively.



(a) $p = 2$ (b) $p = 3$

(c) $p = 4$ (d) $p = \infty$

Fig. 4. Achievable zero rotation static workspaces using different unconstrained p-norm optimization algorithms at $z = 1250$ mm showing the wrench-closure and wrench-feasible workspaces (WCW and WFW respectively).



(a) $p = 2$ (b) $p = 3$

(c) $p = 4$ (d) $p = \infty$

Fig. 5. Achievable zero rotation static workspaces using different unconstrained p-norm optimization algorithms at $z = 2250$ mm showing the wrench-closure and wrench-feasible workspaces (WCW and WFW respectively.

$\mathscr{W}_S$	2-norm	3-norm	4-norm	5-norm	6-norm	7-norm	8-norm	9-norm	10-norm	∞-norm
71.9%	71.9%	71.9%	71.9%	71.9%	71.9%	71.9%	71.9%	71.9%	71.9%	71.9%

Table 4. Feasible percentage of the Cartesian workspace using sign-constrained p-norm solutions.

$\mathscr{W}_S$	2-norm	3-norm	4-norm	5-norm	6-norm	7-norm	8-norm	9-norm	10-norm	∞-norm
71.9%	47.8%	57.6%	60.7%	62.3%	63.1%	63.5%	64.1%	64.4%	64.8%	71.9%

Table 5. Feasible percentage of the Cartesian workspace using unconstrained p-norm minimization of relative cable tensions.

A similar methodology can be used to evaluate the technique introduced in [30], namely minimizing the p-norm of the cable tensions relatively to the middle values of the allowable range. The results are listed in Tables 5 and 6. From these results, it appears that this relative minimization technique is indeed generally beneficial with the notable exception of the Euclidean norm case. Additionally, the lower tension threshold is again the critical constrain to satisfy as evidenced by the numbers listed in Table 6. An interesting result to notice is that the infinity norm of the relative tensions achieves a perfect score even in its unconstrained version. This result has potential consequences since the infinity norm is the only one with the Euclidean norm for which an explicit minimization algorithm exists and a finite number of steps is guaranteed for an arbitrary degree of redundancy. However, the relative tension minimization cannot be transformed with a simple change of coordinates to a simple infinity norm minimization as solved with Cadzow's algorithm. The best approach to deal with the relative infinity norm minimization lacking an adaptation of an explicit

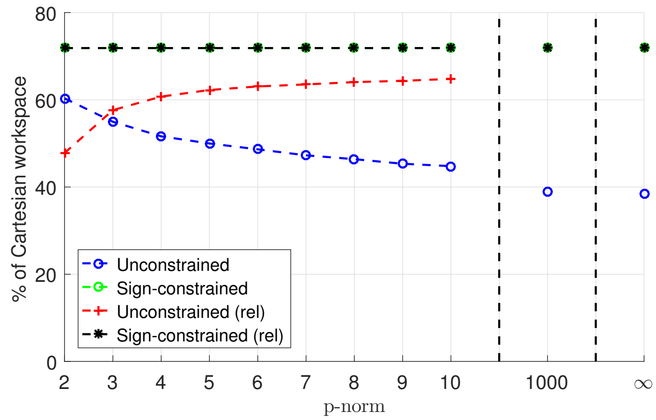


Fig. 6. Percentages of achievable Cartesian workspace, all methods.

$\mathscr{W}_S$	2-norm	3-norm	4-norm	5-norm	6-norm	7-norm	8-norm	9-norm	10-norm	∞-norm
71.9%	71.9%	71.9%	71.9%	71.9%	71.9%	71.9%	71.9%	71.9%	71.9%	71.9%

Table 6. Feasible percentage of the Cartesian workspace using sign-constrained p-norm minimization of relative cable tensions.

algorithm is its reduction to a linear program as detailed in [33] and later adapted to cable robots in [30].

A graphic summary of all the percentage presented in Tables 4-6 is given in Fig. 6 where the results with a higher 1000-norm are also shown. Finally, one can also appreciate in Table 7, the average infinity norm of the relative tension vector produced by the different minimization, i.e.:

$$\mu_i = \frac{\int_{\mathscr{W}_S^i} ||\mathbf{t} - \mathbf{t}_m||_\infty d\mathbf{p}}{\int_{\mathscr{W}_S^i} d\mathbf{p}} \tag{14}$$

where $\mathscr{W}_S^i$ is the achievable percentage of $\mathscr{C}$ as listed in the previous Tables and i is the order of the norm being used. This index is ideally zero which occurs when the cable tensions are always at their mid-values. As can be seen from the numerical results in Table 7, there are very little differences between the different techniques. It is conjectured to be due to the cable-suspended nature of the robot. While fully-constrained cable robot might be able to increase their cable tensions while maintaining a fixed pose, this is more difficult to achieve using a cable-suspended architecture. Indeed, there is intuitively little room for increasing the cable tensions without lifting up the payload for example. This is consistent with the results discussed in [31] for the original CoGiRo system and the numerical results presented in that reference for the selected trajectory.

Type	2-norm	3-norm	4-norm	5-norm	6-norm	7-norm	8-norm	9-norm	10-norm	∞-norm
Unconstrained	91.6	90.7	90.4	90.4	90.4	90.4	90.4	90.4	90.4	90.6
Sign-constrained	92.7	92.7	92.9	93.1	93.3	93.5	93.6	93.8	93.8	94.8
Unconstrained (rel.)	91.2	90.8	90.7	90.6	90.6	90.5	90.5	90.5	90.5	90.5
Sign-constrained (rel.)	94.1	92.4	91.9	91.7	91.5	91.4	91.4	91.3	91.3	90.5

Table 7. Force distribution index μ of the cable tensions depending on the selected minimization.

4. CONCLUSIONS

This paper presented a comparison of different techniques to solve the force redundancy of a cable-suspended robot. These techniques, all based on minimizing p-norms were measured in terms of the percentages of the Cartesian space they allow the robot to reach and how close they can approach the theoretical limit. As it appeared from the simulation results, all sign-constrained p-norm minimization with $p \geq 2$ were very effective. Unconstrained optimizations on the other hand fared not so well but noticeably, the unconstrained relative tension vector minimization technique introduced in [30] can achieve very decent results and was shown to obtain a perfect score if $p = \infty$. Considering its simplicity, the unconstrained Euclidean norm also appears as a very attractive alternative for practical implementation. Additionally, it was shown that the variations of the cable tensions are consistently of similar magnitudes. The limits of this work are that the presented results are only valid for the example given. On the other hand, a large number of different minimizations were analyzed over a whole COW. Quantifying the impact of this choice on the size of the reachable workspace is also a unique contribution of this paper to the best of the authors' knowledge.

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